Math 1450.03

Exam 2 solution

Use the trigonometric numbers of known angles (i.e. 30°, 45°, 60°) to evaluate $\sin(15^\circ)$

Solution

$$\sin(15^\circ) = \sqrt{\frac{1-\cos(30^\circ)}{2}} = \sqrt{\frac{1-(\sqrt{3}/2)}{2}} = \sqrt{\frac{2[1-(\sqrt{3}/2)]}{2\cdot 2}}$$

$$= \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

2nd method

$$\sin(15^\circ) = \sin(45^\circ - 30^\circ) = \sin(45^\circ)\cos(30^\circ) - \sin(30^\circ)\cos(45^\circ)$$

$$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{2}(\sqrt{3}-1)}{4}$$

→ Although the answers look different, they are equal

Use known angles to evaluate $\tan(13\pi/12)$. Rationalize any denominators

Solution

$$\tan^2(13\pi/12) = \tan^2(\pi+\pi/12) = \tan^2(\pi/12) = \frac{1-\cos(\pi/6)}{1+\cos(\pi/6)} =$$

$$= \frac{1-\sqrt{3}/2}{1+\sqrt{3}/2} = \frac{2(1-\sqrt{3}/2)}{2(1+\sqrt{3}/2)} = \frac{2-\sqrt{3}}{2+\sqrt{3}} =$$

$$= \frac{(2-\sqrt{3})^2}{(2+\sqrt{3})(2-\sqrt{3})} \Rightarrow$$

$$\Rightarrow \tan(13\pi/12) = \frac{2-\sqrt{3}}{\sqrt{(2+\sqrt{3})(2-\sqrt{3})}} = \frac{2-\sqrt{3}}{\sqrt{2^2-(\sqrt{3})^2}} = \frac{2-\sqrt{3}}{\sqrt{4-3}}$$

$$= \frac{2-\sqrt{3}}{1} = 2-\sqrt{3}.$$

2nd method

$$\tan^2(13\pi/12) = \dots = \frac{(2-\sqrt{3})^2}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{2^2-2\cdot 2\sqrt{3}+(\sqrt{3})^2}{2^2-(\sqrt{3})^2} =$$

$$= \frac{4-4\sqrt{3}+3}{4-3} = 7-4\sqrt{3} \Rightarrow$$

$$\Rightarrow \tan(13\pi/12) = \sqrt{7-4\sqrt{3}}$$

Show that $\cos^2(\pi/4 - a) - \sin^2(\pi/4 - a) = \sin(2a)$

Solution

$$\cos^2(\pi/4 - a) - \sin^2(\pi/4 - a) = \cos(2(\pi/4 - a)) =$$
$$= \cos(\pi/2 - 2a) = \sin(2a)$$

Show that $\dfrac{\sin(2a)+\sin(3a)}{\cos(2a)-\cos(3a)} = \cot(a/2)$

Solution

$$\frac{\sin(2a)+\sin(3a)}{\cos(2a)-\cos(3a)} = \frac{2\sin((2a+3a)/2)\cos((2a-3a)/2)}{2\sin((3a-2a)/2)\sin((3a+2a)/2)} =$$

$$= \frac{2\sin(5a/2)\cos(-a/2)}{2\sin(a/2)\sin(5a/2)} =$$

$$= \frac{\cos(-a/2)}{\sin(a/2)} = \frac{\cos(a/2)}{\sin(a/2)} =$$

$$= \cot(a/2).$$