Introduction to Maxima

Eleftherios Gkioulekas*

Department of Mathematics, University of Texas-Pan American , Edinburg, TX, United States

I. GETTING STARTED

Computer Algebra Systems are computer programs that can perform simple mathematical calculations. You can use them to solve equations, simplify or factor expressions, calculate limits, derivatives, integrals, Taylor expansions, and series sums. You can also divide polynomials and calculate partial fraction decompositions. The most well-known CAS programs are Mathematica and Maple. Both are very expensive commercial programs, although crippled student editions are available at lower prices. These programs may be needed if you are a research mathematician. For undergraduate level mathematics, there is a free program, Maxima, whose capabilities are sufficient. This tutorial will show you how to use Maxima to confirm your solutions to homework problems for courses ranging from Algebra to Calculus. To get the best benefit from this tutorial, it is recommended that you work through the examples below on your computer as you study this tutorial.

You can freely download and install Maxima to your computer from the following website:

`http://wxmaxima.sourceforge.net/`

The installation details depend on your choice of operating system. After installation, the best way to run Maxima is by running the graphical use interface wxMaxima. Maxima itself is a command-line driven program, so using it without a graphical user interface requires that you learn the needed commands and their syntax. It is also possible to write computer programs in Maxima for very advanced problems. However, at the undergraduate level, the wxMaxima interface is sufficient for your needs, and you can easily learn it very quickly.

It should be noted that while wxMaxima can help you with labor intensive calculations, you have to understand and supply the overall structure and strategy of the argument yourself. This is increasingly the case with the more conceptual types of problems. Another weakness of wxMaxima is that it cannot solve inequalities or complicated types of equations, especially certain types of trigonometric equations and inequalities. It is possible, however, after having solved such equations and inequalities by hand, to use wxMaxima to confirm the solutions found.

II. EVALUATING FUNCTIONS

wxMaxima can be used to define and evaluate a function $f(x)$ for various values of x. An obvious application of this capability, as we will explain later, is in confirming the solutions

*Electronic address: `gkioulekase@utpa.edu`

of equations and inequalities. Let us consider, for example, the function

$$f(x) = \frac{x^2+1}{x^2+2x+5} \tag{1}$$

and let's say that you want to calculate $f(2+3\sqrt{2})$. First, we define the function by entering the following commands:

```
f(x) := (x^2+1)/(x^2+2*x+5)
```
Menu: Edit/Cell/Evaluate Cell

Note that we use `*` for multiplication, `^` for raising to a power (exponentiation), and `/` for division. It is also necessary to use parenthesis to ensure that operations are done in the correct order. Finally, the notation `:=` represents a function definition. Now, to evaluate the function, we enter the commands:

```
f(2+3*sqrt(2))
```
Menu: Simplify/Simplify Expression

This gives the answer:

$$f(2+3\sqrt{2}) = \frac{3\cdot 2^{5/2}+23}{9\cdot 2^{3/2}+31} = \frac{12\sqrt{2}+23}{18\sqrt{2}+31} \tag{2}$$

We note that `sqrt` is a predefined function that evaluates the square root of its argument. Unfortunately, wxMaxima does not seem to know that it should rationalize the denominator in the above result. However, we can easily do this ourselves as follows:

```
(12*sqrt(2)+23)*(18*sqrt(2)-31)
```
Menu: Simplify/Simplify Expression
```
(18*sqrt(2)+31)*(18*sqrt(2)-31)
```
Menu: Simplify/Simplify Expression

Using the output from the above commands, we may now write the entire calculation as:

$$f(2+3\sqrt{2}) = \frac{12\sqrt{2}+23}{18\sqrt{2}+31} = \frac{(12\sqrt{2}+23)(18\sqrt{2}-31)}{(18\sqrt{2}+31)(18\sqrt{2}-31)} \tag{3}$$

$$= \frac{21\cdot 2^{3/2}-281}{-313} = \frac{281-42\sqrt{2}}{313} \tag{4}$$

This is the exact simplified answer.

If, for any reason, you need an approximate answer instead, you can find it using the following command sequence:

```
f(2+3*sqrt(2))
```
Menu: Numeric/To Float

This gives $f(2+3\sqrt{2}) \approx 0.707996690217358$. Unlike more sophisticated programs like Mathematica, it is not possible to increase the accuracy of this approximation to any arbitrary number of decimals. Fortunately, we usually don't care.

Note that in wxMaxima the names of functions do not have to be only one letter long. You can have, for example, definitions like:

```
func1(x) := 31*sqrt(x^2-3)
Menu: Edit/Cell/Evaluate Cell
```

Let us also mention that wxMaxima already has many predefined functions and expressions, such as `%pi, sin(x), cos(x), tan(x), cot(x), sinh(x), cosh(x), tanh(x)` corresponding to π, and the usual trigonometric and hyperbolic trigonometric functions. Also available are the inverse trigonometric and the inverse hyperbolic trigonometric functions: `asin(x), acos(x), atan(x), asinh(x), acosh(x), atanh(x)`. For example, to define the function

$$f(x) = \frac{\cos(3\pi x)}{\sqrt{\sin(\pi x)}} \tag{5}$$

we use the following command:

```
f(x) := cos(3*%pi*x)/sqrt(sin(%pi*x))
Menu: Edit/Cell/Evaluate Cell
```

Last, but not least, wxMaxima has the following predefined functions and variables for the Napier constant e, the natural exponential function $e**x$ and the natural logarithm $\ln x$: `%e, exp(x), log(x)`. For example, to define

$$f(x) = \frac{e^x\sqrt{\cos(ex)}}{\ln(e^{3x}+2)} \tag{6}$$

we use the command

```
f(x) := (exp(x)*sqrt(cos(%e*x)))/log(exp(3*x)+2)
Menu: Edit/Cell/Evaluate Cell
```

III. EVALUATING LIMITS

wxMaxima is very good at evaluating limits. For example, to evaluate the limit

$$L = \lim_{x\to 0} \frac{1-\cos x}{x} \tag{7}$$

we use the following command sequence:

```
(1-cos(x))/x
Menu: Calculus/Find Limit
```

wxMaxima will bring up a pop-up window where you get to indicate the name of the variable and where it is going. Use `+inf` or `-inf` for limits going to $+\infty$ or $-\infty$. A pull-down menu titled "Direction" allows you to specify whether your limit is a side limit. Your options are: "both sides" for $x \to a$, "left" for $x \to a^-$, "right" for $x \to a^+$.

Note that for limits that do not exist, wxMaxima may yield the response `infinity` or `ind`. For examples of this behaviour, try the following limits that do not exist:

$$L_1 = \lim_{x\to 0} \frac{1}{x} \qquad L_2 = \lim_{x\to +\infty} \sin(x) \tag{8}$$

For limits that *do* exist and go to infinity, wxMaxima will respond explicitly with $+\infty$ or $-\infty$.

Note that for limits involving absolute values we use the predefined function `abs`. For example to evaluate the limit

$$L = \lim_{x \to 0} \frac{x^2 + |x|}{x^2 - |x|} \tag{9}$$

we use the command sequence

```
(x^2+abs(x))/(x^2-abs(x))
Menu: Calculus/Find Limit
```

IV. DIFFERENTIAL CALCULUS

wxMaxima can differentiate and factor derivatives. For example, let us consider the function

$$f(x) = (x^2 + 3x + 2)^3 (x + 1)^2 \tag{10}$$

To find the first and second derivative we use the command sequence:

```
(x^2+3*x+2)^3*(x+1)^2
Menu: Calculus/Differentiate
Menu: Simplify/Factor Expression
Menu: Calculus/Differentiate
Menu: Simplify/Factor Expression
```

In the dialog box we are asked how many times to differentiate. Since we want both the first and second derivative, we differentiate one time only at both steps. If, for any reason, you only need the second derivative, you can go ahead and differentiate two times all at once instead. From the above command sequence, we find that

$$f'(x) = (x + 1)^4 (x + 2)^2 (8x + 13) \tag{11}$$

$$f''(x) = 2(x + 1)^3 (x + 2)(28x^2 + 91x + 73) \tag{12}$$

It is now very easy to construct sign tables for $f'(x)$ and $f''(x)$ and determine the monotonicity and convexity of the function $f(x)$. wxMaxima cannot do the sign tables for you. It is possible, however, to use it to confirm your table. To do that, you must capture the derivatives $f'(x)$ and $f''(x)$ into user-defined functions, which we will call `fp` and `fpp`. Then, you can evaluate the derivatives for various numbers on the real line and confirm that the values you get have signs that are consistent with your sign table. To define `fp` and fpp, we use the following command sequence:

```
(x^2+3*x+2)^3*(x+1)^2
Menu: Calculus/Differentiate
Menu: Simplify/Factor Expression
fp(x) := ''%
Menu: Edit/Cell/Evaluate Cell
```

In the differentiation dialog box choose to differentiate only once. Now fp(x) has been defined, and you can calculate, for example, $f'(3/2)$ with:

```
fp(3/2)
```
Menu: Numeric/To Float

Note that `%` refers to the previous expression, which, in the above example, is the command to evaluate the derivative of $f(x)$. The double quote forces wxMaxima to execute the expression and evaluate the actual derivative before assigning it to the function `fp`.

To define `fpp(x)` as the second derivative, we use the already defined function `fp` and differentiate it one more time, as follows:

```
fp(x)
```
Menu: Calculus/Differentiate
Menu: Simplify/Factor Expression
```
fpp(x) := ''%
```
Menu: Edit/Cell/Evaluate Cell

With both `fp` and `fpp` now defined, it is easy to verify your sign table for both $f'(x)$ and $f''(x)$, by checking out the appropriate test values for x, for each interval.

V. EVALUATING INTEGRALS

wxMaxima can evaluate both definite and indefinite integrals. For example, to evaluate

$$I = \int_2^5 x^2\sqrt{2x+1}\,\mathrm{d}x \tag{13}$$

we use the command sequence

```
x^2*sqrt(2*x+1)
```
Menu: Calculus/Integrate

In the dialog box, you must indicate that you are calculating a definite integral, and then enter the limits of the integral. This gives the answer:

$$I = \frac{347\cdot 11^{3/2}}{105} - \frac{2\cdot 5^{5/2}}{21} = \frac{(347\cdot 11)\sqrt{11}}{105} - \frac{(2\cdot 25)\sqrt{5}}{21} = \frac{3817\sqrt{11}}{105} - \frac{50\sqrt{5}}{21} \tag{14}$$

Here, we have to do the conversion to radicals ourselves, but that is rather straightforward, and easily done with a simple calculator.

A similar process can be used for indefinite integrals. You simply do not select the definite integral option. Note that for indefinite integrals, the answer returned by wxMaxima could be different from your answer by a constant number. If that is the case, your answer is still correct. If it turns out, on the other hand, that your answer is wrong, then you should go over your work step by step and try to locate where the error was made. If you can't find the error, then you can use wxMaxima to evaluate the intermediate integrals that appear in your solution. In a correct solution we expect all intermediate integrals to evaluate to the same answer. In a wrong solution, somewhere near the error, the intermediate integrals after the error will yield a different answer than the integrals before the error. By evaluating a few of the intermediate integrals, you can quickly zero in to where the error was made.

For integrals that require a partial fraction decomposition, which tends to be a very cumbersome calculation, you can have wxMaxima perform the decomposition for you. For example, to decompose the function

$$f(x) = \frac{x^2 + 2x + 1}{(2x - 1)^2 (x^2 - 1)^3} \tag{15}$$

we use the command sequence

```
(x^2+2*x+1)/((2*x-1)^2*(x^2-1)^3)
```

Menu: Calculus/Partial Fractions

From the output, we find that

$$f(x) = \frac{-128}{9(2x-1)} + \frac{-16}{3(2x-1)^2} + \frac{-1}{72(x+1)} + \frac{57}{8(x-1)} + \frac{9}{4(x-1)^2} + \frac{1}{2(x-1)^3} \tag{16}$$