

## BRIEF REVIEW OF SETS

### ● Definitions

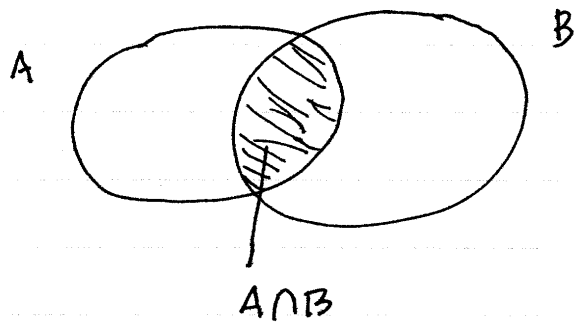
- A set A is a collection of elements x. An element can be a number, a point, or another set.
- A set can be defined by listing its elements:  
e.g.  $A = \{2, 3, 5, 9, 12\}$
- Special sets:
  - a)  $\mathbb{R}$  = the set of all real numbers.
  - b)  $\emptyset = \{\}$  = the empty set (it has no elements).
- Notation:
  - a)  $x \in A$  : x is an element of A (x belongs to A)
  - b)  $x \notin A$  : x is not an element of A
  - c)  $A = B$  : A and B have the same elements
  - d)  $A \subseteq B$  : The elements of A all also belong to B.

### ● Set operations

Let  $A, B$  be two sets. They can be combined into defining a new set via the following operations:

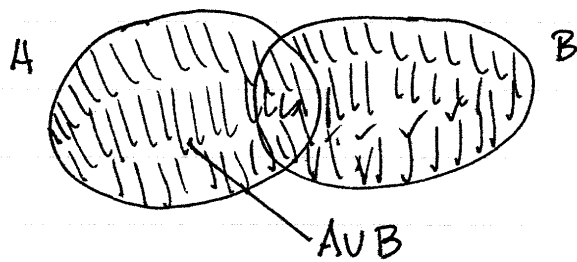
a) Intersection :  $C = A \cap B$

C has all the elements that belong to both sets A and B.



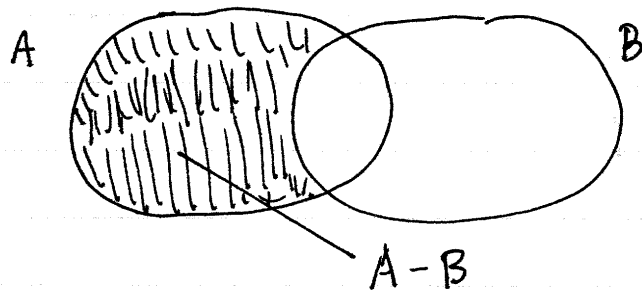
b) Union :  $C = A \cup B$

C has all the elements of A and all the elements of B



c) Difference :  $C = A - B$

C has all the elements of A except for any elements of A that also belong to B.



### EXAMPLE

For  $A = \{1, 2, 3, 5, 9, 10\}$  and  $B = \{3, 4, 5, 10, 11\}$ ,  
evaluate  $A \cup B$ ,  $A \cap B$ ,  $A - B$ ,  $B - A$ ,  $(A - B) \cap (B - A)$

Solution

$$\begin{aligned} A \cup B &= \{1, 2, 3, 5, 9, 10\} \cup \{3, 4, 5, 10, 11\} \\ &= \{1, 2, 3, 4, 5, 9, 10, 11\} \end{aligned}$$

$$\begin{aligned} A \cap B &= \{1, 2, 3, 5, 9, 10\} \cap \{3, 4, 5, 10, 11\} \\ &= \{3, 5, 10\} \end{aligned}$$

$$\begin{aligned} A - B &= \{1, 2, 3, 5, 9, 10\} - \{3, 4, 5, 10, 11\} \\ &= \{1, 2, 9\} \end{aligned}$$

$$\begin{aligned} B - A &= \{3, 4, 5, 10, 11\} - \{1, 2, 3, 5, 9, 10\} \\ &= \{4, 11\} \end{aligned}$$

$$(A - B) \cap (B - A) = \{1, 2, 9\} \cap \{4, 11\} = \emptyset.$$

## EXERCISES

- ① Identify the following statements as TRUE or FALSE:
- |                              |   |   |
|------------------------------|---|---|
| a) $3 \in \{1, 2, 4\}$       | e) $3 \notin \mathbb{R}$                  | i) $\{2, 4, 6\} \subseteq \{1, 2, 4, 5\}$ |
| b) $5 \in \{2, 5, 6\}$       | f) $\sqrt{2} \notin \emptyset$            | j) $\emptyset \subseteq \mathbb{R}$       |
| c) $2 \notin \{1, 3, 7, 9\}$ | g) $\sqrt{5} \in \emptyset$               | k) $\mathbb{R} \subseteq \mathbb{R}$      |
| d) $5 \in \mathbb{R}$        | h) $\{1, 2, 5\} \subseteq \{1, 2, 3, 5\}$ | l) $\mathbb{R} \subseteq \emptyset$       |

- ② Evaluate the sets  $A \cap B$ ,  $A \cup B$ ,  $A - B$ , and  $B - A$ , with the sets  $A$  and  $B$  defined as follows:

- $A = \{1, 2, 3, 5\}$  and  $B = \{2, 4, 6\}$
- $A = \{2, 3, 5, 9\}$  and  $B = \{3, 5\}$
- $A = \{3, 5, 8\}$  and  $B = \{2, 4, 6\}$
- $A = \emptyset$  and  $B = \{1, 3, 7\}$
- $A = \emptyset$  and  $B = \mathbb{R}$
- $A = \mathbb{R}$  and  $B = \mathbb{R}$
- $A = \emptyset$  and  $B = \emptyset$ .

- ③ Evaluate the set  $D = (A \cap B) - C$  with the sets  $A$ ,  $B$ , and  $C$  defined as:

- $A = \{1, 3, 8, 9\}$ ,  $B = \{2, 3, 4, 8\}$ , and  $C = \{1, 3, 4\}$
- $A = \{2, 3, 4, 5\}$ ,  $B = \{4, 5, 7\}$ , and  $C = \{4, 5, 6\}$
- $A = \{1, 2, 3, 4\}$ ,  $B = \{2, 4, 6\}$ , and  $C = \{1, 2, 3\}$

④ Evaluate the set  $D = A \cup (B - C)$  with the sets  $A, B,$  and  $C$  defined as:

a)  $A = \{1, 2\}, B = \{2, 3, 5\},$  and  $C = \{1, 2, 3\}$

b)  $A = \{2, 3, 4\}, B = \{3, 4, 5\},$  and  $C = \{1, 2, 5\}$

c)  $A = \{2, 3, 7\}, B = \emptyset,$  and  $C = \{1, 2\}$

d)  $A = \emptyset, B = \{2, 4\},$  and  $C = \{4, 5\}$

e)  $A = \{1\}, B = \{2, 5, 7\},$  and  $C = \emptyset.$