Homework 05: Linear and nonlinear autonomous dynamical systems

1. Consider the dynamical system

$$\begin{cases} \mathrm{d}x_1/\mathrm{d}t = \mu x_1 - x_2 \\ \mathrm{d}x_2/\mathrm{d}t = x_1 + (\mu + 1)x_2 \end{cases}$$

   (a) Show that $(x_1, x_2) = (0, 0)$ is the unique fixed point of the system for all $\mu \in \mathbb{R}$.

   (b) Show that if $2\mu + 1 < 0$, then $(x_1, x_2) = (0, 0)$ is an asymptotically stable fixed point.

   (c) What happens when $2\mu + 1 = 0$?

2. Consider the dynamical system

$$\begin{cases} \mathrm{d}x_1/\mathrm{d}t = ax_1 + bx_2 \\ \mathrm{d}x_2/\mathrm{d}t = ax_2 \end{cases}$$

   with $a \neq 0$ and $b \neq 0$. Show that $(x_1, x_2) = (0, 0)$ is a degenerate node and determine its stability.

3. Find and classify the fixed points for the following dynamical system

$$\begin{cases} \mathrm{d}x_1/\mathrm{d}t = x_2 + x_1 - x_1^3 \\ \mathrm{d}x_2/\mathrm{d}t = -x_2 \end{cases}$$

4. Show that the following dynamical system is conservative and then find and classify all fixed points

$$\begin{cases} \mathrm{d}x/\mathrm{d}t = x - xy \\ \mathrm{d}y/\mathrm{d}t = 3xy - 3y \end{cases}$$

5. Show that the following dynamical system is reversible and then find and classify all fixed points

$$\begin{cases} \mathrm{d}x_1/\mathrm{d}t = -x_2 \\ \mathrm{d}x_2/\mathrm{d}t = x_1 \cos x_1 \end{cases}$$

6. Show that the following dynamical system does not have any closed trajectories

$$\begin{cases} \mathrm{d}x_1/\mathrm{d}t = x_1^4 + x_2^2 \\ \mathrm{d}x_2/\mathrm{d}t = (x_1 - 1)(x_2 - 3) \end{cases}$$