Math 1450.03

Exam 1 Solution

Fall 2014

Evaluate and simplify the following expression and combine as one fraction

$A = \cos(\pi/4) + 2\tan(\pi/3) - 2$

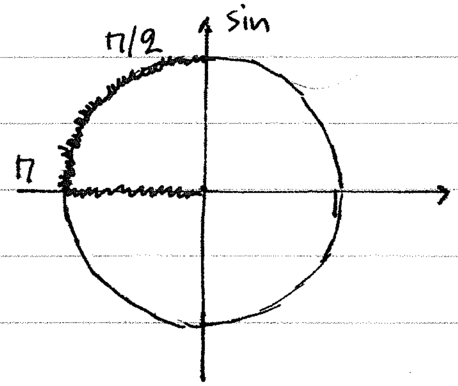
Solution

$$A = \cos(\pi/4) + 2\tan(\pi/3) - 2 =$$

$$= \frac{\sqrt{2}}{2} + 2\sqrt{3} - 2 = \frac{\sqrt{2} + 4\sqrt{3} - 4}{2}$$

If $\tan x = -1/2$ and $\pi/2 < x < \pi$, then evaluate $A = 2 - 3\cos x$ and combine as one fraction.

Solution



We note that

$\pi/2 < x < \pi \Rightarrow \cos x < 0$ (1)

and also that

$$\frac{1}{\cos^2 x} = 1 + \tan^2 x = 1 + \left(\frac{-1}{2}\right)^2 =$$

$$= 1 + \frac{1}{4} = \frac{4+1}{4} = \frac{5}{4} \Rightarrow$$

$$\Rightarrow \cos^2 x = \frac{4}{5} \overset{(1)}{\Rightarrow} \cos x = -\sqrt{\frac{4}{5}} = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5} \Rightarrow$$

$$\Rightarrow A = 2 - 3\cos x = 2 - 3\left(\frac{-2\sqrt{5}}{5}\right) = 2 + \frac{6\sqrt{5}}{5} =$$

$$= \frac{10 + 6\sqrt{5}}{5} = \frac{2(5 + 3\sqrt{5})}{5}$$

Simplify the following expressions and rationalize any denominators

$$A = \frac{\cot(3\pi/4)}{2+\sin(5\pi/6)}$$

Solution

We note that since

$$\cot(3\pi/4) = \cot(\pi-\pi/4) = \cot(-\pi/4) = -\cot(\pi/4) = -1$$

$$\sin(5\pi/6) = \sin(\pi-\pi/6) = (-1)\sin(-\pi/6) = \sin(\pi/6) = 1/2$$

it follows that

$$A = \frac{\cot(3\pi/4)}{2+\sin(5\pi/6)} = \frac{-1}{2+1/2} = \frac{2(-1)}{2(2+1/2)} =$$

$$= \frac{-2}{4+1} = \frac{-2}{5}$$

Show that

$(\sin x+\cos x)^4-(\sin x-\cos x)^4=8\sin x\cos x$

Solution

$$(\sin x+\cos x)^4-(\sin x-\cos x)^4=$$
$$=\left[(\sin x+\cos x)^2-(\sin x-\cos x)^2\right]\left[(\sin x+\cos x)^2+(\sin x-\cos x)^2\right]$$
$$=\left[(\sin x+\cos x)-(\sin x-\cos x)\right]\left[(\sin x+\cos x)+(\sin x-\cos x)\right]$$
$$\times\left[\sin^2 x+2\sin x\cos x+\cos^2 x+\sin^2 x-2\sin x\cos x+\cos^2 x\right]$$
$$=(\sin x+\cos x-\sin x+\cos x)(\sin x+\cos x+\sin x-\cos x)$$
$$\times(2\sin^2 x+2\cos^2 x)$$
$$=(2\cos x)(2\sin x)\cdot 2=8\sin x\cos x$$

Graph the following function: $f(x) = 1 - \sin\left(\frac{3x - \pi}{4}\right)$

Solution

We solve:

$$\frac{3x-\pi}{4} = \frac{k\pi}{2} \Leftrightarrow 2(3x-\pi) = 4k\pi \Leftrightarrow 3x - \pi = 2k\pi \Leftrightarrow$$

$$\Leftrightarrow 3x = 2k\pi + \pi \Leftrightarrow 3x = (2k+1)\pi \Leftrightarrow x = \frac{(2k+1)\pi}{3}$$

For $k=0$: $\begin{cases} x_0 = \pi/3 \\ y_0 = 1 - 0 = 1 \end{cases}$

For $k=1$: $\begin{cases} x_1 = (2\cdot 1+1)\pi/3 = 3\pi/3 = \pi \\ y_1 = 1 - (+1) = 0 \end{cases}$

For $k=2$: $\begin{cases} x_2 = (2\cdot 2+1)\pi/3 = 5\pi/3 \\ y_2 = 1 - 0 = 1 \end{cases}$

For $k=3$: $\begin{cases} x_3 = (2\cdot 3+1)\pi/3 = 7\pi/3 \\ y_3 = 1 - (-1) = 2 \end{cases}$

For $k=4$: $\begin{cases} x_4 = (2\cdot 4+1)\pi/3 = 9\pi/3 = 3\pi \\ y_4 = 1 - 0 = 1 \end{cases}$