Review of equations

1) <u>The linear equation</u> $ax+b=0$

a) If $a \neq 0$, there is a <u>unique solution</u>

$$x = -b/a$$

b) If $a=0$ and $b \neq 0$ then solutions do not exist.

c) If $a=0$ and $b=0$ then every number is a solution.

2) <u>The quadratic equation</u> $ax^2+bx+c=0$

First calculate the discriminant

$$\Delta = b^2 - 4ac$$

Then

a) If $\Delta > 0 \Rightarrow$ Two solutions $x_{1,2} = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

b) If $\Delta = 0 \Rightarrow$ One solution $x = -\dfrac{b}{2a}$

c) If $\Delta < 0 \Rightarrow$ No REAL solution.

Sign Charts Review

We use sign charts to determine the intervals where a function is positive or negative.

1) For $f(x) = ax+b$ with $a \neq 0$.

x		$-b/a$	
$f(x)$	$-$	0	$+$

if $a > 0$

x		$-b/a$	
$f(x)$	$+$	0	$-$

if $a < 0$

- Note that $f(x) = (ax+b)^{2n}$ is always POSITIVE and $f(x) = (ax+b)^{2n+1}$ has the SAME SIGN as $f(x) = ax+b$.
- For product of linear factors we build a sign chart for every factor and for the function itself.

<u>example</u> : $f(x) = (2x+1)(x-2)^3(x+1)^2$

- Find zeroes: $-1/2,\ 2,\ -1$
- Sort zeroes: $-1,\ -1/2,\ 2$
- Make sign chart:

x		-1		$-1/2$		2	
$2x+1$	$-$		$-$	0	$+$		$+$
$x-2$	$-$		$-$		$-$	0	$+$
$x+1$	$-$	0	$+$		$+$		$+$
$f(x)$	$-$	0	$+$	0	$-$	0	$+$

- 1 Place factors / zeroes on sign chart
- 2 Place the 0 zero markers in the correct locations
- 3 Enter signs for each factor.
- 4 Multiply signs to obtain sign for factor.

<u>example</u> : $f(x) = (2-x)(x+1)^2(x+3)^3$

Zeroes: $2,\ -1,\ -3$

Sort: $-3,\ -1,\ 2$

x		-3		-1		2		
$2-x$	$+$		$+$		$+$	0	$-$	
$(x+1)^2$	$+$		$+$	0	$+$		$+$	← Always positive
$(x+3)^3$	$-$	0	$+$		$+$		$+$	← Same sign as without the power 3.
$f(x)$	$-$	0	$+$	0	$+$	0	$-$	

2) <u>For $f(x) = ax^2 + bx + c$</u> with $a \neq 0$

The sign depends on a and $\Delta = b^2 - 4ac$

a) If $\Delta > 0 \Leftrightarrow f(x)$ has the same sign as a outside of the two zeroes x_1, x_2 and the opposite sign between the zeroes

$a > 0$

x		x_1		x_2	
$f(x)$	$+$	0	$-$	0	$+$

$a < 0$

x		x_1		x_2	
$f(x)$	$-$	0	$+$	0	$-$

b) If $\Delta = 0 \Leftrightarrow f(x)$ has the same sign as a when $x \neq -b/2a$

$a > 0$

x		$-b/2a$	
$f(x)$	$+$	0	$+$

$a < 0$

x		$-b/2a$	
$f(x)$	$-$	0	$-$

c) If $\Delta < 0 \Leftrightarrow f(x)$ has the same sign as a for all $x \in \mathbb{R}$.

$a > 0$

x	
$f(x)$	$+$

$a < 0$

x	
$f(x)$	$-$

↳ We see that $f(x)$ always has the same sign as a EXCEPT when $\Delta > 0$ (two zeroes x_1, x_2) and x is BETWEEN the two zeroes.

- For n integer, $n>0$:

a) $f(x) = (ax^2+bx+c)^{2n}$ is ALWAYS positive

b) $f(x) = (ax^2+bx+c)^{2n+1}$ has the same sign as $g(x) = ax^2+bx+c$.

<u>example</u>: $f(x) = (x+1)(x^2+x-3)$.

$$\left.\begin{aligned} \text{For } g(x) &= x^2+x-3 \\ \Delta &= 1^2-4\cdot 1\cdot(-3) = \\ &= 1+12 = 13 \end{aligned}\right\} \Rightarrow x_{1,2} = \frac{-1\pm\sqrt{13}}{2} = \begin{cases} \dfrac{-1-\sqrt{13}}{2} \\ \dfrac{-1+\sqrt{13}}{2} \end{cases}$$

zeroes: $\dfrac{-1-\sqrt{13}}{2}$, -1, $\dfrac{-1+\sqrt{13}}{2}$

x		$(-1-\sqrt{13})/2$		-1		$(-1+\sqrt{13})/2$	
$x+1$	$-$		$-$	0	$+$		$+$
x^2+x-3	$+$	0	$-$		$-$	0	$+$
$f(x)$	$-$	0	$+$	0	$-$	0	$+$

- To compare the zeroes, argue as follows:

$$\underline{\underline{\frac{-1-\sqrt{13}}{2}}} < \frac{-1-\sqrt{9}}{2} = \frac{-1-3}{2} = -2 < \underline{\underline{-1}}$$

$$\underline{\underline{-1}} < -\frac{1}{2} < \underline{\underline{\frac{-1+\sqrt{13}}{2}}}$$

- If you have a fraction, then the zeroes of the denominator are points of singularity of the function:

<u>example</u>: $f(x) = \dfrac{x-1}{x^2+5x+6}$

- For $\left.\begin{array}{l} g(x) = x^2+5x+6 \\ \Delta = 5^2 - 4 \cdot 1 \cdot 6 = \\ \quad = 25 - 24 = 1 \end{array}\right\} \Rightarrow x_{1,2} = \dfrac{-5 \pm 1}{2} = \begin{cases} -2 \\ -3 \end{cases}$

Zeroes: $-3, -2, +1$

x		-3		-2		1	
$x-1$	$-$		$-$		$-$	0	$+$
x^2+5x+6	$+$	0	$-$	0	$+$		$+$
$f(x)$	$-$	$\nparallel$	$+$	$\nparallel$	$-$	0	$+$

singular points.

- The expression $\exp(f(x)) = e^{f(x)} > 0$ is ALWAYS positive and NEVER zero

<u>example</u> : $f(x) = e^x (x-2)^2 (x+1)^3$

Zeroes: $-1, 2$

x		-1		2	
e^x	+		+		+
$(x-2)^2$	+		+	0	+
$(x+1)^3$	−	0	+		+
$f(x)$	−	0	+	0	+