

## Intermediate Algebra review

### ▼ Basic Identities

$$\begin{aligned}(a+b)^2 &= a^2 + 2ab + b^2 \\ (a-b)^2 &= a^2 - 2ab + b^2\end{aligned}$$

Perfect squares

$$(a+b)(a-b) = a^2 - b^2$$

Square Difference

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$\begin{aligned}(a+b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\ (a-b)^3 &= a^3 - 3a^2b + 3ab^2 - b^3\end{aligned}$$

Perfect cube

$$\begin{aligned}a^3 + b^3 &= (a+b)(a^2 - ab + b^2) \\ a^3 - b^3 &= (a-b)(a^2 + ab + b^2)\end{aligned}$$

Sum/Diff of  
cubes

Note that:

$$(-a+b)^2 = (b-a)^2$$

$$(-a-b)^2 = -(a+b)^2 = (a+b)^2$$

$$(-a+b)^3 = (b-a)^3$$

$$(-a-b)^3 = -(a+b)^3$$

## EXERCISES

① Simplify the expressions:

a)  $(2x+3)^2$

b)  $(-5x+2)^2$

c)  $(3-2x)^2$

d)  $(x^2+2x+3)^2$

e)  $(2x+3)(2x-3)$

f)  $(x^2+x)(x^2-x)$

g)  $(x^2+3x+2)^2$

h)  $(x^3+5x-2)^2$

② Simplify the following expressions

a)  $(x+1)^3 + (2x-1)^2$

b)  $(x^2-2x)^3 + x(-x+2)^2$

c)  $2(2x-1)^2 - 3(x+2)(-x+2) - (-x+2)^2$

d)  $(x^3+2)^2 - (x^3-2)(x^3+2) + (1-2x^3)^2$

e)  $(2x+1)^3 + (2x-1)^3$

f)  $(3x-2)^3 - (3x+2)^3$

► Fast multiplication:  $(x+a)(x+b) = x^2 + (a+b)x + ab$

③ Simplify the following:

a)  $(x+2)(x+3)$

b)  $(x-3)(x+4)$

c)  $(x+5)(x+7)$

d)  $(x-3)(x-6)$

e)  $(x+2)(x-3)(x+1)$

f)  $(x-3)(x-4)(x-2)$

## ▼ Factoring

### Case 1: Common Factors.

④ Factor the following:

a)  $(x^3 + 2x^2)(x^2 - x)$

b)  $(2x+1)^2(3x-2) - (x-4)(2x+1) - (2x+1)^2$

c)  $(3x-2)^3(x+2) + (2x+1)^2(3x-2)^2$

d)  $3(x-1)(x-2)^2 - (x-1)^2(2-x) + 2(1-x)(x-1)$

e)  $2(2x+1)(3-2x)^2 + (1+2x)^2(2x-3)^3$

f)  $(5x+3)^4(2x+3)^3 + 3(5x+3)^3(2x+3)^4$

### Case 2: Difference of squares

$$\boxed{a^2 - b^2 = (a+b)(a-b)}$$

⑤ Factor the following:

a)  $(2x+3)^2 - (4x-1)^2$

b)  $(4x^2+3x+3)^2 - (3-4x^2)^2$

c)  $28(x+3)^2 - 7(1-2x)^2$

d)  $(2x^2-8) - (4-2x)^3$

e)  $(3x-6)^2 - 2(x^2-4) - 5(4-x^2)$

f)  $(x^2-16)^2 + 4(x+4)^2$

Case 3: Perfect Square

$$\boxed{a^2 \pm 2ab + b^2 = (a \pm b)^2}$$

↕ → 4th-order factorization

$$\begin{aligned} a^4 + a^2b^2 + b^4 &= a^4 + \underline{2a^2b^2} + b^4 - \underline{a^2b^2} = \\ &= (a^2 + b^2)^2 - a^2b^2 = \\ &= [(a^2 + b^2) - ab][(a^2 + b^2) + ab] \end{aligned}$$

### EXERCISES

⑥ Factor the expression:

- a)  $(x+1)^2 + 6(x+1) + 9$
- b)  $(2x-3)^2 - 4(2x-3) + 9$
- c)  $-8(x+3) + 1 + 16(x+3)^2$
- d)  $x^4 + x^2 + 1$
- e)  $4x^4 - 21x^2y^2 + 9y^4$
- f)  $16x^4 + 4$
- g)  $4a^4 - 13a^2 + 1$
- h)  $4x^4 - 37x^2y^2 + 9y^4$
- i)  $9x^8 - 15x^4 + 1$

$$\text{Case 4 : } a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Other cases:

$$a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)$$

$$= (a-b)(a+b)(a^2 + b^2)$$

$$a^6 - b^6 = (a^3 - b^3)(a^3 + b^3)$$

$$= (a-b)(a^2 + ab + b^2)(a+b)(a^2 - ab + b^2)$$

etc.

### EXERCISES

⑦ Factor the expressions:

a)  $8x^3 - 27$

b)  $27(x+1)^3 - 1$

c)  $8(x+1)^3 + (x-1)^3$

d)  $3x^4 - 3$

e)  $ab^4 - a^4b$

f)  $40x^4 - 5x$

g)  $3(x+1)^3 + 81(x-1)^3$

h)  $16ax^4 - 81(x-1)^4 a$

i)  $64x^6 - 1$

⑧ Factor the expressions by grouping:

a)  $x^3 + y^3 + x^2 - y^2$

b)  $x^2 + 2xy + y^2 + x^3 + y^3$

c)  $x^3 - y^3 - 3ax + 3ay$

d)  $xy(x+y) + y^2(x+y) - x^3 - y^3$

e)  $x^3 - xy(x-y) - y^3$

f)  $a(x^4 - 1) + bx(x^2 - 1)$

g)  $(x^3 - y^3) - (x^2 - y^2) - (x - y)^2$

h)  $a^3 + b^3 - a - b - a^2b - ab^2$

i)  $3(x^4 - 16) + x(x^3 - 27)$