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On some problems of physical economics

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Abstract. **Attempts of designing economics along the lines of natural sciences (in particular, physics) with the use of mathematical modeling are reviewed. This area of research has come to be known as physical economics. Some topical questions of market economics are discussed; specifically, whether the market equilibrium is unique, whether transitions between stationary states are possible, and, if so, how these transitions proceed. By analogy with physics, the apparatus of mathematical modeling is widely used in answering these questions. It is shown that, under given external conditions, a self-sufficient country can be in two stationary, stable states — either in a high-productivity (HP) or in a low-productivity (LP) state. Transitions between them appear to be either an 'economical crisis' or an 'economical miracle'. It is shown that, for contemporary Russia, the crisis is already over, and the country is now in a stable LP state. Possible transitions to a HP state are discussed. The distributions of social elements over liquid accumulations and incomes are considered. It is shown that, in present-day Russia, these distributions are bimodal, meaning the coexistence of the poor and the wealthy with virtually no middle layer in between. In the tail of the distribution, a very small number of very wealthy people are present.**

1. Introduction

Why did physicists get interested in economics? The answer is obvious: the events of the past several years in the world and especially in Russia have appeared to be unexpected for many professional economists. This prompted the natural desire to understand the situation and to explain it using the language accepted in natural sciences, which is the goal of the present review. The authors do not claim to survey the state of economical science as a whole, since this state is not clear. Nevertheless, we believe that several remarks on this topic are in order and could elucidate the place of the discussed mathematical models in contemporary theoretical economics.

The term 'physical economy' (we also call it 'physical economics') was proposed by the economist Lyndon H LaRouche Jr [1]. He is known to be an associate of the US President Ronald Reagan and the originator of so-called Reaganomics, which implies (among other things) an increased role of the state in the economy. Today, LaRouche is the leader of a school in the economical science, which embraces several institutes and social institutions. By physical economy, LaRouche means economical science constructed in similitude of exact and natural sciences. Such an economical theory is far from complete, but some results have already been achieved, and we will discuss them in our review.

There are several avenues of research in theoretical economics. Most developed is classical (and neoclassical) economics (Nelson and Winter [2] also call it 'orthodox economics'). This theory is well equipped mathematically and constitutes a closed system, with its own conceptual tools, axiomatics, and methodology [3]. This area of research stands separate from natural sciences, including physics. Such a situation with theoretical, mathematical economics also causes a negative reaction from economists for the following reasons.

First, self-isolation hinders the development of any science and today, in the era of integration of sciences and the development of allied sciences, this has become especially evident.

Second, neoclassical economics has failed to predict or explain the development of real-world economy over the last

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few decades. Twenty years ago, Nelson and Winter [2] mentioned this fact, and the recent events have proved the validity of their expectation.

Third, natural sciences have gained a lot of experience in constructing and studying dynamical models of evolving systems, to which human society also belongs.

This brought about alternative areas of research in economics, united by the idea that theoretical economics should not be isolated from the natural sciences — on the contrary, it should develop together with these sciences and use their achievements [4]. This primarily concerns the theory of evolving systems (also known as synergetics), which has approved itself in physics, chemistry, and especially biology. This theory uses diverse tools of modern mathematics, giving preference to the theory of dynamical systems. In economics, this field is represented by the works of Pu [5], Zhang [6], and Lebedev [7], and is even called 'synergetic economics' (the title of a book by Zhang [6]). To do them justice, however, dynamical models were used in economics even earlier [8 – 11].

Evolutionary economics has been developed in the same avenue. It was pioneered by Schumpeter [12] and continued in the works of Nelson and Winter [2], Saviotti and Mani [13], and Silverberg and Verspagen [14]; its current state is discussed in Silverberg's review [4]. In Russia, it is represented by the works of Maevskiĭ [15]. Evolutionary economics, as implied by this term, deals with evolving systems. Compared to other nonorthodox fields, it has the longest history and is more developed.

Evolutionary economics differs from orthodox economics, first, in its axiomatics and, second, in the subject of research.

The prime postulates of classical economics are the following:

1. People (both producers and consumers) act rationally and purposefully. The aim of the producer is to make profit, while the aim of the consumer is to satisfy his needs.

2. Market equilibrium, i.e. a balance between the supply and the demand of commodities, money, and labor, is achieved by balancing the goals of producers and consumers.

In mathematical models of orthodox economy, the goals are formulated in terms of goal functions and additional conditions.

The subject of orthodox economics is market equilibrium for fixed parameters. Hence, orthodox economics can be called *static*. Nonequilibrium processes are primarily considered near the equilibrium, where the result of the process is predetermined.

The prime postulates of evolutionary economics are as follows:

1. People behave according to behavioral reactions. Sometimes, this behavior can be interpreted as the drive for maximum profit, but sometimes it cannot, since other motivations for the behavior are possible, perhaps religious, moral, political, etc. In mathematical models of evolutionary economics, behavioral reactions are formalized in terms of demand, supply, income, and expenditure functions.

2. Market equilibrium can be achieved through balancing supply and demand, and also income and expenditures. However, these functions vary with time due to the development of science and technology. Hence equilibrium can never be achieved, although the system is constantly approaching it. This warrants the use of the term 'evolutionary economics'.

3. It is reasonable to develop evolutionary economics based on the theory of evolving systems and of biological evolution.

In view of the last statement, it is worth recalling the essentials of the theory of evolving biological systems. Development is not monotonic in time. Periods of gradual evolution alternate with crises. In the course of gradual evolution, a species is improved due to the selection of the most adaptable individuals. During crises, new forms come into being and/or a transition to another steady state occurs.

Models of different types are used to describe these stages. For instance, adiabatic approximations are used to model the gradual stages [1]. In this case, it is assumed that the system is constantly near the same stable state but the parameters of this state vary slowly with time. For crisis phenomena, a model represents bifurcations, i.e. transitions to other states.

Both orthodox and evolutionary economics are based on exchange (market) relations, which means that both describe market economies. Actually, the above three features of evolutionary economics are also adopted in other nonorthodox economics, physical and synergetic. Hence, these two theories can be considered different versions of evolutionary economics.

The above-mentioned approaches compliment rather than contradict each other. Let us exemplify this point in classical physics.

When solving mechanical problems, we can, on the one hand, use the Newton equations. In this case, we must specify the force field, which can be done for particular conditions and always implies the presence of a hypothetical element. An analog of the force field in economics (and other evolving systems) is represented by behavioral functions. On the other hand, we can use the minimum-action principle. This requires specifying the form of the Lagrangian, which is also done according to the conditions of the problem and also includes a hypothetical element. These two approaches are equivalent in the sense that, once the Lagrangian is specified, we can derive the equations of motion and the form of the force field, and vice versa.

In economics, the goal function is an analog of the action. Given these functions, we can 'derive' the corresponding behavioral functions (which is not always a simple task). The equivalence of these approaches does not mean, however, that the results of modeling will coincide. The coincidence is possible only if the hypotheses concerning the form of the behavioral functions completely correspond to the hypotheses concerning the goal functional.

The choice of a particular approach is dictated by the convenience of this approach in solving the problem.

The second approach is used in problems of controlling technical facilities. In this case, the goal functional can be uniquely specified, since a technical facility operates on the basis of well-studied laws of physics, chemistry, and other natural sciences. The goal of the engineer is to adjust the parameters of the device in such a way that the goal be achieved with minimum expenditures (of assets, energy, etc.). Therefore, the second approach is quite constructive.

[1] To avoid misunderstanding, we note that the term 'adiabatic' is here in no way related to physical entropy. The only similarity is that, in heat engines, the adiabatic variation of the main parameter — the heat content (or enthalpy) — is slow compared to changes in the pressure and volume.

For living (and hence evolving) systems, the behavioral functions cannot be derived from the fundamental principles of physics, so that the particular choice of an approach should depend on the form in which the hypotheses corresponding to the real behavior of the system are simpler and more convenient. Here, the first approach has more advantages.

The behavioral functions can be inferred from empirical data fairly easily. In parallel, a number of 'unexpected' phenomena can be described and predicted, such as the emergence of several steady states, transitions between states, the emergence of unstable states, and transition to chaos. To describe these phenomena in terms of the second approach, a 'Lagrangian' should be chosen in an appropriate form, which is very difficult to do *a priori*, without knowing the dynamical equations.

Evolutionary economics uses both approaches. Examples of employing the first approach can be found in Refs [5–7] and also in our studies [16, 17]. The second approach has been successfully implemented by Petrov, Pospelov, and Shananin [18]. The results based on different methods largely overlap. Some results can easily be obtained in one approach, while it is difficult to obtain them in the other. Below we will give examples of solving important problems of Russian economy using primarily the first approach.

The aforesaid refers to mathematical and theoretical economics and contradicts the idea that economics belongs to the Humanities. A humanitarian aspect of economics really exists, and, thanks to politicians and mass media, it is this aspect that is most widely known to the general public. This field is full of dogma and myth rather than rigorous statements. Here are some examples of these myths.

1. Market equilibrium is unique. Actually, all attempts to prove this statement have not succeeded (see Ref. [4]). The possibility that there may be several stable states has been discussed, for instance, by Polterovich [19]. Below we will show that contemporary Russia can be in at least two stable states, and this fact plays an important role.

2. Emission (or currency issue) always leads to inflation. This statement also lacks substantiation. Under certain conditions, a reverse effect may also take place, and examples of this will be given below.

3. The state should not interfere with economics; in other words, the market itself will regulate everything (Adam Smith's 'invisible hand'). This statement has not been substantiated by theory and is false. Below we demonstrate that state regulation is a necessary condition for the existence and development of the state, especially in crises, which is especially important in contemporary Russia. We stress this fact to show that orthodox economics proved to be isolated not only from the natural sciences but also from the important problems of practical macroeconomics. In this situation, we believe that the incorporation of physical methods into economics is quite justified.

It follows from the above that the science of economy is now embroiled in controversy, and this is reflected by the present publication, which is also controversial. In our review, we use physical (and mathematical) notions. We will attempt to explain all economic terms using a language intended for a broad readership. As examples, we will use, among others, the results of our studies. This fault (or, on the contrary, merit) is characteristic of almost all authors of review articles in *Physics–Uspekhi*.

2. Behavioral functions in economics

2.1 Demand function

In an economical aspect, the behavioral functions are primarily reflected by the so-called demand function $Q(U, p)$. This function represents the dependence of the amount of commodities acquired per unit time on the available amount of money U and the price p. The available amount of money U intended for purchasing commodities either coincides with the total savings of the consumer or is determined by the consumer's income (y) per unit time (if, for some reason, there are no savings). In the second case, the demand function depends on the income and the price p. In both cases, the qualitative properties of the demand function are the same. What is important is that $Q(U, p)$ does not change under proportional variations in p, available funds U, and income y; this property is used, in particular, in the denomination of currency. The function $Q(U, p)$ is a rank-zero function [20], which depends on a single variable, the purchasing power r equal to the ratio $r = U/p$.

Demand functions may refer to demands for goods and services of prime necessities (Q_{I}), for durables (Q_{II}), and for elite goods (Q_{III}). The first group (or category) includes food, clothing, housing, heating, and transportation. The second group embraces most industrial goods. The dividing line between the goods of the first, second, and third categories is fuzzy and is different for different countries. Elite goods and services include luxury goods, as well as goods and services symbolizing a great reputation and power (image).

The demand function for goods and services of prime necessities, $Q_{\mathrm{I}}(r)$, is shown in Fig. 1. Here $Q_{\mathrm{I},0}(r)$ represents the subsistence level, the irreducible minimum necessary to support life. It is primarily determined by physiological factors rather than by the human factor. Nobody can do without goods and services of the first category, even if the income is small.

After the level $Q_{\mathrm{I},0}(r)$ is reached, the demand function grows slowly. This growth is caused by the changes in the assortment, prestige, etc., i.e. what is known as the human factor. This implies that the function $Q_{\mathrm{I}}(r)$ vanishes at $r = 0$ and thereafter increases, but more slowly than a linear function does, i.e. is convex everywhere (which is shown in

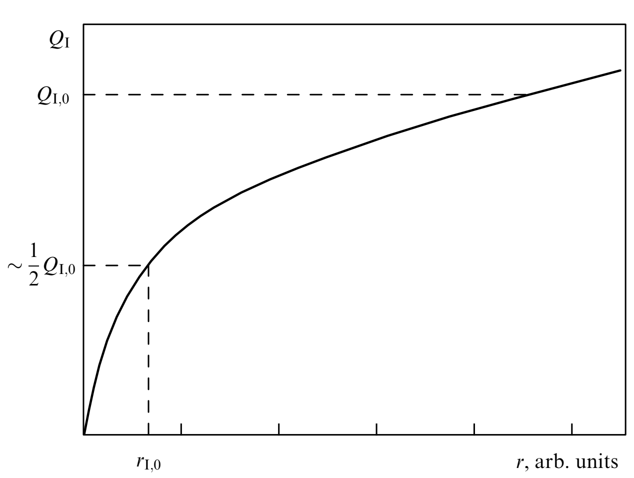

Figure 1. Dependence of the demand function for goods of prime necessities (first category) Q_{I} on the purchasing power of savings $r = U/p$ (U is the amount of money available, and p is cost).

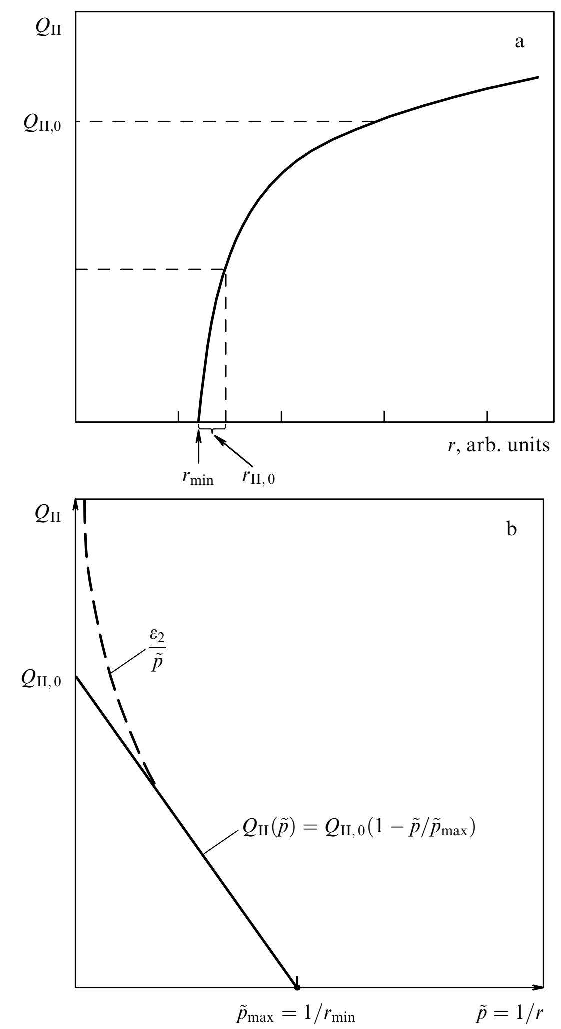

Figure 2. (a) Dependence of the demand function for goods and services of the second category and elite goods, $Q_{\rm II}$, on the purchasing power of savings r (present-day Russia). (b) Dependence of the demand function on the conditional cost $\tilde p = 1/r$. The straight line represents the threshold nature of the demand function for goods and services of the second category, while the dashed line illustrates the departure from the linear dependence, which corresponds to unsaturability.

Fig. 1). This function can be conveniently represented in the analytic form

$$Q_{\rm I}(r) = Q_{\rm I,0}\left[\frac{r}{r + r_{\rm I,0}} + \varepsilon_1 r\right]. \tag{1}$$

The parameter ε_1 reflects the availability of goods of a given type, differing in quality and price. It is small and only slightly affects the form of $Q_{\rm I}(r)$ for small r.

The demand function for goods and services of the second category, $Q_{\rm II}(r)$, whose diagram is shown in Fig. 2a, can be described analytically by the formula

$$Q_{\rm II}(r) = \Theta(r - r_{\min})\left[Q_{\rm II,0}\,\frac{r - r_{\min}}{(r - r_{\min}) + r_{\rm II,0}} + \varepsilon_2(r - r_{\min})\right], \tag{2}$$

where

$$\Theta(x) = \begin{cases} 0 & \text{for} \quad x \leqslant 0\,, \\ 1 & \text{for} \quad x > 0\,. \end{cases}$$

The function $Q_{\rm II}$ has the following properties:

(1) it is of a threshold nature, which means that the consumer refuses purchasing goods of the second category in the case of a deficiency of funds (or if the price is too high, i.e., $r < r_{\min}$);

(2) the function virtually never reaches saturation (unsaturability); this property plays an important role in evolutionary economics [21], and we will discuss it here in greater detail.

Goods and services of all categories (especially the second one) are highly diversified. The demand for a certain commodity can be saturated and may even fall with the increase of the purchasing power due to the supersession of the good by newer goods of the same category. This happens when new goods functionally equivalent to the old ones appear on the market. Usually, these new goods are of a higher quality. Very often, however, the supersession occurs without increases in quality, solely due to advertising and fashion (the bandwagon effect). The aggregate-product demand function includes all goods, including new ones. The fact that this function is not saturable follows from the renewal of the range of goods or, which is the same, from progress in science and technology. Without such progress, saturation is possible [21]. As the purchasing power r grows, the consumption of goods and services of the second category gradually changes into the consumption of goods of the elite category.

An important characteristic of $Q_{\rm II}(r)$ is its rate of increase, or its behavior near $r = r_{\min}$. This parameter reflects the sensitivity of the demand to changes in the prices and/or savings. In economics, the measure of sensitivity is a special quantity, the demand elasticity with respect to purchasing power, E_r. In the case at hand,

$$E_r = \frac{r}{Q}\,\frac{{\rm d}Q}{{\rm d}r} = \frac{{\rm d}\ln Q}{{\rm d}\ln r}\,. \tag{3}$$

According to equation (2), the quantity E_r is (formally) infinite at $r = r_{\min}$. At $r = 2r_{\min}$, we have

$$E_r(r \approx 2r_{\min}) \simeq \frac{r_{\min} r_{\rm II,0}}{(r - r_{\min})(r - r_{\min} + r_{\rm II,0})} = \frac{2r_{\rm II,0}}{r_{\min} + r_{\rm II,0}}\,. \tag{4}$$

Thus, the elasticity of the function $Q_{\rm II}(r)$ is determined by the parameter $r_{\rm II,0}$.

The demand elasticity with respect to price, E_p, is related to E_r in the following way:

$$E_{\tilde p} = \frac{\tilde p}{Q}\,\frac{{\rm d}Q}{{\rm d}\tilde p} = -E_r\,. \tag{5}$$

The demand function is often represented by the dependence of Q on the conditional price $\tilde p$ (which is inversely proportional to the purchasing rate: $\tilde p = p/U = 1/r$) in its simplest linear form [22, 23]. In this case, $Q_{\rm II}(r)$ can be written as

$$Q_{\rm II} = Q_{\rm II,0}(1 - r_{\min}\tilde p)\,, \tag{6}$$

which is illustrated by the diagram in Fig. 2b in the $(\tilde p, Q)$ coordinates. What is important is that equation (6) does not describe unsaturability; therefore, it can be used only within a limited interval of p values. For comparison, we have also depicted curve (2) in Fig. 2b (the dashed curve), which reflects the unsaturability.

The parameters of the function $Q_{\rm II}(r)$ reflect different aspects of the human factor, i.e., they depend on the social customs, advertising, fashion, propaganda, etc.

The parameter $r_{\min}$ is a reflection of the 'dividing line' for the consumption of goods and services between the first and second categories.

The quantity $r_{\rm II,0}$ characterizes the behavior of the middle class. People from this group are content with goods of moderate quality, which belong to the second category, and usually do not tend to pattern themselves on the elite. This means that the function $Q_{\rm II}(r)$ varies smoothly and gently. In Russia, there is no middle class to speak of, and the people who have just barely achieved economic success behave as 'nouveau-riche Russians', imitating the elite and spending money to form their image. Therefore, the function $Q_{\rm II}(r)$ ascends fairly steeply. What stimulates such behavior is the propagandist slogan that being poor is a disgrace. As noted earlier, such behavior in society is collective, and a person cannot refrain from it even if he or she is personally very modest.

The parameter $Q_{\rm II,0}$ describes the living standard of the well-to-do. In developed countries, this is the level of living of the middle class, while in Russia, this is the level of living of the social group that imitates nouveau-riche Russians.

The parameter ε_2 describes the increase in the demand for elite goods and services with the increase in the amount of accumulated income (or savings). It differs from country to country and depends on the situation in the particular country.

There are several methods used to determine the demand function (and/or its parameters).

1. Sociology uses interrogation methods, which provide direct but often incorrect information and are extremely laborious. At present, the demand function for Russia cannot be obtained by the interrogation methods alone.

2. The method of indirect estimates is based on the use of statistical data on the incomes, prices for various goods, and production volumes for these goods. This information is easier to get and makes it possible to estimate some of the parameters of the demand function.

3. The Delphi approach, which is commonly used in both economics and sociology, is similar to a council of physicians diagnosing the patient's ailment. For instance, a panel of economists and sociologists is brought together, and they are asked to estimate the parameters of the demand function for a given country on the basis of their intuition and experience. In many cases, the experts make estimates independently. If most of their estimates coincide, the common value is assumed to be close to the real one.

Using all the three approaches, one can reconstruct with certainty the demand function for goods and services of prime necessities, $Q_{\rm I}(r)$. The quantities that are commonly used in this context are the cost of living, or subsistence level (the parameter $Q_{\rm I,0}$), and 'food basket'. Expenditures for food in Russia amount to about 50% of the total expenditures (including those for clothes and utilities). Hence we can assume that the purchasing power corresponding to the 'basket' is equal to the parameter $r_{\rm I,0}$. A more complicated task is to estimate the parameters of the demand function $Q_{\rm II}(r)$, and the Delphi approach plays an important role in this case.

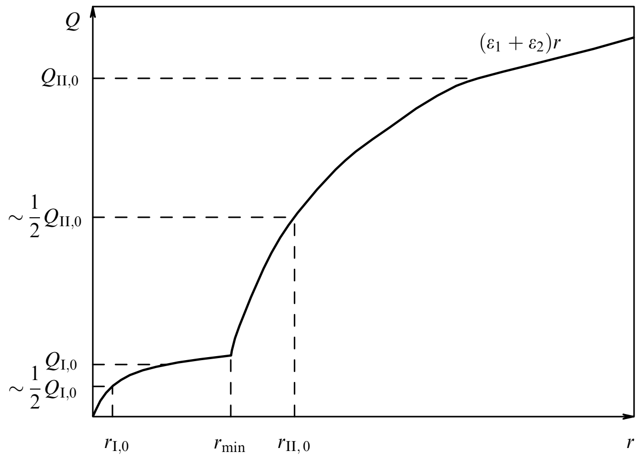

Figure 3. Total demand function $Q(r) = Q_{\rm I}(r) + Q_{\rm II}(r)$.

It is convenient to introduce the total demand function

$$Q(r) = Q_{\rm I}(r) + Q_{\rm II}(r)\,, \tag{7}$$

which is depicted in Fig. 3. The vertical axis represents the total demand for goods of the first and second categories. It can be said that this is the superaggregate-product demand function. The fractions of the goods and services of the two categories in the superaggregate depend on the purchasing power. Figure 3 shows that, when $r < r_{\min}$, only goods and services of the first category are in demand. If, however, $r > r_{\min}$, goods and services of both the first and the second category, as well as elite goods, are consumed.

For our further analysis, it is important to note that the function $Q_{\rm I}(r)$ is always convex, while the function $Q_{\rm II}(r)$ is convex only in its domain of existence, while in the interval $0 < r < \infty$ it is, strictly speaking, sigmoid rather than convex. Generally, the function $Q(r)$ is not everywhere convex.

2.2 Production function

The production function $F(r, n, \tau)$ is defined as the amount of product produced per unit time τ as a function of the number of people n participating in the manufacturing process and of the invested finances. The unit of time is the duration of the manufacturing cycle (it is also called the turnover time τ). The invested finances that are used to cover the manufacturing expenditures are called the working capital and are measured in monetary units.

In economics, the invested finances are commonly broken down into those used to pay wages and salaries, T, and the capital K. The latter implies the finance used to buy new machines. Such a division is convenient when we want to know what is more profitable — either (i) to increase labor productivity by introducing new technologies or (ii) by intensifying labor and/or by increasing the number of employees. In such an approach, the common expenditures for the initial product (raw materials), electricity, and transportation, which are not related to innovations, should be included in the 'labor' expenditures, since they provide necessary conditions for productive labor rather than replace it.

In the 1930s, Charles W Cobb and Paul H Douglas suggested what is now known as the empirical Cobb–Douglas formula [20, 24]

$$F(r, n, \tau) = {\rm K}^{\alpha}\,{\rm T}^{1-\alpha}\,, \tag{8}$$

where $\alpha \ll 1$ is an empirical parameter, and

$$\mathrm{T} = n(P + P^*) \,. \tag{9}$$

Here, P represents the wages and salaries, and P^* represents the traditional production expenditures per employee.

Formula (8) implies that the production function increases with T more slowly than linearly, so that the ratio F/T decreases. The meaning of this is simple: if a capacity is available (i.e. T is of order K), the amount of the produced goods is proportional to the invested finances. However, for a given permanent technology and increasing traditional expenditures, the growth in productivity slows down.

Actually, the increase in the amount of goods and services produced by one worker not only slows down but also has an upper limit, determined by the state of technology and the level of management. This effect cannot be described by the Cobb – Douglas formula. Below, we will use a simpler form of the production function:

$$F(V, n, \tau) = n\tilde{F}\left(\frac{V}{n}, \tau\right) = \begin{cases} n\,\dfrac{V}{pn\tau} = \dfrac{r}{\tau} & \text{for} \quad \dfrac{r}{n\tau} \leqslant F_{\max}\,, \\ nF_{\max} & \text{for} \quad \dfrac{r}{n\tau} \geqslant F_{\max}\,, \end{cases} \tag{10}$$

where $F_{\max}$ is the maximum amount of goods produced by one worker, V is the working capital, and V/n is the working capital per worker, with $r = V/p$.

In equation (10), we have assumed that the function $F(r)$ increases in proportion to r if the working capital is scarce and the capacity is underutilized. This growth, however, continues until a certain limit is reached, which is determined by the state of technology, the level of management, and the physical capacity of the workers. The diagram of the production function $F(r)$ defined by equation (10) is shown in Fig. 4.

The human factor also plays an important role in the production function. This role manifests itself in work ethics and labor management. These factors are reflected by the parameters $F_{\max}$ and τ.

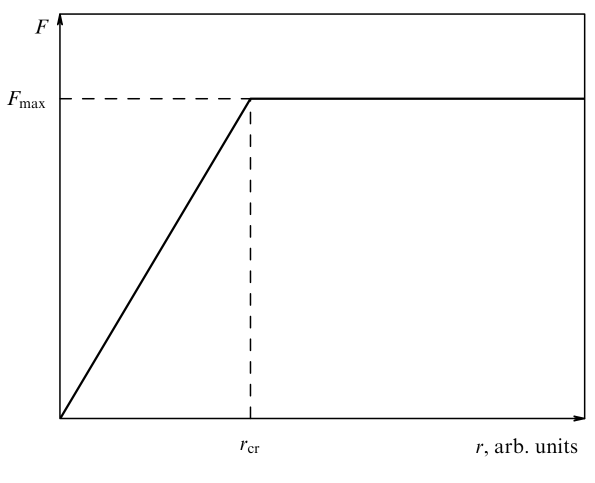

Figure 4. Dependence of the production function on the working capital r. The slanted section reflects the growth in production with increasing investment. This growth is limited from above, and $F_{\max}$ is the maximum level of production that is possible at the given state of technology if the productive capacity is completely activated.

3. Economical structure of society

The economical structure of society (ESS) is the distribution of society elements (i.e. families) over liquid accumulations, $\rho(U)$ [25, 27]. Here, U stands for the accumulations (in arbitrary units), and $\rho(U)$ is the fraction of people whose savings are in the range from U to $U + \Delta U$ (i.e. the distribution density). Liquid accumulations are the savings, in monetary units and securities that can be converted into money easily and without losses. Property, i.e. cars, flats, and houses, are not included in liquidities.

Frequently, a different (but similar in meaning) characteristic is also used: the income distribution of families, $\rho(y)$, where y is the income of the families per unit time. The two distributions, $\rho(U)$ and $\rho(y)$, are different but interrelated (see below).

The two functions, $\rho(U)$ and $\rho(y)$, can be found employing the above-mentioned methods [interrogation (polls), analysis, and the Delphi method]. In the developed countries, which are countries with a well-developed tax system, the income distribution can be calculated using the data from the tax returns. The income distribution can also be found by analyzing bank deposits. In Russia, such methods are sure to yield wrong results, and a more effective way to reconstruct the ESS is to analyze indirect data and employ the Delphi approach.

Sociology uses the so-called Lorenz curve instead of the distributions $\rho(U)$ and $\rho(y)$ [22]. This curve is shown in Fig. 5 and is constructed as follows. Society is broken down into parts (or groups) with equal numbers of elements (families or people)[2]. Usually, the number of parts is five (then each part is called a quintile) or ten (then each part is called a decile). Accordingly, the horizontal axis is divided into five or ten segments. Each segment of the horizontal axis contains elements whose savings are larger than the minimum for the given group but smaller than the maximum for this group.

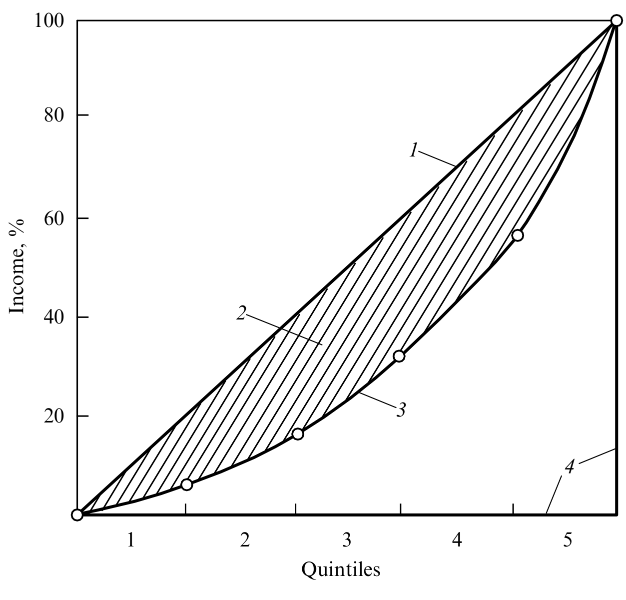

Figure 5. Lorenz curve [22]: *1*, line of absolute equality; *2*, deviation from absolute equality; *3*, actual income distribution in the United States (1959); and *4*, line of absolute inequality.

[2] Generally, these parts may not be equal (see Ref. [22]). However, the common practice is to divide society into equal parts.

The savings are plotted along the vertical axis. Thus, we get a broken line and, after smoothing it, a Lorenz curve.

By construction, the Lorenz curve represents the dependence of the savings $U(N)$ on the number of families N whose savings are equal to or less than U. The inverse is

$$N(U) = \int_0^U \rho(u')\,\mathrm{d}u',$$

whence

$$\frac{\mathrm{d}N}{\mathrm{d}U} = \rho(U) = \frac{1}{\mathrm{d}U(N)/\mathrm{d}N}. \tag{11}$$

This relationship specifies a link between the Lorenz curve $U(N)$ are the savings distribution. A Lorenz curve can, in a similar way, be constructed for the income distribution.

In sociology, the Lorenz curve is used to characterize the economic polarization of society. This is done by introducing the polarization index (the Gini index) equal to the ratio of the average savings in the richest decile to those in the poorest decile. It is assumed that if this ratio is greater than 15, social tension is present in society.

Until recently, the role of the ESS in economics has been constantly underrated. Below, we will show the consequences to which such an attitude leads.

It is simpler to find the income distribution $\rho(y)$, and this was done by Aĭvazyan [28], although the reliability of this distribution in the high-income region is doubtful.

In Refs [25–27], a mathematical model has been proposed that makes it possible to reconstruct $\rho(U)$ from indirect data. This model, being dynamical, can provide the trend in the ESS evolution, but its main purpose is to serve as an instrument in reconstructing the ESS. The model uses the following equation for the balance between incomes and expenditures of families with allowance for random factors:

$$\frac{\mathrm{d}U}{\mathrm{d}t} = P(U) - Q_{\mathrm{I}}(U) - Q_{\mathrm{II}}(U) + g\xi(t), \tag{12}$$

where $Q_{\mathrm{I}}(U)$ and $Q_{\mathrm{II}}(U)$ are the demand functions, $P(U)$ is the family income, $\xi(t)$ is a random function of unit amplitude, and g is a coefficient that accounts for accidental losses (or acquisitions). The income function $P(U)$ depends on the line of activity and, therefore, is different for different social groups.

For factory and office workers who have fixed wages and salaries, P is constant and independent of U. The income of the population varies from group to group. Two groups can conventionally be distinguished, a low-income and a high-income group. The data on wages and salaries are available and are used in the model as indirect data.

The profit of enterprisers depends on the invested finances and, to a first approximation, is proportional to these finances:

$$P(U) = A\left[1 + a\,\xi(t)\right] U + P_0. \tag{13}$$

Here, AU is the difference between the proceeds of activity and the expenditures, i.e. the surplus value, and the parameter A is the added-coast coefficient. The value of A is different for different people (i.e. A is also a distributed parameter), but the differences are small (i.e. the distribution is fairly narrow). The characteristic value of A acts as an indirect factor, and information on it is also available. The quantity P_0 is a small constant profit. Prior to reforms, all people in Russia, including illegal enterprisers, were in the government's employ and earned salaries P_0 small compared to their real incomes.

The quantity $a\,\xi(t)$ reflects random processes in producing and/or realizing goods.

To reconstruct $\rho(U)$, it is sufficient to solve equation (12) for each social group and then average the results over the groups. Equation (12) is a Langevin equation equivalent to the Fokker–Plank equation

$$\frac{\partial \rho_i(U)}{\partial t} = \frac{\partial}{\partial U}\left\{\frac{\partial V(U)}{\partial U}\,\rho_i(U) + \frac{1}{2}\frac{\partial}{\partial U}\left[G_i^2\,\frac{\partial}{\partial U} - \rho_i(U)\right]\right\}, \tag{14}$$

where $\rho_i(U)$ is the savings distribution over the ith group, and

$$V(U) = \int_0^U \left[P(U') - Q_{\mathrm{I}}(U') - Q_{\mathrm{II}}(U')\right]\mathrm{d}U'. \tag{15}$$

The function $V(U)$ is called the potential (as in statistical physics), although in our case, it is in no way related to energy. The quantity $G_i^2 = AaU + g$ is an analog of the diffusion coefficient. Information about the coefficients a and g can be obtained using the Delphi approach. The properties of the demand functions $Q_{\mathrm{I}}(U)$ and $Q_{\mathrm{II}}(U)$ have already been discussed.

The steady-state solution of equation (14) has the form

$$\bar{\rho}_i(U) = \rho_{i,0}\exp\left[-\frac{2V(U)}{G^2}\right], \tag{16}$$

where $\rho_{i,0}$ is a normalization factor such that $\int \rho_i(U)\,\mathrm{d}U = v_i$ (v_i is the fraction of the given group in society). For constant-income groups, $\bar{\rho}_i(U)$ is a unimodal distribution close to a normal distribution. Its peak corresponds to the minimum of U, i.e. the zero integrand in (15). For the low-income groups of the population ($P = P_1 < Q_{\mathrm{I},0}$), this point corresponds to the intersection of the line $P_1 = \mathrm{const}$ with the curve $Q(U)$. People in these groups spend their entire income for goods and services of prime necessities.

For the high-income groups ($P = P_2 > Q_{\mathrm{I},0}$), this point corresponds to the intersection of the line $P_2 = \mathrm{const}$ with the curve $Q(U) = Q_{\mathrm{I}}(U) + Q_{\mathrm{II}}(U)$. The people from these groups spend their income not only for daily necessities but also on goods and services of the second category.

For the groups consisting of enterprisers, the peaks of $\bar{\rho}_i(U)$ correspond to the points of intersection of the straight line representing the function $P_3(U) = AU + P_0$ and the curve of the demand function $Q(U)$. Figure 6 shows a diagram of these functions. Clearly, there are several points of intersection. Points marked by crosses correspond to the minima of the potential, while points marked by solid circles correspond to the maxima.

The function $\rho(U)$ has two peaks, at $\bar{U}_1$ and $\bar{U}_2$. The first peak refers to the enterprisers who were not able to obtain start-up capital (their mode of life corresponds to the low-income groups of the population). The second maximum refers to the successful enterprisers (their mode of life is that of the high-income groups of the population).

Figure 6 shows that a third group of active population may emerge, with accumulations exceeding $\bar{U}_3$. According to the model, there is no stationary state for this group, since the incomes are always higher than the expenditures included in

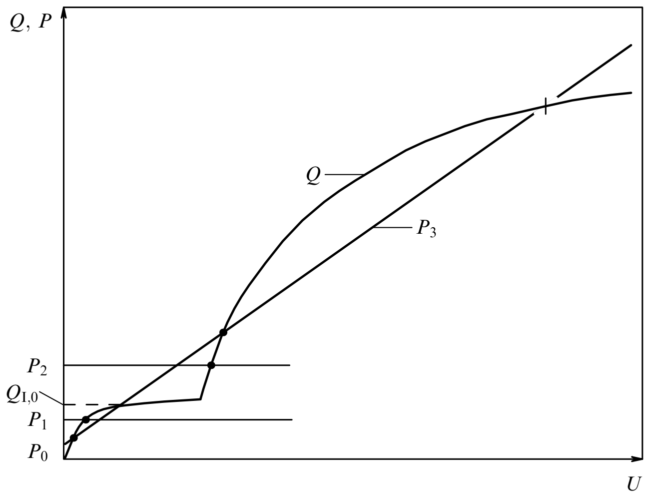

Figure 6. Income function $P(U)$ and demand function $Q(U)$ for the groups of people with a low constant income, $P_1 < Q_{I,0}$ (low-income group), a high constant income, $P_2 > Q_{I,0}$ (high-income group), and a low constant income P_0 plus an income from enterprise activities proportional to the invested finances U; •, stable stationary states; +, unstable states.

the demand function $Q_{II}(U)$. This group could conventionally be called the 'runaway tail', by analogy with a similar phenomenon in physics. Actually, the income and savings in this group are either restricted by law or are transferred to other countries (capital flight), or are converted into other forms of accumulation of wealth (jewelry, luxury goods, etc.). If all these factors are taken into account, the distribution in the tail region can be considered stationary, but formula (16) does not describe it. This phenomenon has fairly long been known in economics. The famous Italian economist and sociologist Vilfredo Pareto (1848 – 1923), known for his theory of mass and elite interaction as well as for his application of mathematics to economic analysis, proposed the following empirical distribution function for the tail region:

$$\rho_i(U) = \frac{\chi}{U^{\nu}} , \qquad (17)$$

where ν is the order exponent ($\nu = 1$ to 2). Pareto distributions are used not only in economics but also in biology and in the physics of nonequilibrium processes [29, 30].

The most commonly used Pareto distribution in economics is that with $\nu \simeq 2$ [30], and it is this distribution that we will use here.

In conclusion of this section, we note that, by its properties, the function $\rho(U)$ is similar to the distribution of particles in a potential $V(U)$ at a 'temperature' $kT = G^2$. In other words, people tend to 'gather' in the regions of potential minima. But why do people, with their free will, ambition, and strivings, act like soulless molecules (spheres)? The answer is simple: naturally, people are not simply spheres (although resemble them in some respects), so that their strivings and ambition should be taken into account, which is done in forming the demand function. After (and only after) making these remarks, we can say that it is reasonable to use the methods of the physics of developing systems to describe the behavior of people. This is not surprising, since these methods were evolved to solve various problems in widely differing branches of science.

3.1 Examples of reconstruction of the ESS for the USSR and Russia

The above-discussed mathematical model has been used to reconstruct the ESS in the Soviet Union and Russia from 1987 to the present day. Naturally, the parameters of the model have changed over the years, which made it possible to follow the ESS dynamics. A detailed description of this work can be found in Refs [25, 31, 32]; here we discuss the results.

In the Soviet Union, prior to its disintegration and the reforms, private enterprises were formally prohibited. During the *perestroĭka*, some of the bans were lifted, but this had no serious effect on the ESS. Actually, a small group of low-paid but active people existed in the Soviet Union. It included people working in trade and procurement agencies and the 'owners' of black-economy enterprises. The point is that trade and procurement require unrestricted stockpiles of supplies, but this was prohibited by the law. Nevertheless, the country continued to operate, and the reserves in the hands of commercial laborers amounted to about 30 – 40 thousand roubles. In any country, trading entails risks, but in the Soviet Union these risks were of a criminal nature. Hence, the number of people involved in trade was somewhat lower than in the developed countries and in present-day Russia.

In a country with a planned economy, the prices p are relatively stable, with the result that the purchasing power $r = U/p$ is proportional to the savings. Since, in the Soviet Union, the ESS formed over many years, we assume it to be stationary; its structure is shown in Fig. 7. Clearly, it is bimodal, and the humps are far apart, so that the ratio between the positions of the humps is $D = U_{II}/U_I \simeq 70$. Below, we will show that the bimodality of the ESS is very important for transient processes. Two parameters played the main role in the bimodal ESS: the wages (and salaries), which were below $Q_{I,0}$ for most people, and the threshold r_{min} for purchasing goods of the second category (this threshold was relatively high). The groups that formed the first and second humps differed in social composition and function: the 'rich' group included people involved in trade, science, culture, and military officers. There was practically no middle class. Neither was there a 'runaway tail' before the reforms.

For comparisons, in Fig. 8 we show the ESS of developed countries calculated by the same method. The structure is unimodal, i.e. most people (irrespective of the income

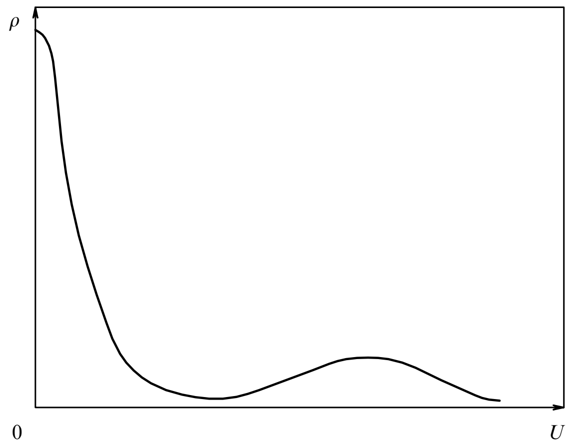

Figure 7. ESS in the Soviet Union prior to the reforms (1987). Reconstructed by the model used in Ref. [25].

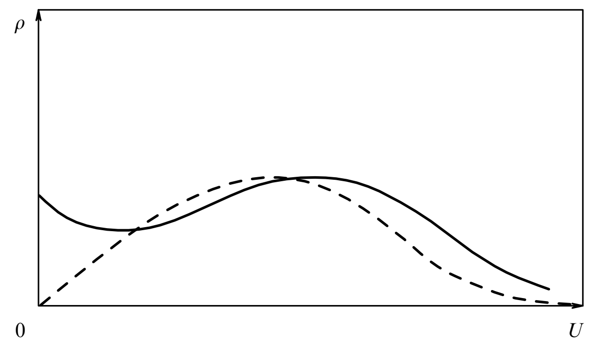

Figure 8. ESS in developed countries reconstructed by the model used in Ref. [25] (solid curve) and liquidity distribution for Japan according to the data from Ref. [33] (dashed curve).

structure) belongs to the middle class. This result stems from the fact that the wages and salaries in the developed countries are higher than $Q_{\mathrm{I},0}$, and $r_{\min}$ is lower than it was in the Soviet Union.

The ESS for Japan (see Fig. 8) was calculated on the basis of data on bank deposits [33]. Qualitatively, the two curves coincide. These results demonstrate that the model is valid over a wide range of parameters and that it yields qualitatively different results for different parameters.

The ESS for Russia in 1993 and 1995 (Fig. 9) has been calculated by the same model. The horizontal axis gives the savings in relative units (in this period, the prices changed very rapidly). The unit is here the price of the 'food basket'. In these units, the savings U coincide with the purchasing power r almost perfectly.

During the years of reforms, the parameters of earnings and spending changed considerably, and, accordingly, the structure of society also changed. Society became more polarized and, in addition, a 'runaway tail' formed in 1992–1993. According to our estimates, the 'runaway tail' consists of the owners of more than 50% of all liquid accumulations in the country. The richest people in this 'tail' are called oligarchs. The income of this group was estimated, and it was concluded that the oligarchs transfer the most part of their profits from Russia to other countries, since investing in Russia's economy is not profitable. Our estimates yield a figure of about \$20 billion for the yearly capital flight from Russia over the period from 1992 to 1995.

In the subsequent years, the economic polarization in Russia increased further.

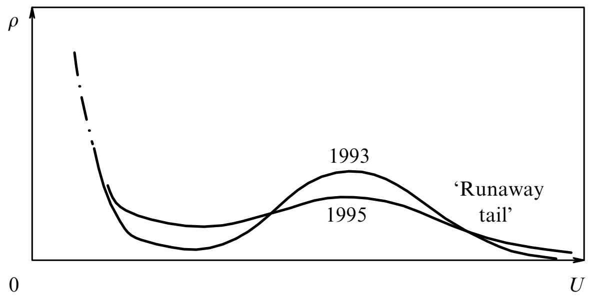

Figure 9. ESS in Russia for 1993 and 1995 reconstructed by the model in Ref. [36].

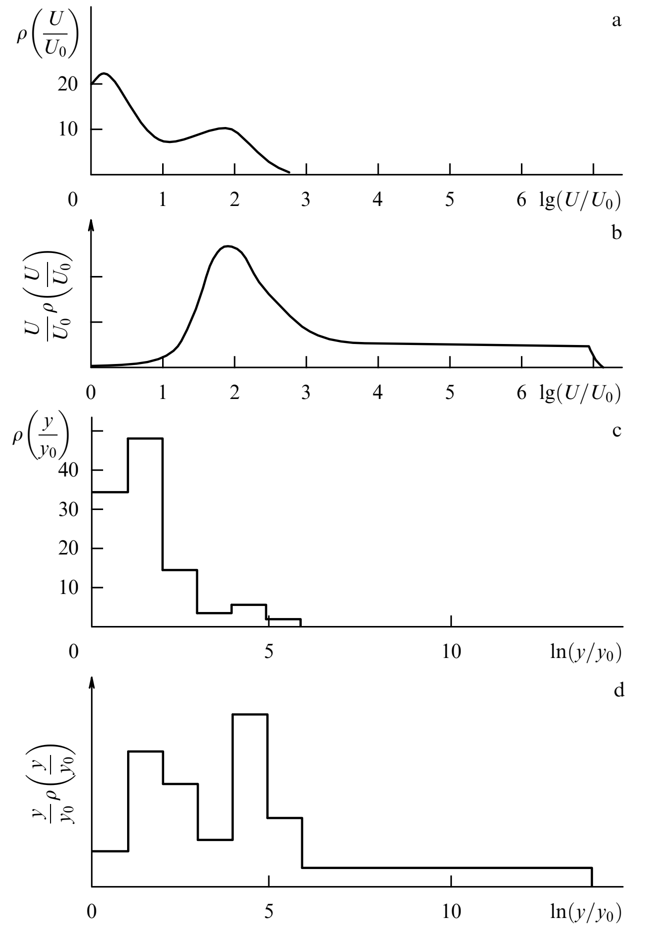

Figure 10. (a) ESS for Russia in 1999. Abscissas: $\lg(U/U_0)$, where U_0 is the monthly subsistence level; ordinates: $\rho(U/U_0)$, i.e. the number of families in the interval $\Delta\lg(U/U_0) = 3.16$ (millions). (b) The first moment of the distribution, $(U/U_0)\,\rho(U/U_0)$, in arbitrary units. (c) Income distribution of families, $\rho(y/y_0)$, based of the data in Ref. [33]. Abscissas: $\ln(y/y_0)$, where y_0 is the income equal to the subsistence level; ordinates: the number of people in the interval $\Delta\lg(y/y_0) = 1$, per cent. (d) Ordinates: the first moment of the distribution, $(y/y_0)\,\rho(y/y_0)$. It is assumed that, in the interval of $\ln(y/y_0)$ from 6 to 13, the distribution obeys the Pareto law with $\nu = 2$.

Figure 10 shows the ESS for Russia in 1999. Abscissas in Fig. 10a are the ratios of the savings U to the monthly subsistence level U_0. The 'runaway tail' in Fig. 10a is indistinguishable, since the people in the corresponding group are few in number. The tail is more pronounced in the function $U\rho(U/U_0)$ (Fig. 10b), where the distribution in the tail region is assumed to obey the Pareto law:

$$\rho\left(\frac{U}{U_0}\right) \approx \frac{\rho_0}{U^2}\,. \tag{18}$$

Clearly, the 'runaway tail' extends over several orders of magnitude in the interval from $U_1/U_0 = 10^3$ to $U_2/U_0 = 10^6$.

For comparisons, in Fig. 10c we show the income distribution $\rho(y/y_0)$ for Russia at approximately the same time, as obtained by Aĭvazyan [28]. The distribution is also bimodal, although the 'rich' hump is here less pronounced than in the savings distribution. The meaning of this feature is clear: in the model under discussion, the people with an income at least slightly in excess of $Q_{\mathrm{I},0}$ accumulate their savings and move into the 'rich' hump. The bimodality clearly manifests itself in the distribution $(y/y_0)\,\rho(y/y_0)$, which is shown in Fig. 10d.

The people whose income exceeds $Q_{\mathrm{II},0}$ 'land' in the 'runaway tail'. Aĭvazyan's study [28] contains no direct information about this tail. It is assumed in Fig. 10d that the tail of the distribution obeys the Pareto law with $\nu = 2$ and extends from $y/y_0 = \mathrm{e}^6$ to $y/y_0 = \mathrm{e}^{14}$. This interval qualitatively agrees with the above savings interval, provided that the annual income amounts to about 10% of the savings.

In recent years (1999 – 2001), the savings distributions and income distributions have not substantially changed: only U_0 and y_0 increased due to the rise in the minimum incomes (in money terms). In contemporary Russia, $U_1 = \$300$ thousand, and $U_2 = U_{\max} = \$300$ million. In the region where $U > U_2$, the distribution loses all meaning, since beyond this limit, there are only 30 families or so whose savings are of the order of \$1 billion. In the income distribution, the 'runaway tail' extends from $y = \$30$ thousand to $y = \$30$ million annually.

3.2 Role of the ESS in economics

Income and savings distributions are widely used in sociology and economics when solving the following problem. In sociology, the polarization of society (and the Gini index) is considered an indicator of social tension. This subject is interesting by itself but is beyond the scope of the present review. In economics, these distributions play an important role in the problems of taxation, pricing, and targeted emission. Below, we examine all these problems.

The goal of a tax system is to ensure maximum tax returns in the budget provided additional conditions are met, such as ensuring a high profitability of businesses, effectual demand, living standards, and social stability. The subjects of taxation are legal entities (i.e. organizations doing business in manufacturing, trade, and other types of business activity) and individuals.

The taxation of legal entities has a direct effect on the profitability of businesses, but it depends on the ESS only indirectly. We will return to this problem later.

The taxation of private persons (the tax on individual property, the tax on income, etc.) directly depends on the ESS. The meaning of this dependence is simple — to tax those who have money but not those who do not have it.

All countries (except present-day Russia) adhere to an ascending tax scale. This means that the profit fraction $\chi(y)$ taken away in the form of income taxes increases with the income. The same is true for taxes on property and capital. The optimal taxation (in the above-mentioned sense) depends on the distributions $\rho(y)$ and $\rho(U)$. This means that the greater the polarization of society, the higher the rate of ascendance of the taxation should be. Clearly, the tax system should track the changes in the distribution functions $\rho(y)$ and $\rho(U)$, i.e. it should change with them.

Usually, in quiet periods of the country's history, these distributions change gradually, and the tax system is virtually stable. This is important, since the stability of the tax system enables people to plan their economical behavior well in advance. But, in crises, the situation changes rapidly, and the tax system should react to these changes. It is especially important to track the situation in the right part of the distributions, in the tail region.

The question is: What tax system is best suited for the situation in contemporary Russia? The primary problem here is that of the income tax on individuals. This problem has already been discussed by Braginskiĭ [34]. The extreme solution — confiscating superhigh savings — is possible and reasonable, but could lead to social tension. A variant with the highest taxation amounting to 80% was tried out in the United States during the Great Depression and did not lead to destabilization. Our estimates show that, in this case, the taxation of the 'runaway tail' in Russia of 2001 would have brought \$7 to 10 billion. This, of course, is only a fraction of the capital flight and would not lead to social tension. Neither would it have a negative effect on Russia's economy.

We note, however, that the reader, knowing the income and savings distributions, can work out his or her own version of ascending taxation and make the necessary estimates.

The above examples show how important the ESS and especially the 'runaway tail' are in the taxation problems. Note that the methods of analyzing the ESS commonly used in economics — the Lorenz diagram and the Gini index — do not reflect the role of the 'runaway tail', since the people in the group described by the 'tail' are few in number.

Here we do not discuss the social, ethical, and political aspects of the problem, although they do play a crucial role here.

3.3 Role of the ESS in pricing

Let us discuss the conditions under which the free market can regulate the prices and the conditions under which the state and/or society must interfere. Following Ref. [35], we examine the problem of pricing in conditions where the maximum-profit principle (or, which is the same, the principle of equality of the marginal gain and marginal cost) operates. For the time being, we ignore the effect of competition for the following reasons. First, there can be no real competition if goods and services are produced by natural monopolies. By goods, we mean here electric energy, combustibles, and transportation and communication services. Second, if a good or service is produced by several companies (in conditions of oligopoly), the optimal strategy of these companies is to come to a collusive agreement, or an oligopolistic convention [36], which means their acting as a monopoly. To prevent such situations, antitrust laws are enforced. Note that such laws themselves are a means of state regulation, although not the unique possible and not the most effective means. Other measures can also be taken, and we will discuss them later. Finally, the absence of competition is a limiting case, and it is useful to know the properties of this case when discussing the problem as a whole.

The profit from manufacturing and marketing a good or service depends on the price, the demand function, and the savings distribution of families (i.e. the ESS) $\rho(U)$ or the income distribution of families $\rho(y)$. Both $\rho(U)$ and $\rho(y)$ are used to determine the effectual demand in different groups of society. The poor spend almost all their income to support their life (purchasing goods and services of prime necessities). In this case, the purchasing power depends on the income. However, in this group, the income distribution almost coincides with the savings distribution, so that either can be used in analyses. In the middle class and in the rich group of the population, most part of the means is used to purchase durables and elite goods and services, for which savings are needed. Hence, the purchasing power of these groups of the population depend on the savings [i.e. $\rho(U)$] rather than the income at the given time [i.e. $\rho(y)$]. Below, we will use the distribution function $\rho(U)$.

The profit Π is equal to the gross return R_T minus the total charges C_T:

$$\Pi = R_T - C_T\,. \tag{19}$$

The gross return depends on the price p, the quantity Q_T, and the total charges C_T, where Q_T is the total amount of the produced (and sold) good (or service):

$$Q_T = \int_0^\infty Q(U, p)\,\rho(U)\,\mathrm{d}U\,. \tag{20}$$

The total charges C_T consist of fixed charges (C_F) and variable charges (C_V): $C_T = C_F + C_V$. The former include the charges for keeping the production process in an operating state. The variable charges include the wages and salaries, the charges for raw materials, componentry, and their transportation. To a first approximation, the variable charges are proportional to the amount of the product, $C_V = sQ_T$, where s represents the variable charges for unit product. Taking all these facts into account, we obtain

$$\Pi = pQ_T - sQ_T - C_F\,. \tag{21}$$

Generally, s and C_F depend on the price p, but this dependence is weak, and we will assume s and C_F to be constant.

The maximum profit is achieved at the optimal price p_{opt} that obeys the condition

$$\frac{\mathrm{d}\Pi}{\mathrm{d}p} = \int_0^\infty Q(U, p)\,\rho(U)\,\mathrm{d}U + p\int_0^\infty \frac{\mathrm{d}Q}{\mathrm{d}p}\,\rho(U)\,\mathrm{d}U - s\int_0^\infty \frac{\mathrm{d}Q}{\mathrm{d}p}\,\rho(U)\,\mathrm{d}U = 0\,. \tag{22}$$

It is convenient to examine the problem of pricing separately in the cases of the demand functions for goods and services of the first and second categories.

The demand for goods and services of prime necessities is described by the function $Q_{\mathrm{I}}(r)$, which, as noted before, is everywhere increasing and convex, i.e. satisfies the conditions

$$\frac{\mathrm{d}Q_{\mathrm{I}}}{\mathrm{d}r} > 0\,, \qquad \frac{\mathrm{d}Q_{\mathrm{I}}(r)}{\mathrm{d}p} < 0\,, \qquad \frac{\mathrm{d}^2Q_{\mathrm{I}}(r)}{\mathrm{d}r^2} < 0\,. \tag{23}$$

It is obvious that, in this case, the maximum profit can never be achieved (condition (22) cannot be met). Indeed, equation (22) can be written as

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}p}\,\Pi &= \int_0^\infty Q_{\mathrm{I}}(U)\,\rho(U)\,\mathrm{d}U + p\int_0^\infty \frac{\mathrm{d}Q_{\mathrm{I}}}{\mathrm{d}p}\,\rho(U)\,\mathrm{d}U - s\int_0^\infty \frac{\mathrm{d}Q_{\mathrm{I}}}{\mathrm{d}p}\,\rho(U)\,\mathrm{d}U \\ &= p\int_0^\infty Q_{\mathrm{I}}(r)\,\rho(pr)(1-E_r)\,\mathrm{d}U - s\int_0^\infty \frac{\mathrm{d}Q_{\mathrm{I}}}{\mathrm{d}p}\,\rho(U)\,\mathrm{d}U\,,\end{aligned} \tag{24}$$

where E_r is the demand elasticity with respect to the purchasing power; it reaches its maximum at

$$\frac{\mathrm{d}E_r}{\mathrm{d}r} = \frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\mathrm{d}Q_{\mathrm{I}}}{\mathrm{d}r}\,\frac{r}{Q_{\mathrm{I}}}\right) = \frac{\mathrm{d}^2Q}{\mathrm{d}r^2}\,\frac{r}{Q_{\mathrm{I}}} + \frac{\mathrm{d}Q_{\mathrm{I}}}{\mathrm{d}r}\,\frac{1}{Q_{\mathrm{I}}}(1-E_r) = 0\,. \tag{25}$$

This yields the following expression for the maximum value of the demand elasticity:

$$(E_r)_{\max} = 1 + \frac{\mathrm{d}^2Q_{\mathrm{I}}}{\mathrm{d}r^2}\,r\left(\frac{\mathrm{d}Q_{\mathrm{I}}}{\mathrm{d}r}\right)^{-1}. \tag{26}$$

Conditions (23) indicate that the second term in the right-hand side of (26) is negative, and the demand elasticity is always less than unity; therefore, $1 - E_r$ in (24) is always positive. The last term in (24) is also always positive. Thus, for the production of goods and services of prime necessities, the profit as a function of price has no extremum and monotonically increases with price. This statement holds true even if the fixed and variable charges depend on the price p [35], and for any ESS; only minor details of the dependence vary. In a unimodal society, the profit rapidly grows up to the value $p = x_{\max}$, where $x_{\max}$ stands for the savings corresponding to the maximum of the distribution function $\rho(x)$; thereafter, the growth slows down, and the profit tends to an asymptotic limit.

Qualitatively, the picture is the same in a bimodal society, but the profit growth slows down near the price $p = U_{\max,2}$, where $U_{\max,2}$ denotes the savings corresponding to the right hump in the distribution; beyond this point, the growth continues at a lower rate.

In both cases, the maximum profit is achieved at the limitingly high price. This means that it is profitable to produce a very small amount of a good but sell it at a very high price. Obviously, such a strategy is not acceptable in any society, irrespective of the type of the ESS.

At first sight, this conclusion seems paradoxical, but a more detailed analysis shows that in many countries, including developed ones, nonmarket measures of state regulations are taken. Without such measures, the prices for goods and services of prime necessities grow constantly, which leads to price inflation.

The type of measures adopted by the state depends on the nature of the goods (and services) and on the structure of society. For instance, in most developed countries, the services in communication and transportation are regulated by the state and the prices for such services are fixed. In the power industry and in food production, depending on the situation, the state either takes economic measures (subsidies and preferential and/or classified taxation) or administrative measures (limitations on prices, antitrust laws, etc.). Using only antitrust laws (in order to artificially create competition) is clearly insufficient.

As concerns the prices for durables, the demand function is not everywhere convex: it has an inflection point or, to be more exact, a purchasing-power threshold $r_{\min}$ (see Fig. 3). An analysis of condition (22) similar to the above-discussed one leads to the following results (for more details, see Ref. [35]).

The price corresponding to the maximum profit, p_{opt}, can set in and become stable without an interference of the state. However, this price essentially depends on the structure of society, and the situations in unimodal and bimodal societies are qualitatively different. A bimodal society is characterized by four parameters: two positions of maxima, $U_{\max,1}$ and $U_{\max,2}$, and two dispersions, σ_1 and σ_2. This type of society differs greatly from the unimodal type, if $U_{\max,2} \gg U_{\max,1}$, $\sigma_1 \simeq U_{\max,1}$, and $\sigma_2 < U_{\max,2}$. We will consider this case here.

Profit as a function of price also has two maxima. The first one is attained near $p_{\text{opt}} \simeq U_{\max,1}$ (or $p_{\text{opt}} \simeq \sigma_1$) and the second near $p_{\text{opt}} \simeq U_{\max,2}$. However, two very different prices for the same good cannot coexist in one society[3]. Hence, a single price settles so as to ensure the highest profit.

When $N_2 U_{\max,2} \gg U_{\max,1}$ (where N_2 is the number of people who form the 'rich' hump), the second price is most profitable, and it is this price that is established in society. In this case, the effectual demand includes only the right hump. Production adjusts to the effectual demand and drops to a level that satisfies only the rich group of the population. Thus, the people who form the left hump become excluded from both production and consumption. Avoiding such a situation requires strict state regulation.

The above considerations are important for practice in the following aspects:

1. The statement that market relations always lead to price stabilization is erroneous. The lack of state regulation of production and sales of goods and services results in a constant growth in prices, i.e. in price inflation. This is not the only source of inflation, but in certain conditions it becomes predominant.

2. The character of the ESS becomes especially important during the transition from a planned economy to a market economy. In a unimodal society, the market prices for goods of the second category settle at a level acceptable for the majority of people. If, prior to this phase, the 'planned' prices were also acceptable for most members of society, then the transition to market economy does not lead to degradation. In a bimodal society, the situation is qualitatively different — the transition from a planned economy to a market economy leads almost immediately to the establishment of prices acceptable only for the 'rich' group of the population. As already noted, in the Soviet Union, the savings distribution was bimodal, with the peak values differing by a factor of about seventy. The inflation jump in 1992 was of the same order of magnitude. After that, the inflation continued growing, as it continues right now in Russia, although for another reason (see above).

3.4 Role of the ESS in monetary emission (noninflationary emission)

A commonly accepted view of monetary emission is that it always leads to inflation. This view is correct only partially. It is correct in an unimodal society if the emitted money is evenly distributed. Then the distribution $\rho(U)$ moves to the right, and the market prices rise, i.e. price inflation occurs.

A different situation is possible in a bimodal society when the emission is targeted to a certain segment of society. This leads to a drop in prices rather than a raise. Such monetary emission can be called noninflationary. This case has been studied in detail in Ref. [37]. Here are the main results of this study.

If the emitted money is targeted at the right edge of the left hump in a sufficiently large amount, the people in this segment move to the left edge of the right hump. The peak of the right hump shifts to the left, which, according to the aforesaid, should lead to a drop in prices for goods of the second category. The targeted segment could be civil employees (in education, science, culture, medicine), military officers, highly qualified workers — in short, all those who formed the 'rich' hump prior to the reforms.

A positive effect of targeted emission is possible if the following fairly stringent conditions are met:

(1) the emitted money reaches its addressees;

(2) there is a stock of goods and/or productive capacity in the country sufficient to rapidly increase the output of goods after an increase in the demand.

The fulfilment of the first condition completely depends on the law enforcement in the country and, ultimately, on the effectiveness of state management. Obviously, in a criminal situation this condition cannon be met.

The second condition depends on the state of the industry and, in certain stages of industrial development (or degradation), can certainly be met. In the Russia of 1995, this condition was actually met: there was a stock of goods and a sufficient productive capacity (see Ref. [37]), but the effectual demand was low, and the output of consumer goods was dropping. At that time, targeted emission could have boosted demand and revived Russia's economy.

Figure 11 shows the results of calculations of possible changes in the ESS in Russia of 1995 brought on by targeted emission [37]. Clearly, even a fairly moderate emission could lead to tangible results — a revival in

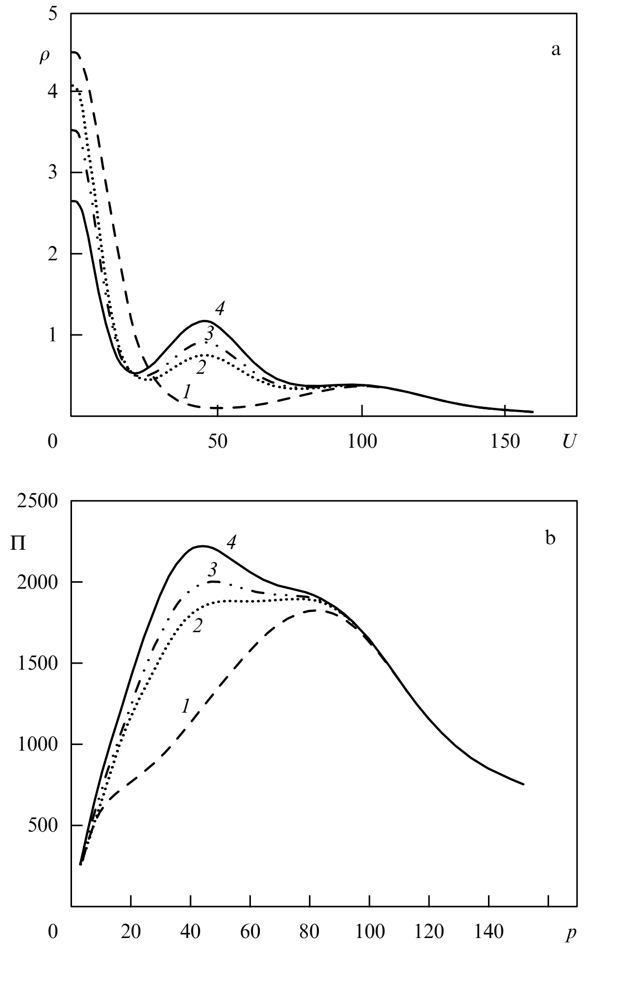

Figure 11. (a) Effect of targeted emission on the ESS in Russia of 1995 for different volumes of emitted money: *1*, no emission; *2*, emission amounting to 30 billion roubles; *3*, 40 billion roubles; *4*, 50 billion roubles. Calculations were done with the model used in Ref. [35]. (b) Dependence of the profit on prices for the corresponding emission volumes.

[3] There are cases where differentiation in society by property status is accompanied by other types of differentiation (by estate, nationality, etc.). Then the 'double-price' situation proves to be possible. We, however, will not consider such cases here.

industry by 30% accompanied by a 40% drop in prices [37]. Such optimistic estimates seem unrealistic, but they demonstrate the possibilities of targeted emission without a raise in prices (noninflationary emission). We present these results in support of the statement that targeted emission in a bimodal society does not necessarily lead to inflation and that a positive effect of targeted emission is possible. Today, the situation in Russia is different, but nevertheless the problem of targeted emission remains pressing, and we will certainly come back to it.

4. Dynamical models of macroeconomics

To describe a phenomenon of some sort, the theory of evolving systems first constructs so-called base (or minimal) models. They are intended for comprehending the essence of the phenomenon. More detailed (imitation) models are constructed on the basis of the minimal models. Such a sequence in constructing models is successfully used in biophysics [38].

Base models should be as simple as possible and should contain a minimum number of dynamical variables and parameters. In the theory of catastrophes, this requirement corresponds to the minimum-codimension condition [39]. For this reason, we will first consider a base model.

4.1 Base model of market economy in a closed society

In our case, the model is needed to reveal the states in which a self-sufficient country with a market economy can function without influences from other countries, to determine the number of such states, and to describe transitions between them. Self-sufficiency means that the country in question has enough resources and does not need exporting or importing goods and services.

The base model is intended to describe the crisis phenomena, such as the disappearance of one stationary state and a transition to another. The latter may occur, in particular, as a result of changes in the parameters. The base model does not describe processes caused by slow variations of the parameters (due to technological innovations). Slow processes of this type are studied by evolutionary economics.

In physics, a similar formulation of the problem appears in studying phase states and phase transitions in an isolated system.

The term ‘market economy’ means the following.

First, the price of a product is determined by the balance between supply and demand.

Second, the means of production are in the hands of private ‘owners’. This group of people (we denote their number as m) profits from selling the produced goods. Their expenditures include those used for personal needs and production charges. Another group of people, the ‘workers’, get their income in the form of wages and spend it for personal needs. The roles of owners and workers in society are different: the people in the first group are active, i.e. they can adjust some parameters to maximize their profit or their savings, while the second group is passive, i.e. the workers cannot affect the parameters. The expenditures for personal needs are determined by the demand function, the production charges incurred by the owners are determined by the production function, and the savings of both owners and workers by the balance between incomes and expenditures.

Third, the state does not directly control the economic processes.

In constructing the base model, we make a number of simplifying assumptions.

1. We adopt the so-called one-commodity approximation, which is frequently used in theoretical economics (see above). This means that the raw materials, products of all categories, and services (transportation, communication, etc.) are combined into one aggregate product. The demand for this single product is determined by the total demand function $Q(r)$. Production costs in such a model reduce to paying the wages.

2. We assume that the amount of money in society is fixed and equal to M. This means that the law of conservation of the sum of the savings of owners (U_m) and workers (U_n) holds in such a society:

$$nU_n + mU_m = M\,, \qquad n + m = N\,, \tag{26a}$$

where N is the number of economically active people, n is the number of workers, and m is the number of owners.

The parameter M has the following properties.

If M varies so slowly (adiabatically) that the price p and savings can follow these changes, the state of society does not change. Such a variation is in fact equivalent to denomination. A rapid variation in M can significantly change the state of society, especially if the emitted money is targeted to a certain group of the population (targeted emission).

With allowance for all these remarks, the base model can be written as

$$\begin{aligned} \frac{\mathrm{d}U_m}{\mathrm{d}t} &= \frac{p}{m}\left[nQ\left(\frac{U_n}{p}\right) - nP_1\right], \\ \frac{\mathrm{d}p}{\mathrm{d}t} &= \gamma\left[nQ\left(\frac{U_n}{p}\right) + mQ\left(\frac{gU_m}{p}\right) - F_m\right]. \end{aligned} \tag{27}$$

Here $P_1(U_m/p)$ represents the wages (in natural units) paid by the owners to the workers. In the case of piecework pay, the wages are proportional to the amount of the manufactured product:

$$nP_1 = hmF\left((1-g)\,r_m\right). \tag{28}$$

The factor h is smaller than unity, and the quantity $(1-h)$ is the surplus product; g is the fraction of the owners' savings used for personal needs; and F is the production function, which depends on the working capital [see Eqn (10)]. In our case, the working capital is

$$r = (1-g)\,\frac{U_m}{p} = (1-g)\,r_m\,,$$

where γ is the rate of establishment of the market price.

The variable U_n is not independent, and, according to (26a),

$$U_n = \frac{M - mU_m}{n}\,. \tag{29}$$

We introduce the dimensionless variables

$$\begin{aligned} &n' = \frac{n}{N}\,, \quad m' = \frac{m}{N}\,, \quad U'_n = \frac{U_n}{\tilde{U}} = \frac{M - mU_m}{n\tilde{U}} = \frac{1 - m'U'_m}{n'}\,, \\ &U'_m = \frac{U_m}{\tilde{U}}\,, \qquad p' = \frac{p}{p_0}\,, \end{aligned} \tag{30}$$

where $\tilde{U} = M/N$ are the average savings, $p_0 = \tilde{U}/r_0$, and r_0 is a parameter in the demand function $Q(r)$ [see Eqn (1)]. Next, we assume that $r_0 = 1$, i.e. we will measure the purchasing power in units corresponding to the subsistence level.

To study the model (27) qualitatively, it is convenient to introduce the variables $x = U'_m$ and $y = p^{-1}$. Then

$$\frac{dx}{dt} = \frac{1}{m'y}\left[n'Q\left(\frac{1-m'x}{n'}\,y\right) - m'hF\big((1-g)xy\big)\right],$$

$$\frac{dy}{dt} = -y^2\gamma\,\frac{N^2}{M}\left[n'Q\left(\frac{1-m'x}{n'}\,y\right) + m'Q(gxy) - m'F\big((1-g)xy\big)\right]. \qquad (31)$$

In terms of the new variables, $U'_n = (1-m'x)/n'$; from here on, we will drop the primes.

The phase portrait of the system (31) is shown in Fig. 12. The heavy solid lines represent the principal isoclines (along which $dx/dt = 0$ and $dy/dt = 0$), with the kinks corresponding to the values at which the production function is maximum. Clearly, at the values of the parameters used here ($m = 0.3$, $n = 0.7$, $h = 0.1$, $\tau = 5$, $F_0 = 8$), there are several stable stationary states. The first one is at $\bar{x} = x_{\rm I}$ and $\bar{y} = y_{\rm I}$, above the kinks. In this state, the entire productive capacity is in use, and the state can be called a high-productivity (HP) state.

The stationary state at $\bar{x} = x_{\rm II}$ and $\bar{y} = y_{\rm II}$ is unstable, with the separatrix passing through it (the dot – dashed line).

The state at $\bar{x} = x_{\rm III}$ and $\bar{y} = y_{\rm III}$ is stable. It lies below the kinks; therefore, the production function is far from its maximum at this point. This state can be called a low-productivity (LP) state, in which the productive capacity is underutilized, the working capital of the owners is smaller than in the HP state, and the price of the product ($p = 1/y$) is higher. Thus, the levels of both production and consumption of all groups of society in the LP state are lower than those in a HP state.

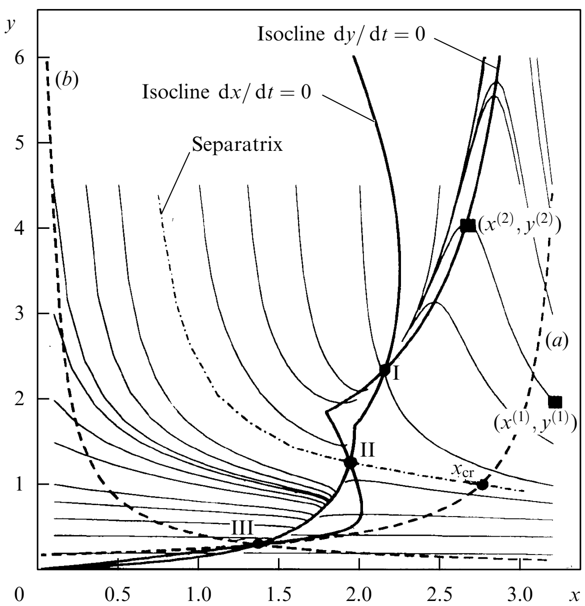

Figure 12. Phase portrait of the base model (31). The heavy solid lines represent the principal isoclines, the light solid lines represent the trajectories, the dot – dashed line is the separatrix, and the dashed lines show the trajectories of the displacements of the representative point in the case of emission targeted to owners (*a*) and to workers (*b*).

At small values of x and y, there are two more states (they merge in the phase portrait): an unstable state at $\bar{x} = x_{\rm IV}$ and $\bar{y} = y_{\rm IV}$, and a stable state at $\bar{x} = x_{\rm V} = 0$ and $\bar{y} = y_{\rm V} = 0$. The last one corresponds to natural economy, in which there is no commodity production. This state is separated from the LP state by a separatrix that passes through the point $(x_{\rm V}, y_{\rm V})$. The model does not claim to describe natural economy in detail, but it can describe a possible transition to such economy.

The light solid lines in Fig. 12 depict the trajectories of the representative point of the system (31) in the x, y plane for different initial states. Clearly, the dynamics of approaching one stationary state or another are not always monotonic. For instance, as the point moves from the initial state $(x^{(1)}, y^{(1)})$ where an excess working capital is available, y first increases [before reaching $(x^{(2)}, y^{(2)})$] and the price $p = 1/y$ drops, but thereafter [between points $(x^{(2)}, y^{(2)})$ and I], the price grows again. The meaning of this effect is simple: the working capital first flows into production, a surplus of the product is manufactured, and its price drops. Then, after a balance between supply and demand is reached (on the isocline $dy/dt = 0$), the demand continues to grow by inertia, while the supply drops. The price grows again until reaching an equilibrium at point I.

Other initial conditions correspond to other trajectories in the phase portrait in Fig. 12. The interested reader can trace the movement of the representative point along any trajectory in Fig. 12 and interpret the results. The most interesting cases appear when the initial conditions are close to the separatrix, and even weak perturbations (due to an intervention by the government or other countries) can produce serious effects.

Note that the principal isoclines and their intersection points do not depend on the parameter γ. However, the slopes of the trajectories (including the separatrix) depend on γ very strongly. The trajectories become steeper as γ increases and flatten as γ decreases. At the same time, the sign of the angle of inclination of a trajectory does not depend on the parameter γ. This parameter determines the rate at which equilibrium prices set in and depends on the rate of acquisition and processing of information on demand, supply, distribution of means in society, etc.

In our case (see Fig. 12) we have assumed that $\gamma = 1$. This means that the rate of establishment of equilibrium prices is of the order of the rate at which a balance between supply and demand settles. Actually, the characteristic parameters of these processes may differ severalfold.

Figure 12 contains two heavy dashed curves, (*a*) and (*b*). They are important for the assessment of the effect of targeted emission, which will be discussed below.

It is convenient to carry out a parametric analysis of the model (31) using the equations for stationary states. We assume that $dx/dt = dy/dt = 0$ and find from (31) that

$$Q(gr_m) = (1-h)\,F\big((1-g)\,r_m\big). \qquad (32)$$

This equation contains only one combination of the variables x and y, i.e. $r_m = xy$. The left-hand side represents the expenses of the owners for personal needs, while the right-hand side represents the owners' returns, $R(r_m)$, i.e. profits minus production charges. Thus, equation (32) describes a balance between 'the desirable' [the function $Q(gr_m)$] and 'the possible' [$R(r_m)$].

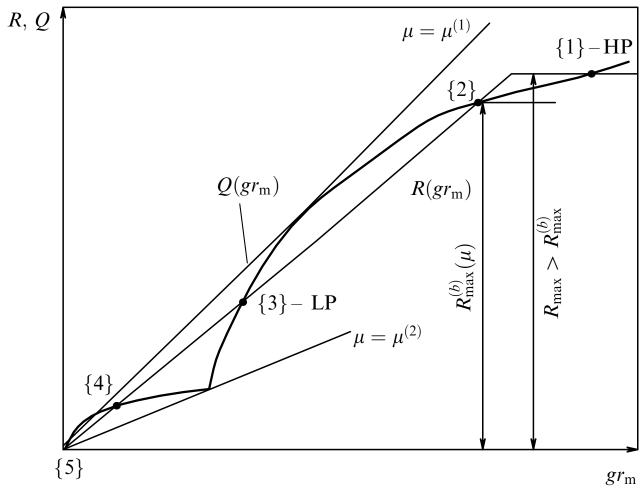

Figure 13. Balance diagram (details in the text).

In view of the properties of the production function, we can write equation (32) in the form

$$Q(gr_m) = \begin{cases} \mu g r_m & \text{for} \quad r_m < \dfrac{F_{max}}{1-g}\,, \\ (1-h)F_{max} = R_{max} & \text{for} \quad r_m \geqslant \dfrac{F_{max}}{1-g}\,, \end{cases} \tag{33}$$

which contains the parameter $\mu = (1-h)(1-g)/\tau g$.

A graphic solution to equation (33) is given in the balance diagram (Fig. 13). On the horizontal axis is the quantity $gr_m = xy$ and on the vertical axis, the demand function $Q(gr_m)$ and the return function $R(r_m)$. The parameter μ is the slope of the linear segment of the return function.

The state $\bar{x} = x^{\{5\}} = 0$, $\bar{y} = y^{\{5\}} = 0$ (natural economy) and the unstable state $(x^{\{4\}}, y^{\{4\}})$ are already separated in this diagram.

Thus, the model contains two control parameters, μ and R_{max}. Their bifurcation values are related, i.e. $R_{max}^{(b)} = R_{max}^{(b)}(\mu)$. The following states are possible for different values of these parameters.

$\{1\}$. For $\mu \geqslant 1$ and any R_{max}, one stationary state exists.

$\{2\}$. For $1 > \mu > \mu^{(1)}$, two stable stationary states are possible — HP and natural economy $(x^{\{5\}}, y^{\{5\}})$. However, the basin of attraction of the HP state is much broader. Hence, mode $\{2\}$ differs little from mode $\{1\}$. At $\mu = \mu^{(1)}$, there is a fold bifurcation: the states $(x^{\{2\}}, y^{\{2\}})$ and $(x^{\{3\}}, y^{\{3\}})$ merge. For $\mu > \mu^{(1)}$, these states disappear and for $\mu < \mu^{(1)}$, appear.

$\{3\}$. For $\mu^{(1)} > \mu > \mu^{(2)}$, the situation depends on the parameter R_{max}. Two variants are possible here.

$\{3a\}$. For $R_{max} > R_{max}(\mu)$, all five states are possible. Among these, the HP and LP states and natural economy are stable. The phase portrait in Fig. 12 corresponds precisely to this case. The bifurcation value $R_{max}^{(b)}(\mu)$ depends on the parameter μ and increases with μ.

$\{3b\}$. For $R_{max} < R_{max}^{(b)}(\mu)$, there is no HP state, and only the LP state and natural economy are possible. At $R_{max} = R_{max}^{(b)}(\mu)$, there is a fold bifurcation. The states $(x^{\{1\}}, y^{\{1\}})$ and $(x^{\{2\}}, y^{\{2\}})$ merge.

$\{4\}$. For $\mu \leqslant \mu^{(2)}$, a bifurcation is present. For $\mu \leqslant \mu^{(2)}$, the states $(x^{\{3\}}, y^{\{3\}})$ and $(x^{\{4\}}, y^{\{4\}})$ merge and then disappear. Two variants are possible here.

$\{4a\}$. For $R_{max} < R_{max}^{(b)}(\mu)$, only one stationary state remains, natural economy.

$\{4b\}$. For $R_{max} > R_{max}^{(b)}(\mu)$, two stable states are possible — the HP state and natural economy. However, the basin of attraction of the former is small, and mode $\{4b\}$ differs little from mode $\{4a\}$.

The above is true for the given parameters of the demand function. Generally, the bifurcation values of the control parameters μ and R_{max} depend on the parameters of the demand function.

Let us discuss variants of transient processes. Generally, transitions can be initiated by external factors. Two types of switching factors are distinguished, forced and parametric ones.

Forced switching occurs as a result of changes in the dynamical variables (in our case, the owners' savings x and/or prices $p = 1/y$). The parameters of the model do not change, and the phase portrait remains the same, but the representative point is displaced.

By way of an example, let us discuss a possible transition from the LP state to the HP state due to, say, targeted emission. In the case of nontargeted emission, the parameter M increases, and the savings and prices change proportionally. However, the dimensionless (renormalized) variables, as well as the structure of the phase portrait remain the same. In the case of targeted emission, the dimensionless variables change. If the emission is targeted to owners, the point x moves to the right along the line (a), which is represented by a heavy dashed curve in Fig. 12. If the displacement is so large that the point crosses the separatrix (i.e. the point x_{cr}), the system will approach the HP state by itself. In economics, this phenomenon appears as an 'economic miracle'.

If the targeted emission is not sufficiently intense, and the point remains in the basin of attraction of the LP state, the system, after a certain revival of the economy, returns to the LP state. The dimensionless variables do not change, but the dimensional quantities (prices and savings) increase in proportion to the emission.

If the emission is targeted to workers, the dimensionless variable x decreases (due to the decrease in the fraction of the owners' savings). Then the point x moves along the dashed curve (b) in Fig. 12. An 'economic miracle' is also possible in this case, if the representative point is 'thrown' over the separatrix. This is possible if the separatrix slope in the left half of the portrait is sufficiently gentle (the variant depicted in Fig. 12 does not satisfy this requirement). This means that the production rapidly grows, following targeted emission, and outruns the increase of prices. We discussed above precisely this variant in the context of the problem of targeted noninflationary emission in a 'double-humped' society.

An inverse process, i.e. the transition from the HP state into the LP state as a result of the withdrawal of working capital from the owners, is also possible. It is perceived as an economic crisis. We will discuss this case below in greater detail.

Parametric switching results from changes in the control parameters μ and F_{max}, which lead to a bifurcation. The representative point (x, y) remains in place, but finds itself in the basin of attraction of another state.

The transition from the LP state into the HP state (economic miracle) takes place due to an increase in the parameter μ to $\mu \geqslant \mu^{(1)}$. As a result, the LP state disappears, and the system finds itself in the basin of attraction of the HP state and moves by itself toward the HP state. An inverse parametric switching from the HP state to the LP state is also possible. For this transition, it is sufficient to reduce the parameter μ to a value at which $F_{\max}(\mu)$ is smaller than the bifurcation value. As a result, the HP state disappears, and the system moves by itself toward the LP state — an economic crisis sets in.

It is important that both the forced and the parametric switching are hysteretic. This means that changes in the dynamical variables and in the parameters are different in the case of switching from the HP state to the LP state and in an opposite process. Such properties are well known in physics and are a characteristic feature of first-order phase transitions.

Let us now discuss the important question of who controls the parameters. The parameter $F_{\max}$ depends on the technological level of society and is fixed in the base model.

The parameter μ is a combination of the parameters τ, g, and h. The duration of the manufacturing cycle τ depends on the levels of management and technology and is also fixed. The fraction g of the profit used by the owners for their personal needs can be varied by the owners. The fraction h of the profit spent for wages depends on both the owners and the workers. In the model, h is a free parameters that can vary within certain limits that are determined by the balance of interests of the owners and workers.

The money supply M is controlled by the state (here we do not distinguish between cash and cashless money supply). However, the state is not included in our base model. It is reasonable to assume that all decisions on the governmental level are made by owners and in their own interests. It is useful to formulate what are the interests of the owners. It is commonly assumed that the aim of the owners is to get the maximum profit. In our model, the profit is

$$\Pi = (1-h)\,F\big((1-g)\,r_m\big) = \begin{cases} (1-h)\,\dfrac{(1-g)\,r_m}{\tau} & \text{in the LP state,} \\ (1-h)\,F_{\max} & \text{in the HP state.} \end{cases} \qquad (34)$$

The profit grows if either the wages are cut or g is reduced. The wages can be cut only within certain limits, which depend on social factors. The reduction of g means a reduction of personal expenses by the owners, which runs contrary to their interests. Hence, the maximum-profit principle is limited by the additional condition that the expenses for personal needs are maintained at a high level.

It is also important to distinguish between short-term goals and long-term goals. The former means that the owners may vary the parameters near a given stationary state. If this is the LP state, a transition to the HP state is not included in the short-term goals.

Long-term goals call for a transition to the HP state, where the profit and the satisfaction of personal needs (of both owners and workers) are higher than in the LP state. Thus, the owners and workers may sacrifice the short-term goals for the long-term goals. For instance, the owners may reduce, by mutual consent, their personal needs (i.e. g) and/or, through an agreement with the workers, reduce the piecework pay (i.e. h) so that the parameter μ grow to the bifurcation value. This will initiate a transition into the HP state (economic miracle), and everyone will be happy.

Switching to the HP state caused by targeted emission does not conflict with the short-term goals of the owners or workers. Such a situation is quite possible. However, this is true only for a closed society, where the interests of all groups of the population are linked solely to the situation in the country rather than the situation in other countries. Within an extended model, which allows for the interests of exporters, importers, and the financial elite, targeted emission would run contrary the interests of some social groups and countries. We will discuss this problem later.

In conclusion of this section, let us dwell on the relation between the base model and the structure of society. Within the dynamical approach, the savings distribution should follow from the model. If we know the owners' and workers' savings, we can compare them and estimate the degree to which society is polarized. In the dynamical model, the ESS has two peaks along the savings axis, and the distance between them reflects the degree of bimodality. In reality, both peaks are smeared due to random processes.

The result can easily be predicted: the peaks will broaden accordingly to the 'noise' level. Thus, the ESS in the model is a dynamical characteristic of society; it is determined by the model and changes with the state of society.

4.2 Dynamical macroeconomical model of contemporary Russia

Let us now discuss an extended model constructed around the base model with allowance for the conditions of contemporary Russia (we follow our study [17]). This model is not highly detailed (imitating) but it answers the most important questions concerning the macroeconomic strategy:

(1) what stationary states are possible in contemporary Russia?

(2) in which of them is Russia's economy now?

(3) in what state was Russia's economy before the reforms and what was the transient process?

(4) what are the possible scenarios of the future evolution of Russia's economy?

(5) what will be the implications of one measure or another taken on the governmental level?

In other words, the model may serve as an instrument for making decisions on the country-management level. Let us specify the initial assumptions of the model.

As in the base model, we assume that the means of production are private property in the hands of legal entities, or 'owners'. They organize production, command the finances, incur the charges of manufacturing and marketing the product, and pay taxes. They are also supposed to sell the product. This means that the legal entities in the field of commercial business also belong to the category of owners.

Raw materials (energy sources and metals), electric energy, and transportation services are not included in the aggregate product and are considered separately. The reasons for this are as follows. In contemporary Russia, the raw-materials sector works mostly for export. The prices for raw materials are determined by the world market rather than the domestic market. The world prices for raw materials are much higher than those admitted by the balance of supply and demand on the domestic market. Nevertheless, the metallurgical industry sells its product (both ferrous and nonferrous) inside Russia for world prices [31, 32]. The prices for energy sources inside the country are partially regulated by the state (but not by the domestic market). They are somewhat lower than the world prices, but are also anomalously high. In both cases, the prices for raw materials are not determined by the domestic market — they are fixed from outside, which can (and must) be regulated by the state.

The prices for electric energy and transportation rates largely depend on the prices for energy sources. Furthermore, such goods and services belong to the first category (prime necessities) and are produced by natural monopolies. We have demonstrated above that, without an interference from the state and/or human institutions, the market is unable to regulate the prices for these goods and services. Hence, in the present model, they are similar to prices for raw materials, being fixed from the outside. In principle, they can (and should) be regulated by the state. Thus, the model describes the state and dynamics of the manufacturing industry in Russia.

The export of industrial products from Russia is insignificant compared to the export of raw materials. After the 1998 crisis, the volume of imports has reduced substantially and still remains relatively small. Hence, to a first approximation, we will ignore these factors. Further developments of the model can take them into account fairly easily.

The society under investigation consists of N persons and is divided into eight groups (clusters):

nonworking pensioners (their number is $n_0 N$);
workers in the production sector of the economy ($n_1 N$);
workers in budget organizations ($n_2 N$);
pensioners working in budget organizations ($n_3 N$);
pensioners working at private enterprises ($n_4 N$);
workers in the raw-materials sector ($n_5 N$);
owners of private enterprises ($n_m N$);
the elite, or the owners and top managers of raw-materials enterprises and commercial banks, top officials, etc. (lN).

The quantities n_i $(i = 0-5)$, m, and l are the relative populations of the groups: their sum is equal to unity. In order to associate quantities with social groups, we will assign corresponding indices to these quantities.

The members of each group have savings U_i $(i = 0-5, m, l)$, which are dynamical variables of the model and are determined by the balance between incomes and expenditures. Money savings have corresponding purchasing powers, $r_i = U_i/p$. It is assumed that the incomes within each group are the same, while the incomes of different groups differ. The incomes of pensioners, workers in budget organizations, and workers in the raw-materials sector are fixed and equal to P_0, P_2, and P_5, respectively. Working pensioners receive their pay and pensions. The income of workers at private enterprises that do not belong to the raw-materials sector depends on the production volume, while the income of the owners is determined by the profit from selling their product.

The quantities P_i $(i = 1, 2, 5)$ are the imputed wages and salaries. The income of workers (in a broad sense) is P_i minus the income tax $\varkappa_0 P_i$. Pensions P_0 are not taxed.

The owners spend a fraction of their income ($g \ll 1$) for personal needs, which include 'authority' and 'image' costs. These expenses are equal to $g r_m$, and the volume of consumption by the owners is described by the demand function $Q(g r_m)$. The other part, $(1-g)\,r_m$, is used by the owners as working capital, which covers the production charges. Out of these charges, the wages and salaries amount to $(1+\varkappa_1)P_1$, where $\varkappa_1$ is the tax on the salary schedule, which includes contributions to various funds (primarily the pension fund).

The wages of workers in the private sector are proportional to the volume of the product they produce. By analogy with (32), we can write:

$$(n_1 + n_4)P_1(r) = hF\big((1-g)\,r\big)\,. \tag{35}$$

In addition to the charges in the form of wages and salaries, the owners also incur the charges for raw materials, energy, and transportation (production charges) and pay taxes. In the model, they are assembled into one group of charges proportional to the volume of the product and equal to $(\lambda + \varkappa_2)\,F\big((1-g)\,r_m\big)$. Here, λ is a factor reflecting the production charges, and $\varkappa_2$ is the taxation level proportional to the volume of product.

The total charges incurred by the owners per unit time is

$$\big[h(1+\varkappa_1) + \lambda + \varkappa_2\big]\,F\big((1-g)\,r_m\big)\,, \tag{36}$$

and the profitability of the investments is $\alpha = \big[h(1+\varkappa_1)+\lambda+\varkappa_2\big]^{-1}$.

The owners' returns are equal to the profit earned from selling the product minus production charges. The dynamics of the working capital is determined by the balance between the income and expenditures of the owners.

The dynamics of prices is determined by the balance between supply and demand on the domestic market.

It is assumed that the amount of money in society, M, is a constant. This condition imposes a restriction on the savings in each group:

$$N\bigg[\sum_{i=0}^{5} n_i U_i + mU_m + lU_l\bigg] = M = \text{const}\,. \tag{37}$$

As a result, only seven out of the eight dynamical variables U_i are independent.

The price of the product p is another dynamical variable, and its dynamics is determined by the balance between supply and demand for this product on the market.

The model is represented by a system of eight ordinary differential equations describing possible stationary states of the economy and transitions between them:

$$\begin{aligned}
&(1)\quad \frac{\mathrm{d}U_0}{\mathrm{d}t} = p\left[P_0 - Q\left(\frac{U_0}{p}\right)\right],\\
&(2)\quad \frac{\mathrm{d}U_1}{\mathrm{d}t} = p\left[P_1(1-\varkappa_0) - Q\left(\frac{U_1}{p}\right)\right],\\
&(3)\quad \frac{\mathrm{d}U_2}{\mathrm{d}t} = p\left[P_2(1-\varkappa_0) - Q\left(\frac{U_2}{p}\right)\right],\\
&(4)\quad \frac{\mathrm{d}U_3}{\mathrm{d}t} = p\left[P_2(1-\varkappa_0) + P_0 - Q\left(\frac{U_3}{p}\right)\right],\\
&(5)\quad \frac{\mathrm{d}U_4}{\mathrm{d}t} = p\left[P_1(1-\varkappa_0) + P_0 - Q\left(\frac{U_4}{p}\right)\right],\\
&(6)\quad \frac{\mathrm{d}U_5}{\mathrm{d}t} = p\left[P_5(1-\varkappa_0) - Q\left(\frac{U_5}{p}\right)\right],\\
&(7)\quad \frac{\mathrm{d}U_m}{\mathrm{d}t} = \frac{p}{m}\left[\sum_{i=0}^{5} n_i Q\left(\frac{U_i}{p}\right) + Q_b + lQ\left(\frac{U_l}{p}\right)\right.\\
&\qquad\qquad - P_1\big((1-g)\,r_m\big)(n_1+n_4)(1+\varkappa_1)\\
&\qquad\qquad \left. - m(\lambda+\varkappa_2)\,F\big((1-g)\,r_m\big)\right],\\
&(8)\quad \frac{\mathrm{d}p}{\mathrm{d}t} = \gamma\left[\sum_{i=0}^{5} n_1 Q\left(\frac{U_i}{p}\right) + Q_b + lQ\left(\frac{U_l}{p}\right)\right.\\
&\qquad\qquad \left. + mQ\left(\frac{gU_m}{p}\right) - mF\big((1-g)\,r_m\big)\right].
\end{aligned} \tag{38}$$

The parameter γ in the eighth equation of system (38) determines the relative rate of price stabilization on the market, and Q_b is the governmental award for the product in question (it is assumed that the size of the award represented in kind does not change in time).

The phase space of system (38) is eight-dimensional, so that it is impossible to represent the phase portrait as simply as in the previous case. However, the balance equation for the owners' returns $R(gr_m)$ and expenditures $Q(gr_m)$ has the same form as before:

$$R(gr_m) = Q(gr_m)\,. \tag{39}$$

Equation (39) follows from the seventh and eighth equations of the system (38) with zero time derivatives (i.e. for stationary states). A graphic solution to equation (39) is represented by the diagram in Fig. 13. Here, the parameter μ depends not only on the wages but also on the charges for raw materials and taxes and is equal to

$$\mu = \frac{1-g}{g\tau}\left\{1 - \left[(1-\varkappa_1)h + \lambda + \varkappa_2\right]\right\}. \tag{40}$$

In view of this, all the qualitative results inferred from the base model remain valid. This result could be expected, since base models are constructed in such a way that their results remain valid in a more realistic extended model. At the same time, we can now study transitions from the LP state to the HP state (and back) depending on the prices for energy and raw materials, tariffs and taxes. Thus, the balance diagram in Fig. 13 together with equation (40) can even serve as an instrument for strategic decision making.

The model makes it possible to comprehend what is the present-day state of Russia's economy and how this state has been reached.

Based on the data from the (Russian) State Statistics Committee [40, 41] and other sources, we have estimated the parameters and have arrived at the following conclusion: today Russia is in the LP state (more precisely, in a low-productivity stagnation state). A transition from this state into the HP state requires surmounting a barrier. A spontaneous transition, i.e. a transition without special measures taken in order to overcome the barrier, from the LP state to the HP state is impossible.

Let us discuss how Russia has found its way to the LP state. Prior to the reforms, the level of manufacturing industry in Russia was approximately three times higher than today. In terms of the present model, this means that the country was in the HP state.

It is well known that, as a result of the liberalization of prices and the inflation that followed, the working capital in the manufacturing industry shrunk almost instantly (within a month) by a factor of 3.5. As noted before, a reduction in working capital is equivalent to a forced switching, which is the inverse of targeted emission. The factor of 3.5 was taken as the measure of the displacement of the representative point. As a result of such a displacement, the system found itself within the basin of attraction of the LP state and moved into this state. The results of model calculations of this process are presented in Fig. 14. The year 1992 was taken as the starting time. We can see that the model calculations agree with the data on the dynamics of the gross domestic product (GDP) and the decline in production in the manufacturing industry and the machine-building industry.

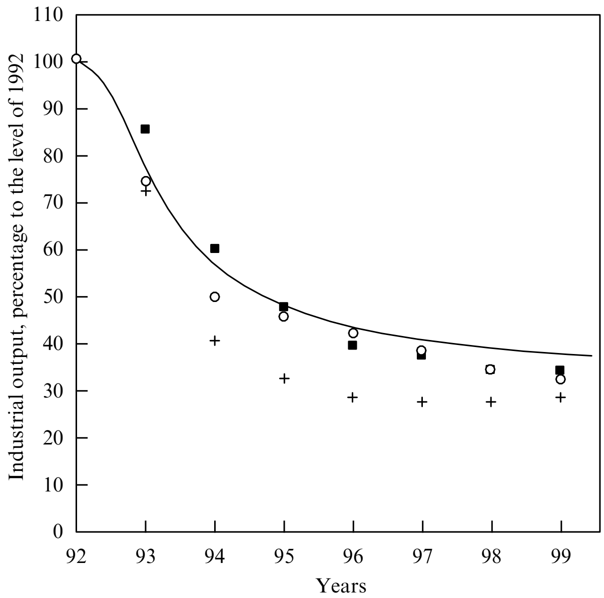

Figure 14. Yearly dynamics of the decline of production in the manufacturing industry in Russia as result of the withdrawal of working capital, beginning with 1992; solid curve, the results of model calculations in Ref. [27]; ■, GDP in percentage to GNP in 1992; +, output in machine-building industry; and ○, manufacturing industry (according to the data of the (Russian) State Statistics Committee [40]).

The balance diagram makes it possible to answer the question of whether it is possible to switch from the LP state to the HP state by reducing taxes $\varkappa_2$, transportation charges, and prices for raw materials λ. Such combined measures lead to an increase in μ and, as a result, to a parametric switching. We recall that, to land on the development trajectory, the system must cross the dividing ridge (separatrix). This means that only a sufficiently large increase in μ can pull the system out of stagnation. A small change (increase) in the angle of slope will lead only to a slight upswing of the economy, but the country will remain in the LP state.

Figure 15 shows the results of model calculations for different values of the parameter $\lambda + \varkappa_2$. Clearly, to reach the switching threshold, the costs $\lambda + \varkappa_2$ must be reduced by a factor of 1.5. However, the transition to the HP state will be very slow in this case and will last for several decades. The transition will speed up if the production costs are reduced more significantly, but even in this case the recovery will be fairly slow.

Figure 16 illustrates the dynamics of the transition from the LP to the HP state in the event of a forced switching due to targeted emission to the real (production) sector of the economy. It can be seen that the transition proceeds much faster than in the previous cases (it takes years rather than decades). A slight growth in prices (i.e. inflation) is unavoidable, but it will then be compensated by an increase in supply as a result of the production growth. The effect of targeted emission is also of a threshold nature, since in this case, again, the system must cross a dividing ridge. Insufficient emission does not make it possible for the representative point to reach the separatrix, and the country, after some revival in the economy, will return to the LP state. The growth in prices persists and is not balanced by an increase in the production of goods and services.

Such a behavior of the system in the HP state illustrates the limitations of counterposing the Keynesian doctrine [42]

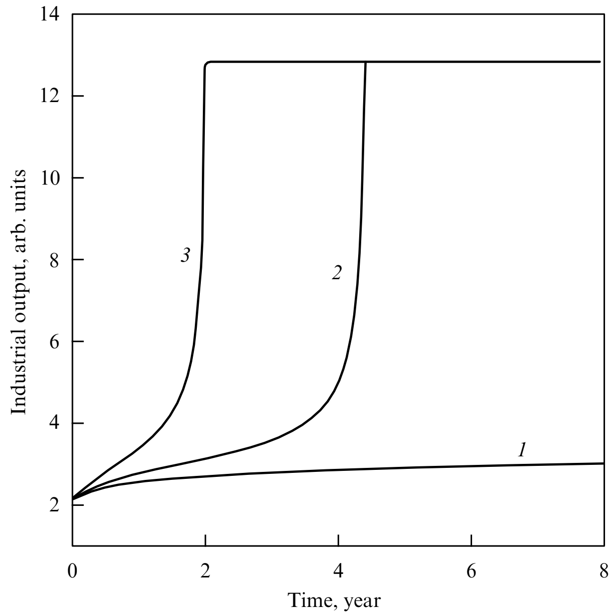

Figure 15. Model calculations of the dynamics of the manufacturing industry after a reduction in the production charges for raw materials and transportation (the parameter λ) and taxes (the parameter $\varkappa$). Curve *1* represents the dynamics in the case where $\lambda + \varkappa_2$ is reduced by a factor of 1.5 (which corresponds to the bifurcation), and curves *2* and *3* correspond to the cases where the costs have been reduced by factors of 1.75 and 2.3, respectively. On the horizontal axis are the years from the time of costs reduction.

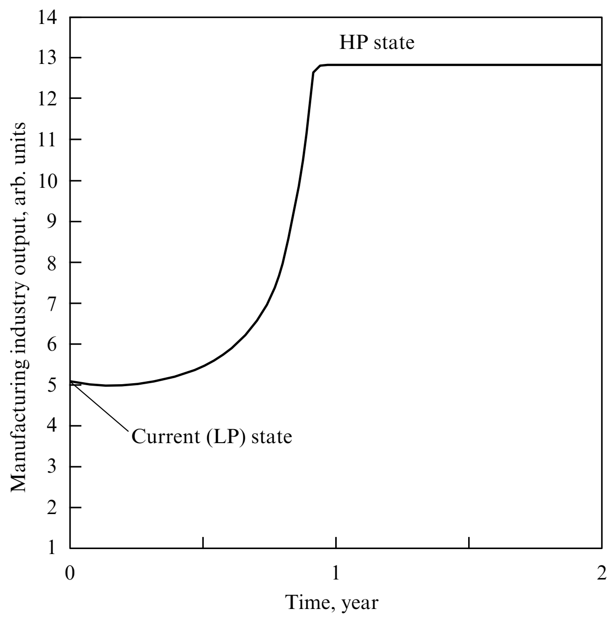

Figure 16. Model calculations of the dynamics of the manufacturing industry after a targeted emission in a volume sufficient to cross the separatrix. On the horizontal axis are the years from the time of emission.

to monetarism [43]. If the targeted emission is sufficiently high, the model behaves in agreement with the Keynesian doctrine; otherwise, the model behaves in agreement with monetarism.

It should be noted that such a rapid revival of the economy as depicted in Fig. 16 is possible if the productive assets (industrial facilities, equipment, etc.) are in good condition, i.e. there is a productive capacity reserve. Actually, during the years of reforms, the wear and tear of industrial facilities have been great in Russia (and have not been taken up), and this process is continuing. In view of this, additional capital and time should be allotted to restore the equipment. The model can easily allow for this factor, but nevertheless this problem merits special attention.

On the whole, we believe that targeted emission can result in an economic miracle in Russia, although not as rapidly as predicted by the present model. One of the main difficulties in actualizing this possibility is determining the correct target and ensuring that the necessary funds reach their addressees.

The transition to the HP state is also possible by changing the parameters of the demand function while preserving the economic parameters. The human factor plays a leading role in this case. The measures that would lead to changes in the demand function are of a political and social, rather than purely economical nature. For instance, slogans like 'Shame on you to be poor!' or 'Live the way people live in the West!' make the function steeper. In contrast, slogans like 'Shame on you to be rich when the country is in peril!' or 'Do not try to live the way people live in the West — help Russia get on its feet again!' or 'Russian goods come first!' facilitate reduction of consumption and makes the function $Q(r)$ flatter. In this case, the country will find itself in the basin of attraction of the HP state and will pass to the development trajectory.

Such a change in the demand function is equivalent to an increase in the angle of slope of the return function without, however, a reduction in taxes and costs and without economic pressure exerted by the state on the raw-materials natural monopolies. This, of course, requires that the nation trusts the state and is ideologically united.

In conclusion of this section, we would like to list the main results obtained on the basis of the present model.

1. Market equilibrium is generally not unique. Even at the same macroeconomic parameters, the country can pass to either the HP state or the LP state, depending on the initial conditions. Both states are stationary and both correspond to equilibrium between prices and savings.

2. The model makes it possible to answer the question of what factors (and to what extent) can cause a crisis (i.e. a transition to the LP state) and, on the other hand, can initiate an inverse transition to the HP state.

3. Contemporary Russia is in the LP state and will not be able, without serious measures taken by the government, to leave this state. Market self-organization alone cannot initiate a transition to the HP state.

4. The balance diagram gives an idea of the economical state and the possible implications of one decision or another made on the governmental level.

Several factors that are traditionally considered important for macroeconomics have not yet been taken into account by the present version of the model. These are:

(i) The export and import of agricultural and manufacturing-industry produce (as noted before, the export of raw materials is taken into account by the model). At present, the volume of exports and imports of these products is relatively small and does not play a significant role. However, this question is important, since the strategy of Russia entering the world market depends on the answer.

(ii) The role of commercial banks in accumulating the savings of the population and, as a result, in providing credits to the real sector of the economy. For various reasons, this factor plays a minor role in contemporary Russia. Moreover, fundamental problems of a global nature arise in this context.

Now, there is a gap between the financial activity in Russia and the real sector of the economy, and this could lead to a serious crisis. It is this danger that is actively discussed by LaRouche and other economists.

(iii) The model uses a one-commodity approximation. Actually, however, the situation is different in different sectors of the economy (say, in agriculture and industry). This aspect cannot seriously affect the qualitative results of the base model but must be taken into account in future studies.

The above factors can (and should) be taken into account in the future development of the model. In doing so, one should preserve, as far as possible, the illustrativeness and simplicity of the base model. However, the dynamical modeling of macro-economic processes is far from completion.

5. Conclusions

In our opinion, two our findings are most important and should be emphasized here — the role of the ESS in economics and the existence of two stable macroeconomical states (the low-productivity and the high-productivity state), with a possibility of transitions between them.

As noted before, the possibility of several states of a market economy has never been refuted by specialists. This feature is new only to those whose knowledge about economics comes only from the mass media. However, it is important to point out this fact, since such people form a majority. For physicists, this result would seem trivial. Indeed, it is hard to imagine condensed-matter physics without phase transitions.

What is new and important is quite different: dynamical models in economics make it possible to qualitatively (and even semiquantitatively) describe the transitions between states and reveal the main parameters controlling these processes.

But to what extent are these results substantiated? There are two aspects to this question, and we will discuss them separately. First, are the starting assumptions of the model properly substantiated by experiment or theory? And, second, do the results agree with reality?

The starting assumptions of 'physical economy' are the behavioral reactions of the objects under investigation. In our case, these reactions are described by the demand and production functions. The form of these functions can, in principle, be determined experimentally. As noted above, this can be done either directly (by interrogation methods) or indirectly (in particular, by the Delphi method). The latter is not considered very reliable, although it is used most widely. It is virtually impossible to get reliable information using interrogations. Hence, the starting assumptions always contain a hypothetical element.

The situation is similar in orthodox economics. There, the starting assumption is the hypothesis that all people act rationally. One could raise the question of an empirical verification of this hypothesis by the interrogation method. Say, passers-by could be asked whether they always act rationally. The statistics of the answers is predictable, but it can hardly be considered reliable. Hence, this assumption is also hypothetical.

True, the situation in natural sciences is exactly the same. New hypotheses and their representation in the form of governing equations (e.g. the Newton, Maxwell, and Schrödinger equations) result from new experimental data. However, these data never prove to be sufficient to formulate these equations [45]. A theory becomes convincing and commonly accepted only after additional experimental data are gathered and only if it agrees with these data.

Finally, we leave the question of whether the results agree with reality to the readers' discretion. In economics, reality is not a laboratory experiment. Instead, this is life itself, in which each person is an observer and, at the same time, a guinea-pig.

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References

1. LaRouche L H (Jr) "The science of physical economy as the Platonic epistemological basis for all branches of human knowledge" Parts 1, 2, 3 *Executive Intel. Rev.* **21** (9 – 10) (1994) [Translated into Russian (Moscow: Nauchnaya Kniga, 1997)]
2. Nelson R R, Winter S G *An Evolutionary Theory of Economic Change* (Cambridge, MA: Belknap Press of Harvard Univ. Press, 1982) [Translated into Russian (Moscow: ZAO 'Finstatinform', 2000)]
3. Ashmanov S A *Vvedenie v Matematicheskuyu Ekonomiku* (Introduction to Mathematical Economics) (Moscow: Nauka, 1984)
4. Silverberg G "Modelirovanie evolyutsionnykh protsessov v ekonomike" ("Modeling of evolutionary processes in economics") *Vestn. Molod. Uchenykh. Ser. Ekonom. Nauki* **2** (6) 76 (2000)
5. Pu T *Nonlinear Economic Dynamics* (Berlin: Springer, 1997) [Translated into Russian (Izhevsk: Udmurtskiĭ Univ., 2000); Trofimov V V "Geometricheskiĭ analiz bol'shikh ekonomicheskikh sistem" ("Geometrical analysis of large economic systems"), in Pu T *Nelineĭnaya Ekonomicheskaya Dinamika* (Nonlinear Economic Dynamics) (Izhevsk: Udmurtskiĭ Univ., 2000) p. 174
6. Zhang W-B *Synergetic Economics*: *Time and Change in Nonlinear Economics* (Berlin: Springer-Verlag, 1991) [Translated into Russian (Moscow: Mir, 1999)]
7. Lebedev V V *Matematicheskoe Modelirovanie Sotsial'no-Ekonomicheskikh Protsessov* (Mathematical Modeling of Socioeconomic Processes) (Moscow: Izograf, 1997)
8. Samuelson P A *Foundations of Economic Analysis* (Cambridge, Mass.: Harvard Univ. Press, 1947)
9. Allen R G D *Mathematical Economics* (London: Macmillan, 1956) [Translated into Russian (Moscow: IL, 1963)]
10. Hicks J R *Value and Capital*: *An Inquiry Into Some Fundamental Principles of Economic Theory* 2nd ed. (Oxford: Clarendon Press, 1946) [Translated into Russian (Moscow: Progress, 1988)]
11. Allais M *Economics as a Science* (Genève: Graduate Institute of International Studies, 1968) [Translated into Russian (Moscow: Nauch.-Izd. Tsentr 'Nauka dlya Obshchestva': Izd. Tsentr Ross. Gumanitarnogo Univ., 1995)]
12. Schumpeter J A *The Theory of Economic Development* (Cambridge, Mass.: Harvard Univ. Press, 1934) [Translated into Russian (Moscow: Progress, 1982)]
13. Saviotti P P, Mani G S *J. Evol. Econ.* **5** 369 (1995)
14. Silverberg G, Verspagen B, in *Evolution und Selbstorganisation in der Ökonomie* (Selbstorganisation. Jahrbuch für Komplexität in den Natur-Sozial- und Geisteswissenschaften, Bd. 9, Hrsg. F Schweitzer, G Silverberg) (Berlin: Duncker & Humbolt, 1998) s. 239
15. Maevskiĭ V I *Vvedenie v Evolyutsionnuyu Ekonomiku* (Introduction to Evolutionary Economics) (Moscow: Yaponiya Segodnya, 1997)
16. Chernavskiĭ D S, Starkov N I, Shcherbakov A V "Dinamicheskaya model' zakrytogo obshchestva" ("Dynamical model of a closed

society''), Preprint No. 36 of P N Lebedev Physics Institute (Moscow: FIAN, 1999); *Mat. Model.* **13** (11) 97 (2001)
17. Chernavskiĭ D S, Starkov N I, Shcherbakov A V ''Bazovaya dinamicheskaya model' ekonomiki Rossii'' (''The base dynamical model of Russia's economy''), Preprint No. 1 of P N Lebedev Physics Institute (Moscow: FIAN, 2000)
18. Petrov A A, Pospelov I G, Shananin A A *Opyt Matematicheskogo Modelirovaniya Ekonomiki* (A Study in Mathematical Modeling of Economics) (Moscow: Energoatomizdat, 1996)
19. Polterovich V M ''Institutsional'nye lovushki i ekonomicheskie reformy'' (''Institutional traps and economic reform''), Preprint 98/004 of Russian Economic School (Moscow: RÉSh, 1998); *Ekonom. Mat. Metody* **35** (2) 3 (1999)
20. Ivanov Yu N, Tokarev V V, Uzdemir A P *Matematicheskoe Opisanie Elementov Ekonomiki* (Mathematical Description of the Economy Elements) (Moscow: Nauka, 1994)
21. Saviotti P P *J. Evol. Econ.* **11** 119 (2001)
22. Samuelson P A *Economics* (New York: McGraw-Hill, 1980) [Translated into Russian (Moscow: NPO 'Algon', Mashinostroenie, 1993)]
23. Fischer S, Dornbusch R, Schmalensee R *Economics* 2nd ed. (New York: McGraw-Hill, 1988) [Translated into Russian (Moscow: Delo, 1993)]
24. Blaug M *Economic Theory in Retrospect* 4th ed. (Cambridge: Cambridge Univ. Press, 1985) [Translated into Russian (Moscow: Delo, 1994)]
25. Chernavskiĭ D S et al. *Izv. Vyssh. Uchebn. Zaved. Prikl. Nelineĭnaya Dinamika* **4** (3) 67 (1996)
26. Chernavskii D S, Starkov N I, Shcherbakov A V *J. Moscow Phys. Soc.* **9** 89 (1999)
27. Chernavskii D S et al. ''The dynamics of the economic society structure'', in *Mathematical Models of Non-Linear Excitations, Transfer, Dynamics, and Control in Condensed Systems and Other Media* (Eds L A Uvarova, A E Arinstein, A V Latyshev) (New York: Kluwer Acad./Plenum Publ., 1999) p. 103
28. Aĭvazyan S A *Ekonom. Mat. Metody* **33** (3–4) 74 (1997)
29. Malinetskiĭ G G, in *Upravlenie Riskom*: *Risk*. *Ustoĭchivoe Razvitie*. *Sinergetika* (Risk Management: Risks. Sustainable Development. Synergetics) (Eds I M Makarov et al.) (Moscow: Nauka, 2000) p. 125
30. Peters E E *Chaos and Order in the Capital Markets* 2nd ed. (New York: Wiley, 1996) [Translated into Russian (Moscow: Mir, 2000) p. 130]
31. Chernavskiĭ D S, Shcherbakov A V, Suslakov B A *Sotsial'no-ekonomicheskiĭ Byulleten'* (Socioeconomic Bulletin) (Moscow: Izd. Sotsial'no-Tekhnologicheskogo Inst., 1999)
32. Chernavskiĭ D S, Shcherbakov A V, Suslakov B A *Sotsial'no-ekonomicheskiĭ Byulleten'* (Socioeconomic Bulletin) (Moscow: Izd. Sotsial'no-Tekhnologicheskogo Inst., 2001)
33. *Rodo Tokay Eran* (Review of Labor Policy in Japan) (in Japanese) (Tokyo, 1988) p. 185
34. Braginsky V B *Priroda* (12) 3 (2000) [*Nature* (Moscow) (12) 3 (2000)]
35. Chernavskiĭ D S et al. *Ekonom. Mat. Metody* **34** (2) 44 (1998)
36. Voronovitskiĭ M M *Ekonom. Mat. Metody* **33** (2) 109 (1997)
37. Chernavskiĭ D S et al. *Zakonodatel'stvo Ekonom.* (7/8) 8 (1995)
38. Romanovskiĭ Yu M, Stepanova N V, Chernavskiĭ D S *Matematicheskaya Biofizika* (Mathematical Biophysics) (Moscow: Nauka, 1984)
39. Poston T, Stewart I *Catastrophe Theory and Its Applications* (London: Pitman, 1978) [Translated into Russian (Moscow: Mir, 1980)]
40. *Sotsial'no-ekonomicheskoe Polozhenie Rossii. Janvar' 2000 goda* (The Socioeconomic State of Russia. January 2000) (Moscow: Goskomstat RF, 2000)
41. *Sotsial'no-ekonomicheskoe Polozhenie Rossii. Janvar' 2001* (The Socioeconomic State of Russia. January 2001) (Moscow: Goskomstat RF, 2001)
42. *The Collected Writings of John Maynard Keynes* 30 vols (London: Macmillan, 1971–1989) [Partial Russian translation: *Izbrannye Proizvedenia* (Moscow: Ekonomika, 1993)]
43. Fridman M *Esli by Den'gi Zagovorili*... (If Money Could Speak...) (Moscow: Delo, 1999)
44. *Krakh Dollara* (Crash of the Dollar) A Collection of Articles (Compiled by A A Nagornyĭ) (Moscow: Izd. N E Chernysheva, 2001)
45. Feĭnberg E L *Dve Kul'tury*: *Intuitsiya i Logika v Iskusstve i Nauke* (Two Cultures: Intuition and Logic in Art and Science) (Moscow: Nauka, 1992) [Expanded German edition: *Zwei Kulturen* (Berlin: Springer-Verlag, 1998)]; *Voprosy Filosofii* (7) 54 (1997)