12/4/2019 Lecture 29, Dimension of span(B)

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- returned exam 3

Given $(V, +, \cdot)$ vector space: Let $B \subseteq V$,

$$\boxed{B \text{ linearly independent} \Rightarrow \dim(\text{span}(B)) = |B|}$$

Theorem: For $p < n$ and:
$$\begin{cases} \{x_1, x_2, \dots, x_p\} \text{ linearly independent} \\ \forall u \in \{x_{p+1}, \dots, x_n\} : \{x_1, \dots, x_p, u\} \text{ linearly dependent} \end{cases}$$

$$\Rightarrow \text{span}\{x_1, \dots, x_n\} = \text{span}\{x_1, \dots, x_p\}$$

Proof since: $\{x_1, \dots, x_p\} \subseteq \{x_1, \dots, x_n\} \Rightarrow$
$$\Rightarrow \text{span}\{x_1, \dots, x_p\} \subseteq \text{span}\{x_1, \dots, x_n\} \quad (1)$$

- Preliminary argument - let $a \in \mathbb{N}$ with $1 \leq a \leq n-p$ and consider x_{p+a}

It follows that $\begin{cases} \{x_1, \dots, x_p\} \text{ linearly independent} \\ \{x_1, \dots, x_p, x_{p+a}\} \text{ linearly dependent} \end{cases} \Rightarrow$

$$\Rightarrow x_{p+a} \in \text{span}\{x_1, \dots, x_p\} \Rightarrow$$

$$\Rightarrow \exists \mu_{a1}, \mu_{a2}, \dots, \mu_{ap} \in \mathbb{R} : x_{p+a} = \sum_{b=1}^{p} \mu_{ab} x_b \dots$$

cont.

12/4/2019 Dimension of span(B)

Prop: Let $B = \{x_1, \dots, x_p\} \subseteq V$ and assume that B is basis of
then...

$$u \in V \iff \{x_1, \dots, x_p, u\} \text{ linearly dependent}$$

Proof: $(\Rightarrow)$: Assume that $u \in V$, then

$u \in V \Rightarrow u \in span(B)$ ← [via $span(B) = V$]

$\Rightarrow u \in span(\{x_1, \dots, x_p\})$

$\Rightarrow \{x_1, \dots, x_p, u\}$ linearly dependent

$(\Leftarrow)$: Assume that $\{x_1, \dots, x_p, u\}$ linearly dependent, sinc
$\{x_1, \dots, x_p\}$ basis of V $\Rightarrow$

$\Rightarrow \begin{cases} \{x_1, \dots, x_p\} \text{ linealy independent} \\ \{x_1, \dots, x_p, u\} \text{ linearly dependent} \end{cases} \Rightarrow$

$\Rightarrow u \in span(\{x_1, \dots, x_p\}) \Rightarrow u \in span(B)$

$\Rightarrow u \in V$ (via $V = span(B)$)

Example $V = span\{x_1, x_2, x_3, x_4\}$ $x_1 = (1, 2, 0, 3)$ $x_2 = (2, 0, 3, 1)$
$x_3 = (-1, 2, -3, 2)$ $x_4 = (3, -2, 6, -1)$

Find $\dim(V)$ condition$(a, b, c, d) \in V$

Solution ▸ check $\{x_1, x_2, x_3, x_4\}$ for linear dependence/independen

$$\det[x_1\ x_2\ x_3\ x_4] = \begin{vmatrix} 1 & 2 & 0 & 3 \\ 2 & 0 & 3 & 1 \\ -1 & 2 & -3 & 2 \\ 3 & -2 & 6 & -1 \end{vmatrix} = \dots = 0 \Rightarrow$$

transpose either works

$\Rightarrow \{x_1, x_2, x_3, x_4\}$ dependent

▸ now check $\{x_1, x_2, x_3\}$

↓ cont.