8/26/2019 Lecture 1 - Lin Alg

Syllabus comments

Proof coverage very brief

★ Exams indicated on syllabus ★

weeks 7 - weeks 10 : proposal project

(4390 m
class n

Review example: ~~interstellar travel, sorting algorith~~
check website for more, needs to be Lin A

★ Sample paper & LaTeX download on websi

New Octave = Matlab, open source
Maxima, can verify answers to HW

| Math | Logic & Sets

a) Propositions (boolean)
b) Sets
c) Quantified statements (combines a & b)

▶ **Propositions:**

proposition p is a statement that is true or false

$p: 3x+1=5$ (true/false depending on x)

$p: 42$ (not a proposition, is an expression)

▶ **Operations:** 6 of them

1) Conjunction: $p \wedge q$ (p and q is true)

p is true and q is true

(system of equations) $p: 3x+y=1 \wedge x+2y=5$ $p: \{ 3x$ $\{ x$

6/2019 Operations

2) Disjunction: $p \vee q$: at least one is true
P OR Q is true

$p: x > 3 \vee x < 2$
$p: x = 1 \vee x = 2$

3) Exclusive Disjunction: $p \underline{\vee} q$: Either p is true or q is true
Both cannot be true / false
opposite truth values

$p: x > 0 \underline{\vee} x = 0 \underline{\vee} x < 0$, stronger claim

4) Negation: $\overline{p}$: p is false

$\overline{x = 0} \iff x \neq 0$

$\overline{x > 3} \iff x \leq 3$

notation: $\overline{x > 3}$ can be written as
$\neg(x > 3)$
↳ friendly computer syntax

5) Implication: $p \Rightarrow q$: if p, then q / p implies q
if p is true then q is true

$\overline{p \wedge \overline{q}}$ ☆ it is not true that p is true and q is false

something to ponder
e.g. $x > 3 \Rightarrow x > 1$, true for all x

$x > 1 \Rightarrow x > 3$, false if $1 < x < 3$

☆ implications should be true for all values of x

8/28/2019 Lecture 2 - Operations continued

- Types of Propositions ~~(operations)~~ (operations)

1) Conjunction $p \wedge q$ "p and q"

2) Disjunction $p \vee q$ "p or q"

3) Exclusive disjunction $p \underline{\vee} q$ "either p or q BUT not both"

4) Negation $\bar{p}$; $\neg p$ "p is false"

5) Implication $p \Rightarrow q$ "p implies q"

"if p is true then q

can be displayed as $\overline{p \wedge \bar{q}}$

"It is not true that p is and q is false."

p	q	$p \Rightarrow q$
true	true	TRUE
true	false	FALSE
false	true	TRUE
false	false	TRUE

table shows what $p \Rightarrow q$ truth value is when truth values of p and q are known.

$x > 3 \Rightarrow x > 1$: if x is greater than 3, x is greater th true for all values of x

$\underset{(p)}{x > 1} \Rightarrow \underset{(q)}{x > 3}$: if $x = 2$, (p) is true, but (q) is fa false for all x between 1 and 3

28/2019 <u>Operations with propositions</u>

6) Equivalence $p \Leftrightarrow q$: "p is true <u>if and only if</u> q is true"

also shown as $(p \Rightarrow q) \wedge (q \Rightarrow p)$

"p is equivalent to q"

both p and q have same truth values, $T \wedge T$ or $F \wedge F$

$$x^2 - 1 = 0 \Leftrightarrow (x+1)(x-1) = 0$$
$$\Leftrightarrow (x-1=0) \vee (x+1=0)$$
$$\Leftrightarrow x = 1 \vee x = -1$$

one of these numbers is the solution

Sets

a set A is an <u>unordered</u> collection of <u>elements.</u>

elements: can be numbers, vectors

sets can be elements of new sets

<u>Notation of sets</u>

a) Finite sets can be defined by listing their elements

$A = \{1, 2, 3, 7, 8\}$

( ) = ordered collection/set

b) Given element x and sets A, B

$x \in A$: x is an element of A

$x \notin A$: x is <u>not</u> an element of A

$A = B$: sets A, B have the same elements

$A \subseteq B$: all elements of A belong to B

} all can evaluate to true or false

almost anything can be an element or a set

8/28/2019 Sets & Notation & Operations

- sets can be defined via belonging condition

$x \in A \iff (x=1 \lor x=2 \lor x=3 \lor x=7$

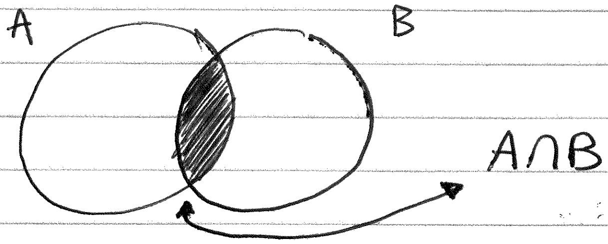
↳ friendlier computer syntax

Set Operations

Given sets A, B, we define:

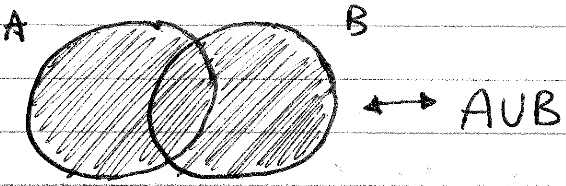
1) Intersection $A \cap B$ $\quad x \in A \cap B \iff (x \in A \land x \in B)$

Can visualize with Venn Diagram



2) Union $A \cup B$ $\quad x \in A \cup B \iff (x \in A \lor x \in B)$

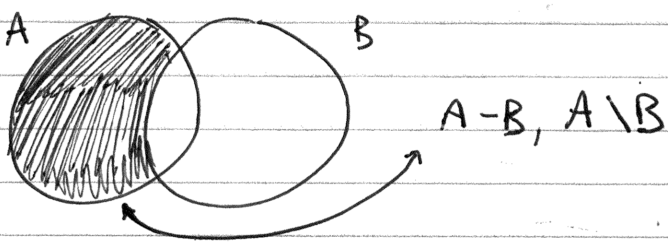
Corresponding Venn Diagram



NOTE: Logical and and or go between statements NOT sets
$\land$ $\lor$
for sets use $\cap$ and $\cup$

3) set difference $A - B$, $A \setminus B$ $\quad x \in A - B \iff (x \in A \land$

Venn Diagram



2019 Number sets & Quantified Statements

Special sets:

$\emptyset = \{\ \} \leftarrow$ empty set $\rightarrow \boxed{x \in \emptyset \Leftrightarrow x \neq x}$

never true so always empty

$\mathbb{N} = \{0, 1, 2, 3, \ldots\} \leftarrow$ set of natural numbers

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} \leftarrow$ set of integers

$\mathbb{Q} \rightarrow$ set of rational numbers ($\mathbb{Z}/\mathbb{N}$ except when $\mathbb{N} = 0$) $\hookrightarrow \mathbb{N} - \{0\}$

$\mathbb{R} \rightarrow$ set of real numbers

$\mathbb{C} \rightarrow$ set of complex numbers (i)

$[n] = \{1, 2, 3, 4, \ldots, n\}$ with $n \in \mathbb{N} - \{0\}$

Quantified statements:

Predicate $p(x) \rightarrow$ a proposition about x whose truth value depends on x

possible that $x = (x_1, x_2, x_3, \ldots, x_n) \leftarrow$ ordered collection of variables

① Universal quantifier $\forall x \in A : p(x)$

"For all $x \in A$, $p(x)$ is true"

written about elements rather than sets, it looks like:

$(\forall x \in \{a, b\} : p(x) \Leftrightarrow (p(a) \wedge p(b))$

$(\forall x \in \{a, b, c\} : p(x) \Leftrightarrow (p(a) \wedge p(b) \wedge p(c))$

good for finite sets
if set is long use set notation

8/28/2019 <u>Quantified Statements</u>

② <u>Existential quantifier</u> $\exists x \in A : p(x)$

"There is at least one $x \in A$ such that $p(x)$ is true"

$(\exists x \in \{a, b\} : p(x) \Leftrightarrow (p(a) \vee p(b))$

$(\exists x \in \{a, b, c\} : p(x) \Leftrightarrow (p(a) \vee p(b) \vee p(c))$