Math 1450.01

Fall 2014

1st Exam

Evaluate and simplify the following expression and combine as one function: $A = 3\sin(\pi/6) + \cos(\pi/3) - \tan(\pi/3)$

Solution

$$A = 3\sin(\pi/6) + \cos(\pi/3) - \tan(\pi/3) =$$

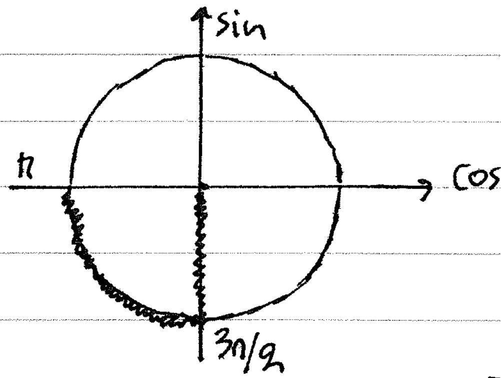
$$= 3 \cdot \frac{1}{2} + \frac{1}{2} - \sqrt{3} = \frac{3+1-2\sqrt{3}}{2} = \frac{4-2\sqrt{3}}{2} =$$

$$= 2 - \sqrt{3}.$$

If $\tan x = 2$ and $\pi < x < 3\pi/2$, then evaluate $A = \sin x + 2$ and combine as fraction.

Solution

We note that $\pi < x < 3\pi/2 \Rightarrow \sin x < 0$ (1)



We also have:

$$\frac{1}{\cos^2 x} = 1 + \tan^2 x = 1 + 2^2 =$$

$$= 1 + 4 = 5 \Rightarrow$$

$$\Rightarrow \cos^2 x = \frac{1}{5} \Rightarrow$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x = 1 - \frac{1}{5} = \frac{5-1}{5} = \frac{4}{5} \overset{(1)}{\Rightarrow}$$

$$\Rightarrow \sin x = -\sqrt{\frac{4}{5}} = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5} \Rightarrow$$

$$\Rightarrow A = \sin x + 2 = \frac{-2\sqrt{5}}{5} + 2 = \frac{-2\sqrt{5} + 2 \cdot 5}{5} = \frac{10 - 2\sqrt{5}}{5}$$

$$= \frac{2(5 - \sqrt{5})}{5}$$

Simplify the following expression and rationalize any denominators

$$A = \frac{\tan(15\pi/4)}{3+\cos(7\pi/3)}$$

Solution

We note that

$$\tan(15\pi/4) = \tan(16\pi/4 - \pi/4) = \tan(4\pi - \pi/4) = \tan(-\pi/4)$$
$$= -\tan(\pi/4) = -1$$

$$\cos(7\pi/3) = \cos(6\pi/3 + \pi/3) = \cos(2\pi + \pi/3) = \cos(\pi/3)$$
$$= 1/2$$

and therefore

$$A = \frac{\tan(15\pi/4)}{3+\cos(7\pi/3)} = \frac{-1}{3+1/2} = \frac{2(-1)}{2(3+1/2)} =$$
$$= \frac{-2}{6+1} = \frac{-2}{7}$$

Show that

$$(\sin x + \cos x + 1)(\sin x + \cos x - 1) = 2\sin x \cos x$$

<u>Solution</u>

$$(\sin x + \cos x + 1)(\sin x + \cos x - 1) = (\sin x + \cos x)^2 - 1 =$$

$$= \sin^2 x + 2\sin x\cos x + \cos^2 x - 1$$

$$= 2\sin x\cos x + (\sin^2 x + \cos^2 x) - 1$$

$$= 2\sin x\cos x + 1 - 1$$

$$= 2\sin x\cos x \qquad \square$$

Graph the following function $f(x) = 1 + \sin\left(\frac{2x+\pi}{5}\right)$

Solution

Solve:

$$\frac{2x+\pi}{5} = \frac{k\pi}{2} \Leftrightarrow 2(2x+\pi) = 5k\pi \Leftrightarrow 4x + 2\pi = 5k\pi \Leftrightarrow$$

$$\Leftrightarrow 4x = 5k\pi - 2\pi \Leftrightarrow 4x = (5k-2)\pi \Leftrightarrow x = \frac{(5k-2)\pi}{4}$$

For $k=0$: $\begin{cases} x_0 = -2\pi/4 = -\pi/2 \\ y_0 = 1+0 = 1 \end{cases}$

For $k=1$: $\begin{cases} x_1 = (5\cdot 1-2)\pi/4 = 3\pi/4 \\ y_1 = 1+1 = 2 \end{cases}$

For $k=2$: $\begin{cases} x_2 = (5\cdot 2-2)\pi/4 = 8\pi/4 = 2\pi \\ y_2 = 1+0 = 1 \end{cases}$

For $k=3$: $\begin{cases} x_3 = (5\cdot 3-2)\pi/4 = 13\pi/4 \\ y_3 = 1-1 = 0 \end{cases}$

For $k=4$: $\begin{cases} x_4 = (5\cdot 4-2)\pi/4 = 18\pi/4 = 9\pi/2 \\ y_4 = 1+0 = 1. \end{cases}$