

A Russian Teacher in America

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I am a Russian mathematician and teacher. For 20 years I did research and taught students at Moscow University. Now I have moved to the U.S., as have many other Russians. This article is about some of my experiences of teaching both in Russia and America.

RUSSIAN CONTRASTS

Americans' ideas about Russia are as contradictory as Russia itself. For many years Soviet Russia was perceived as 'The Evil Empire'. On the other hand, there was a *Sputnik* movement in America which claimed that the Russian educational system was much better than the American one. Obviously, these images did not fit together. A lot of effort is needed to give the real picture. I am just going to make a few comments to explain my background.

Communist rule in Russia emerged from the collapse of the obsolete Tzarist autocracy, under which most people were deprived of education. Early Communists enthusiastically sang the "*International*" which claimed: "Who was nothing will become everything". Nobody ever knew what it meant exactly, but many were excited. Many Russian revolutionaries sincerely believed that it was their mission to redress all the social injustices immediately, but ignorance crippled all their efforts. A telling example is described in the novel *Chapayev* by the Russian writer Furmanov. The hero Chapayev, a Red Army commander insists on giving an official certificate of competence in medicine to a poorly educated man, naïvely thinking that having such a certificate really makes one a doctor.

Communists made promises that looked very democratic, particularly that children of 'proletarians' would be given unlimited educational opportunities. Children of manual workers and poor peasants really were given privileges to enter all kinds of schools, and professors who gave them bad grades might be accused of anti-revolutionary activity. Only a generation later Russia had thousands of hastily coached engineers and scientists of proletarian descent. One of these 'proletarian scientists,' an academician named Lysenko, gave fantastic agricultural promises which he never kept. However, Lysenko impressed Soviet rulers from Stalin to Khrushchev because they were pseudo-educated also. A

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major branch of biology, namely genetics, was declared a 'bourgeois pseudo-science' because Lysenko was against it.

The ambitions of pseudo-educated 'proletarian scientists', their haughtiness towards bourgeois science, their pretensions of superiority because of having had poor parents and being led by 'the world's truest teaching' (that is Marxism) caused a lot of industrial and ecological disasters. However, Communists never admitted their true causes; all of these disasters were attributed to some 'enemies' sabotage. A number of alleged 'enemies' were arrested and reportedly confessed. Masses of people, although declared 'educated' by that time, believed these reports. But disasters continued, and to explain them away the authorities needed more and more 'enemies'. Meanwhile, Russia became the world leader in wasted resources and polluted environment: *Chernobyl* is just one (but not unique) example.

I was 11 when Stalin died. For many years all Soviet people, especially youngsters, had been indoctrinated that they should never doubt the Communist tenets. All media had been filled with verbose praises to Stalin who was called 'the greatest genius of all times and all peoples'.

However, much of Russian and foreign literature was available, including American authors. Foreign authors were published under the pretext that they 'criticized burgeois society'. Mark Twain, Jack London, Ernest Seton, O. Henry, Edgar Allan Poe, Paul de Kruif, Ernest Hemingway, Ray Bradbury were among my favorite authors.

I vividly remember reading a book about a scientist who proved that insects have no reason; they only have instinct. What he actually proved was that the behavior of insects was effective only in situations usual for them. When the experimenter artificially arranged unusual situations, the insects did the same standard movements although they evidently could not be of any use in that new situation, because it was different from those to which the insects had become accommodated through evolution. I was impressed: I understood that propaganda tried to turn us into some kind of insects. I thought then and think now that it is a most important duty of a teacher of humans to teach them to be humans, that is, to behave reasonably in unusual situations. When I taught in Russia, I was thanked most explicitly for this. But I met a lot of resistance from some of my American undergraduate students especially when I tried to give them something unexpected. On tests they wanted to do practically the same as what they had done before, only with different numerical data. This is why I decided to write this article.

Anti-intellectualism always played a major role in Soviet mentality. 'Intel-ligent' (the Russian word for 'intellectual') was a pejorative term throughout Stalin's rule. Soviet leaders felt free to dictate to scientists and artists. Through-out Soviet rule many of the intellectual strata of society had to emigrate, and today's 'democracy' has not changed this, because most key positions in Russian science are still held by those who made their careers by political means.

I always believed that really good education is the most valuable contribution which intellectuals of a country can make towards its democratization. Remember that The Great French Revolution was prepared by The Age of Enlightenment. It was evident that the worst features of Soviet rule were connected with the power of the pseudo-educated who got their certificates for being ‘proletarians’, but cared only for their careers. Understandably, Soviet authorities always were suspicious about independent thought and real intellectuals.

In return, good teaching, intended to develop real competence of students, always had a flavor of resistance to Soviet authorities, as it involved realism, open-mindedness and critical thinking. When a good mathematics teacher tried to move his students to think independently, he was aware that his real influence went far beyond mathematics: he tried and succeeded to keep alive the critical spirit. Learning recipes without thinking was associated with the Communist tyranny, learning to solve non-trivial problems was associated with independence and criticism. For this reason, for example, George Pólya’s writings on teaching were perceived in Russia as books on open-mindedness and critical thinking rather than just on the teaching of mathematics. We knew that Pólya was not alone: he referred to other scientists, for example to Max Wertheimer’s notion of ‘productive thinking’.

In the years of Khrushchev’s liberalism, some new foreign books also became available in Russia. Russian thinkers read very attentively all the foreign authors they could find. Many valuable ideas came from Americans: authoritarian personality (Theodor Adorno), group pressure (Solomon Asch), obedience to authority (Stanley Milgram). Eric Berne’s *Games People Play* moved us to see which dirty games our rulers played with us. Thomas Kuhn’s book about scientific revolutions was about ideological revolutions for us. Milton Rokeach’s idea of open and closed minds opened our minds. John Holt’s criticism of American schools made us understand that our schools deserved much harsher criticism.

ABOUT MYSELF

My parents belonged to artistic circles, and pressure of censure was a constant theme of conversations. If trimming a tree went too far, they would say with regret: “Look, how we have edited this tree!” Exact sciences provided the greatest available degree of independence from authorities, and my parents spoke with envy about mathematicians who could afford to say the exact truth and even be paid for it rather than punished. They could not guide my study of sciences, but they expected intellectual efforts of me, and it was important.

Later, my school teacher of mathematics, Alexander Shershevsky, helped me a lot. He strived to become a mathematician, but could not obtain a research position because in his student years he had gotten into some political trouble. (The trouble must have been minor, otherwise we would never have seen him again.) I was especially impressed by his responsible attitude to his mission. He

urged me to attend informal classes in mathematics at Moscow University. The main business of these classes was solving non-standard problems. Students were free to drop in and out; using this, I changed several groups until I found a teacher, Alexander Olevsky, whom I liked most. Every year students of Moscow University arranged a competition in solving problems for high school students. Every problem was new and unlike others and demanded a non-trivial idea and a rigorous proof to solve. There were five problems and five hours to solve them. Typically, everyone who solved at least one problem, was rewarded. In this way I got several prizes. This convinced me that I could succeed as a mathematician. When I moved from high school to the Mathematics Department of Moscow University, solving problems naturally led me to research.

From my first year in the university I took it for granted that a competent mathematician should participate in the teaching of mathematics because I had excellent examples to follow. The famous Kolmogorov organized a mathematical college affiliated to Moscow University and I taught there. Academician Gelfand organized a School by Correspondence and I instructed its teachers. In the 'Computer Club' I headed the teaching program.¹ Aleksandrov, Arnold, Boltvansky, Dobrushin, Dynkin, Efimov, Kirillov, Postnikov, Sinai, Tzvetlin, Uspensky, the Yaglom brothers, and other first-class mathematicians were willing to lecture and to communicate with students. A lot of new and original problems from all branches of mathematics and at various levels of difficulty were invented for all kinds of students from young children to graduate students and young professionals. Now, I was among those who invented problems. When I advised Ph.D. and other students, I gave them problems which interested me, and we solved them together.

The main pressure which students put upon teachers was to tell them something new. A vivid example was Leonid Vaserstein (then a student) who would declare in the middle of a talk: "All this is trivial." Taken out of situational context this may seem impolite, but actually this was quite *productive*. He pressed lecturers for more competence. Soon he had to emigrate. (Now he is a professor at Penn State University.) His fate is typical: top officials of Moscow University, very poor scientists but bombastic Communists, used all pretexts (notably anti-Semitism) to get rid of competent young scientists to assure their own positions. Now, they do the same without Communist paraphernalia: a few months ago they elected a notorious hard-liner, Sadovnichy, President of Moscow University.

Whenever the purpose of learning was real competence, it had nothing to do with good standing with the authorities, who were feared and despised by intellectuals. Grades were just a nuisance, like any extraneous control. For example,

¹ Many thinkers were read with interest in our club. Reading Seymour Papert's *Mindstorms* we thought: 'If small children are taught to think with such care in America, they must develop into tremendously competent university students.'

when I taught in the college organized by Kolmogorov, I simply gave an A to every student because all of them deserved A according to average Russian standards and I wanted to save them the trouble of dealing with the authorities. But they knew perfectly well that we expected much more of them than of the average student, and they worked very hard.

Every advanced school, where independent and creative thought was cultivated, became a breeding ground for political dissent. The Mathematics Department of Moscow University was no exception. From time to time there were political clashes there and I took part in them. This caused me problems with the Soviet authorities and eventually led to my emigration.

Most of my 60 publications are in mathematics, the others pertain to education and humanities. Not one article of the latter part was published as I wanted it, because of censure restrictions. Most of them would never be published without willingness of a particular editor to take a certain, well-calculated risk. Whenever I brought an article to the newspaper *Izvestia*, my cautiously courageous editor, Irina Ovchinnikova, exclaimed: "Oh, Andrei, do you really think that this is publishable?" And she had to cross out the most critical statements to save the others.

My research in mathematics could not improve my position in the university because Communist bureaucrats always (and correctly) understood that I would never *solidarize* with them. The adviser of my Ph.D., Ilya Piatetski-Shapiro, emigrated to Israel, and thereby became *persona non grata* for the Soviet establishment; even referring to his papers was not easy. My papers were known abroad, but were not recognized as anything valuable by my supervisor, because I gave him too few chances to appropriate my work. I received several invitations from foreign universities, but the authorities never allowed me to go abroad. Only by chance, I got to Italy in 1989 and decided to accept all the invitations I had, without going back and subjecting myself to the same arbitrariness. From Rome I went to Rutgers University, then to other American universities. Finally, I got a regular position at Incarnate Word College, where I am now, but it would be premature to write about my experiences here.

TEACHING 'ALGORITHMS' AT BOSTON UNIVERSITY

It is a common opinion that the United States of America supports democracy. Democracy always was connected in my mind with good education for all people and I knew that American thinkers also believed in this connection. Thus, when I came to this country, I expected to have rich opportunities to teach students to think critically, independently, and creatively and to solve non-standard problems without hindrance from authorities.

My first experience in teaching in this country did not contradict this expectation. It was proposed that I give a course called "Analysis of Algorithms" to graduate students of the Computer Science Department of Boston University

(BU). The textbook *Introduction to Algorithms* by Cormen, Leiserson, and Rivest was excellent. The department applied to me a wise rule—to give full freedom to the lecturer—and I used it to the benefit of my students as I understood it. In one semester I covered most of that rich book. I believe that the mathematical introduction was especially useful: I filled many gaps in my students' former education. My 19 students came from all over the globe, and most of them collaborated with each other in an excellent way. After every lecture they came to one room, discussed the problems which I gave them, and solved them together. Some problems I gave them were from the book, some were invented by me. I tried to miss no opportunity to make my students think, and they accepted it. Also there was no problem with grades. The department gave me *carte blanche* and I used it benevolently: almost all of my students learned a great deal, and I rewarded them with good grades.

'HE SHOULD TEACH FROM THE TEXT . . .'

But in the next year, when I came to a huge state university and started to teach the so-called 'business calculus'² to undergraduates, I got into an absolutely new situation. All my ideas about teaching students to think became completely out of place. Never before had I seen so many young people in one place who were so reluctant to meet challenges and to solve original problems. All they wanted were high grades, and they wanted to get them with a conveyer belt regularity. Suppose that a worker at a conveyer belt gets inspired by some interesting idea, and tries to implement it into his work. You can guess that he will get into trouble. This is what happened to me when I started to teach American undergraduates.

In my student years I hated teachers who simply repeated textbooks: it seemed to me that they wasted my time. Naturally, as a teacher, I avoided to repeat the textbook. This worked well until the last year when I started to teach "business calculus." Then I found quite a different attitude among my students: many of them would be most satisfied if the teacher simply repeated and explained what was written in the textbook. It seems that some of them have problems in reading by themselves what is written there, although most textbooks are quite elementary (but verbose). At first I failed to understand this, and one student wrote about me:

He should teach from the text and give exams based on the text or similar problems.

I was astonished by the fact that I could find absolutely no non-standard problems in the textbook I had to use. But I said to myself: this is a good case for me to show what I can do! *I CAN* invent non-standard problems! And so *I DID!*

²Full title: "Calculus II for Business and Economics".

And my first test was a total failure: it turned out to be so difficult for the students that most of them got very low grades. I had to learn that every technical calculation, which I was used to ignoring, was a considerable obstacle for my students. It took a considerable amount of time for me to understand how poor they were in basic algebraic calculations. Every time I prepared another test, I tried to make it as easy as possible, and still several times I failed: the tests turned out to be too difficult. As time went on, I came to the following rule: as long as a problem was interesting for me, it was too difficult for the students; only when a problem became trivial, might it be given in the test. Thus, when I prepared a test, I had to change problems several times making them still more and more simple. Once I prepared a test in my course of discrete mathematics. One problem involved a checkered square. First I took a square of size 10×10 . Then I diminished it to 8×8 , then to 5×5 , and at last to 3×3 . To diminish it further was impossible, so I gave it as it was, and still some students lost their way in that 3×3 square.

It was good luck for me that one of the students auditing my pre-calculus course, Robert Tufts, was a retired engineer who had lived much in Europe and Japan and had an extensive experience of learning and teaching. For him my style of teaching was not unusual; in fact, he liked it and told other students about it. Thus, they got a label 'European Teacher' for me and this softened their shock. Still, another student wrote:

Please inform Mr. Toom about the grading system and instruction methods of THIS country. Mr. Toom assumes that his students were taught as he was. I earned a grade of A in my college algebra and trigonometry courses so it makes no sense for me to be doing so poorly in this course. Please straighten this man out.

In the next semester I straightened myself out: At every lecture I took the textbook into my hand and explained some examples from it. And nobody complained.

As I had often done before, I gave out to students lists of additional problems arranged by me, and as before, these problems were useful as they moved many students to think. But I had not got used to caring about grades, and this time grades were the center of attention, not problems. My carelessness created a lot of trouble for myself and for the department. Those students who solved my problems, wanted extra credit, while those who did not solve them, wanted full credit also. Several times I was called to the official in charge to clarify my grading system. In the next semester I decided not to give any 'extra-credit' problems, and no trouble arose. The less I teach, the less trouble I have. In Russia we used to joke: 'No initiative will remain unpunished.' Now I saw this rule working in American education.

I had to learn by trial and error, how much of elementary mathematics was taboo in the business calculus course. Not at once I realized that I was lecturing

about exponential functions to students who were not required to know about geometrical progressions. And many were frustrated when I included in tests piecewise-defined functions. (An official told me that this normally should not be done.) Also I confused my business-calculus students by trying to explain errors in the textbook. Many of them would prefer to accept every word of it without criticism.

Another mistake made by me was to include a trigonometrical function in a test problem. I could not imagine that students who take 'calculus' were not supposed to know trigonometry, but it was the case. Of course, I was called to the official in charge and rebuked. Thus, I could discuss the equation $y'' - y = 0$, but not $y'' + y = 0$. In addition, I received a telephone call from someone who had graduated from the school of law; referring to a decision made by the authorities, he accused me of wasting taxpayers' money by teaching students what they did not need to know (trigonometry). After several lapses of this sort the department decided not to invite me for the next year, although they knew that I was a competent scholar, that I was interested in teaching, and that I needed a position. All they wanted was not to have problems with the students.

I noticed that research mathematicians treat the 'business calculus' courses like Russians treated Communist meetings: nobody dares to criticize openly, but everybody tries to sneak away. That is why foreign lecturers like me are needed to do this dirty job. But foreigners adjust to the system pretty soon, so that American students have almost no chances of becoming aware of their ignorance. For me, a few months were sufficient: the pressure of those students who wanted good grades with minimal learning, supported by university officials, made me care more about my safety from complaints and less about the real competence of my students.

One foreigner, experienced in teaching Americans, advised me in a friendly manner: "Listen, don't ask for trouble. Education in this country is not our concern. Nobody will care if you fall short of the syllabus, but never go beyond."³ And he went home with dollars earned honestly; that is, by doing to Americans just what they wanted him to do, both students and officials. Of course, he teaches in a much more productive way in his own country.

GRADE RUSH

Suppose you fly in a plane. What is more important for you: the pilot's real competence or his papers that certify that he is competent? Or suppose you get sick and need medical treatment. What is more important for you: your doctor's real competence or his diploma? I believe that in every case the real competence

³Remember that throughout my business-calculus course, I never went beyond into something more advanced: I simply tried to cover up gaps in my students' basic knowledge. And exactly this caused all the trouble.

is more important. But last year I met a large group of people whose priorities were exactly the opposite: my students. Not all but many. Their first priority was to get papers that certify that they are competent rather than to develop real competence. As soon as I started to explain to them something which was a little bit beyond the standard course, they asked suspiciously: "Will this be on the test?" If I said, "no", they did not listen any more and showed clearly that I was doing something inappropriate.

I had to learn also that American students want to be told exactly from the very beginning of the course what percentages of the total score comes from homework, from tests and from quizzes. First I thought that it was some nonsense, as if I were requested to predict how many commas and colons I would use in a paper I was going to write. But later I understood that these percentages make sense for those students who do not care about the subject and take a course just to get a grade with minimal learning.

Of course, students are different. Many really want to learn, because curiosity is inherent in the human nature. But selfless curiosity is 'illegal' (at least in the business-calculus course) in the sense that it is neither expected nor supported officially. On the contrary, officials cater to those who want to learn as little as possible, and percentages are a telling example of this.

The grade looks like the ultimate value, and neither students, nor parents, nor university officials see anything wrong about it. In fact, all officials completely support the top priority of official records. It seems to be generally taken for granted that students normally learn as little as possible for a certain grade. Only by misunderstanding may they learn more, and when this happens due to undetailed syllabus, they blame the teacher like people who blame an official whose neglect caused them a loss.

It is the basic principle of the market that everybody tries to get as much as possible and to pay as little as possible. There is nothing wrong with this: when I buy something, I try to save money, and everybody does the same. What is wrong is that some students apply the same rule to learning: They seem to think that they BUY grades and PAY for them by learning. And they try to PAY as little as possible! In other words, some students seem to think that it is a loss whenever they learn something. This looks crazy when put in such straightforward terms, but there are students who behave as if they think this way. (I do not know what they really think.) And there are officials who take this behavior as normal and arrange the learning environment according to it.

The attitude 'learn as little as possible' is not totally wrong, however, because a good deal of the stuff students are taught indeed deserves minimization (business calculus, for example). A good deal, but not all—there are excellent books and teachers—but many are not sophisticated enough to discriminate.

After every test I explained correct solutions. Many a student said: "Now I understand". I was glad: the purpose of my teaching was achieved. But some said it with regret which meant: 'This understanding is useless because it came

too late to provide me a good grade'. To me tests were just a means to promote understanding, to them understanding was just a means to get a good grade. To some students it made no sense to understand anything after the test.

Some students are so busy and anxious counting points on tests and predicting grades that they have no 'mental room' left to think about mathematics. It seems even irrelevant both for them and for the university whether they have learned anything at all: what matters for both sides is that the students overcame another barrier on their obstacle-race towards graduation. (And wasted some more months of their young and productive years.)

At one lecture I wrote a theorem on the blackboard and said to the students: "Look what a beautiful theorem it is!" Some laughed. I asked what was the matter. Then one explained: "Professor, it is nonsense, a theorem cannot be beautiful!" And I understood that these poor devils, who had always learned under the lash of grades, never from natural curiosity, really could not imagine that an abstraction might be beautiful.

Any creative activity (including learning) needs at least temporal independence from external rewards and pressures. Peaks of creativity (which are essential in learning and solving non-trivial problems) need so much concentration on the subject that any sticks and carrots can only disturb them. Only when the intimate work of creative faculties is over and have produced a finished result, may one think how to 'sell' this result most profitably. Pushkin, a Russian poet said: 'Inspiration is not for sale, but a manuscript may be sold'. The same applies to learning: those who lack intrinsic motivation and are guided only by external rewards, learn poorly. They are never carried away by the subject's charm for its own sake, as they believe that they must be 'practical', that is never forget their points and grades. As a result they never use the powerful potential of creativity given to them by nature. Everybody's natural abilities are rich, but their use depends on individual priorities.⁴

It seems that some students just can not imagine that learning might be of intrinsic value, besides official graduation. And they may go through many years of schooling, communicate with teachers and officials, graduate from an elementary school, middle school, high school, and a university, and never have a chance to question this! Unless they meet some irritating foreigner!

Foreigners, however, soon understand that to survive in this country they have to adjust to the system rather than to criticize it. At various levels and in various ways, newcomers are shown clearly that this country wants intellectuals, but not those who are too independent. This may be one reason why so many immigrants who were excellent mathematics teachers in Russia, have done much less than their best to reform American education.

In my case, the pressure of students made me deviate from my principle to do my best pretty soon: I was forced to care about my safety from students' com-

⁴Censure of political despots is often only skin deep (many a Russian writer successfully bypassed it), but censure of market goes to the bones.

plaints at the expense of their own best interests. Although my personal experience is limited, I think that this situation is typical. In another state students complained about their mathematics teacher, another newcomer from Russia: 'We pay as much as others, but have to know more than they for the same grade.' Still in another state, another newcomer from Russia found an effective way to calm his students: when they asked how he would give them grades, he answered that he would do it 'on the curve'. I asked him what it meant and he answered that he did not know: what mattered for him was that the students got relaxed and became willing to listen to lectures and solve problems.

Well, I can imagine a situation in which learning for a grade makes sense. If students are ultimately disappointed with the teacher, if they have given up any hope of learning anything valuable from him, if they not only disrespect, but actually despise him,—then, and only then, it makes sense to learn for a grade—to get at least this if there is nothing better to get. In the final analysis, learning for a grade is the deepest offense to the teacher, because it implies the thought: 'I know in advance that nothing valuable will come from real contact with the teacher; so let me at least get a grade.' But, according to my experience, students who learn for grades do it in all courses. They seem not to be aware that they offend teachers, they simply take this mode of behavior for granted. (And most American teachers and officials also take it for granted.) In fact, students want to know the percentages at the outset, before any experience with the teacher. And department officials take this demand absolutely seriously, much more seriously than the actual content of courses.

At one of my lectures of business calculus, when asked why I gave problems unlike those in the book, I answered: 'Because I want you to know elementary mathematics.' I expected to convince students by this answer. In Moscow a university student who was told that he or she did not know elementary mathematics, got confused and checked into the matter immediately. Elementary mathematics was normally taught to children who looked like children. Now imagine my astonishment when right after my answer, an imposing train of well-grown adults stood up and tramped out. They decided (correctly) that they could graduate from the university without knowing elementary mathematics. And that they would easily find a lecturer who would teach them from the text.

And the one who had to change was me. In the next semester I never scared students away by checking into basics. I understood perfectly that teaching an advanced subject like calculus without filling gaps in basics was like building on sand. But I could not afford to care about my students because I had to care about my safety from their complaints.

I have examined the American Constitution and found no statement that guarantees the right of ignorance for students. Nevertheless some students behave as if such a statement existed. And some officials behave as if they had no other choice than to comply with them. Why? One official explained to me that some students had sued universities for better grades, and won. (I have never heard of a student who sued a university for better or more knowledge.) Now the

main concern of officials is not to have this trouble again. One evident result of this is that bright students lose a lot of opportunities to learn more, but they never complain (regretfully), and officials do not need to care about them.

I do not propose to put all the blame on students. In fact, their priorities reflect the cynicism of educators who design courses not for the sake of students' best interests, but for other aims: for example, to put another artificial obstacle in their way, to keep teachers busy, etc. The business-calculus course seems to be deliberately designed just as an obstacle for those who want to graduate in the business school.

Roughly, students may be classified into three groups. Some really want to learn. These are most intelligent, and officials should listen to these students' opinions, but they don't because these students never complain.⁵ A teacher may reduce a course to trivialities, cover only part of the syllabus, and this will never cause any trouble. The second group of students are those who do not know what they really want. They may be allured to learn well or waste their time depending on the situation. And there are a few students for whom all their relations with the university come to overcoming another bureaucratic obstacle. All my attempts to move one of these to become interested in the subject matter failed: I spoke to him about mathematics, he answered about points and grades he should get. Students of this kind are the most successful in dealing with officials, because they may cause trouble.

Indeed, in all times and everywhere there were lazy students, but this time I met something quite different. Traditionally, authorities controlled the situation. Now, I saw education molded by the pressures of those students who did not hide their contempt of the very idea of learning. And this was done at the expense of those (not a few) who really wanted to learn.

CULTURE AND PARENTS

To exclude misinterpretations, let me say that I certainly do not think that some (ethnic, social, etc.) groups of students are 'inferior' or fatally reluctant or unable to learn. What I want to emphasize is importance of cultural factors, such as attitudes and priorities. Most questions of my business-calculus students were about points and grades. Those who asked how to solve a problem (TODAY many of these are of Far-Eastern origin) started by making as many mistakes as others, but they wanted to understand even if it would not give them a better grade. And in the final analysis they got better grades, too.⁶

⁵It is always difficult for a bright student to single out what to complain about. For an ignorant one it is easy.

⁶In my student years it was said sometimes that "Kolmogorov liked Jews." I knew this bright scientist enough to assure that he never favored anybody based on ethnical background. He loved students who loved mathematics. In the Russia of his time many of these happened to be Jewish.

It is a cultural fact that some students respect teachers and the very idea of intellectual effort more than some others (who may respect nothing but money). And those who respect intellectual work, get better results, as long as the system allows. One student may think: 'I can not solve this problem. I must think better or ask the teacher. Anyway, I must understand how to solve it.' Another might think: 'I can not solve this problem. It was the teacher's fault to assign a problem which I can not solve. Anyway, it is not my duty to understand anything. I shall better complain to the department.' And this difference of attitudes is a very important cultural factor.

There is nothing mysterious about culture: one of its most evident components is the influence of parents. It is a well-known fact that the intellectual level of children typically repeats that of their parents, but every experienced observer can easily find many possible explanations for this fact, other than biological inheritance.⁷ Even when students have grown up, parents influence them. This influence is enormous when the students are small children. It seems that some parents urge their offspring to get high grades by any means, but fail to add that they care about actual competence, too. I understand that some students are the first in their families to get a higher education. Their parents did monotonous work all their lives, tried to make more money for less work and were right, of course. Now, their offspring do monotonous exercises at universities, try to make more grades for less work, and nobody in the family sees anything wrong with it. Indeed, parents may perceive this as a great achievement when their offspring graduates, and they may think that they now have an 'intellectual' in the family, while this is simply someone who bought a discounted degree at a university sale! Discounted not in the sense of money, but in the sense of intellectual effort and development.

I understand that I have very little experience with the bulk of the Russian population. Most of my students in Moscow were children of intellectuals, because in Russia (as in most countries) a much smaller percentage of youngsters than in U.S. go into higher education. In fact, what is going on in America is an experiment: to give higher education to those strata of society which remain deprived of it in most other countries. My concern is that this should be really an education, not an imitation.

⁷An example from my memory: Once I rented a room in a provincial Russian town. I was sitting in that room and writing when the owners' son, a preschool boy entered because he was curious about me. Without thinking, I gave him a pen and some paper, because it was common in my family to give children pen and paper. Later his mother apologized that her son had "*spoiled*" my paper. I said that I had given him the paper intentionally and advised her to do the same. She could not understand what was the point in letting a preschool child spoil paper. Stories like this, help us to explain how children 'inherit' (without referring to genes) the intellectual level of their parents. According to my experience, a person who was allowed to 'spoil' paper in childhood has a lot more chances to become an intellectual than one who was not, however uniform might be the educational system.

MARKET VS. MODESTY

It is known from the history of science that great scientists were modest. It was not a sacrifice: to concentrate on new problems one has to put away one's old achievements. The same applies to the manner in which one teaches or writes a textbook: either you concentrate on the real difficulties of the subject and teach how to overcome them, or you reduce the students' task to follow standard rules labeled after famous theories. Really useful textbooks have to be modest to concentrate on the essence.

But modesty is out of place in the world of market and advertisement; instead, pretentiousness and pomposity are inevitable. The pompous and shallow marketing style heavily influences education: courses, syllabuses, textbooks. It was proposed once that I teach a course of computer literacy, using a verbose textbook which pretended to teach to program in BASIC. But most of the lines of codes started with REM (that is, were just commentaries) and there was only one loop in the whole of the book.

The voluminous book I had to use in teaching the business calculus may impress non-professionals, for example parents of students. Its chapters are named after really important mathematical theories. But everything non-trivial is carefully eliminated. In fact, every chapter contains a recipe, as in a cookbook, and problems do not go beyond straightforward applications of the recipe. The book carefully avoids connecting the material of different chapters, presenting the subject from different sides, giving problems in which a student should choose which method to apply. And this book was chosen among others, some of which were quite usable. Why? I see one explanation: Because this book perfectly fits the *max-min* principle of the market: maximal pretensions with minimal content. All the other textbooks are not so perfect in this respect.

Most American students know that the business calculus is a *phony*, and those who really want to learn, prefer other courses. But foreigners take this course naïvely expecting to become competent both in calculus and business, and waste their time. One of my business-calculus students had come to this country from Singapore to learn and get new experience. He regularly solved more problems than I assigned. Regretfully, this was not very exciting, because most of these problems were standard; there were no other problems in the textbook. He liked the problems invented by me and did not give up until he had solved all of them. But other students complained about my innovations, and so I was pressed to reduce my course to trivialities. Some people say that if a student really works hard, he or she will make his or her way. Well, the student from Singapore made his way in the sense that he got an 'A' in my course. But he learned much less than he might have, considering his attitudes and natural abilities. If he attended my seminar as I would have liked to teach it, he might have become a scientist. But this seems not to have happened because of the way the course was designed.

ATTITUDES TOWARDS TEACHING OF AMERICAN MATHEMATICIANS

I was astonished to find that many of my American colleagues, although very competent as scientists and quite decent as persons, had absolutely different ideas about education and teaching than I had. When I spoke to them about education, they answered something like: 'This is not my concern. There are special people to care about all that', as if I spoke about some important but remote activity. According to my experience, the prevailing attitude among American mathematicians is to avoid teaching. When these American mathematicians say that they have a "good position," this typically means that *they do not have to teach*. And if a mathematician with (substantiated or not) research ambitions has to teach, he often tries to do it as mechanically as possible. And students take this for granted, and try to learn as mechanically as possible. The result is a tit-for-tat between teachers and students, which may reduce mathematical education to wasteful bureaucratic mirages. And the system (as any system) is robust: if a recent immigrant, unexperienced in American ways, happens to be different (for example, to love teaching), he or she does not fit into the system, and only causes troubles.⁸

The attitudes of some mathematicians towards teaching form a perfect counterpart to the attitudes of some students towards learning. Some but not all. It certainly is not exciting to teach those who invest more efforts into pushing for grades than for understanding. But, on the other hand, students as a whole are not nearly as hopeless as some smug teachers pretend.⁹ It is true that there are a few nasty students who can put anybody off teaching and it is true that some indifferent bureaucrats prefer to yield to their pressures (at the expense of those who want to learn). But at every course there are students who are really interested and I think that these students are the most valuable. In every one of my courses there were students who were excited by those very non-trivial problems which moved others to complain. My former students came to my office to thank me. They said that after my course the next courses were easy for them. Some asked if I was expected to teach something in the next year and advised me to publish the problems I had invented. But bright students never complain (regretfully), and officials do not care about them. More than once I had to say to one student or another: "You did very well in my course, and I give you an A. But this does not mean much, because what I teach you is not really mathematics."

⁸Most advertisements about positions request what they call 'commitment to excellence in teaching', especially teaching undergraduates. But what does it mean: commitment to teach thinking or commitment to waste one's time for pseudo-teaching? And according to my experience, if an applicant claims that he or she loves teaching, he or she only moves others to think that he or she is failing in his efforts to do research.

⁹A typical game (in Berne's sense): 'It is profanity to make such a genius as I am, waste my precious time on teaching.'

Generally, Americans are very concerned about social problems. Civil rights, environment, abortion and many other important issues attract a lot of attention and many people are willing to spend time and money to assure what they consider to be the right decision. A lot of volunteers work as nurses, social workers, firemen. Perhaps, they dreamed of becoming firemen as boys. Indeed, there is much of 'macho' in being a fireman. However, education remains beyond these concerns and aspirations. Do children dream of becoming teachers? It seems that in this country they do not.

It is a common understanding that by doing one's best in sports students defend the honor of their institutions. But it would seem strange for the majority to connect pride and honor with doing one's best when confronting a mathematical problem. And professional mathematicians seem not to be bothered by this.

It seems to be a common opinion among mathematicians that research is the real thing and that teaching is for those who cannot do research. I think that this attitude is dangerous as it may jeopardize the reputation of intellectuals. Some people excuse bad teaching by saying: Since students buy it, it is OK to sell it. But pushers of drugs say the same. It is the responsibility of specialists to do the right things even if laymen can not discriminate between right and wrong. It is the responsibility of teachers to teach in a way which really develops students' intellect. Imagine that a non-educated person is sick. Is it fair to prescribe him a fake medicine just because he cannot tell it from a good one? Of course not: this is not only inhuman, but also dangerous for the reputation of the medical profession. The same about teaching: fake teaching is unfair and breeds anti-intellectualism. The moral status of those who designed the business calculus course is like that of colonial-time hucksters who sold cheap beads, mirrors and 'fire-water' to ignorants, whose role is now played by students. (I do not blame rank-and-file teachers, because many have no choice.)

For many years, the Soviet authorities tried in vain to reduce scientists' concerns to their job and were irritated when someone interfered with public affairs. Sakharov is the most well-known example of a Russian scientist directly involved with politics, but I am sure that educational efforts of many others were equally important. In this respect, the free American job market seems to intimidate dissenters more effectively than Soviet despots ever could: most American mathematicians try to deal with education as little as possible, because of the existing system of rewards.

Most students are young people. They are not yet quite mature, and their priorities are in the process of formation. Every school not only teaches particular subjects, but also suggests certain ideas of what learning and mental activity should be. In the present situation the idea most often promoted by authorities is that official records are the most important results of learning. Many students are not independent enough to defend themselves against this bad influence, and get phony education at the expense of real time and money. Their motivation shrinks

to external sticks and carrots and they fail to develop independence from external rewards of the social system.

SOME PROBLEMS IN ELEMENTARY MATHEMATICS

I love to start my courses at the precalculus level by asking students to vote on the following question:

Take the infinite decimal fraction $0.999\dots$, that is zero, then decimal point, then an infinite row of nines. Is this fraction less than one or equal to one?

Often the majority votes that the fraction is less than one. Then I ask, how much less, and students give different answers according to which calculators they use. This starts a useful discussion in which all the students participate because they feel that this really pertains to them. Every student tries to prove that his answer is correct, which allows me to convince them that all are wrong: this fraction equals one.

You may ask me: why do I start my courses by provoking students in making a wrong decision in such a dramatic form? Because it is absolutely necessary for a teacher to keep his or her students alert and critical of themselves. If I simply informed my students that this fraction equals one, they would easily agree, but forget it by the next lecture. This is just one example. In fact, when I teach as I want, I try systematically to show that something that seems evident, may be wrong. Experience of this sort, as I believe, is essential as a psychological prerequisite for studying rigorous mathematics. In Russia students were delighted whenever I succeeded to bamboozle them. Even children understood that it was a pedagogical device to make their knowledge and thinking more robust. My graduate students at BU also were excited when I proved that no algorithm can solve the sorting problem in linear time and right after that presented an algorithm which seemed to do this.

This is understandable: wise nature has made people, especially young people, in such a way that they love challenges. That is why people (especially children) enjoy performances of magicians whose job is to cheat. Many love mysteries and detective stories whose authors intentionally mislead the reader. Why shouldn't the teacher use the same device? Creative students are happy to meet something puzzling or misleading, because it gives them a chance to become tougher as thinkers.

But many undergraduate students are over-sensitive to everything which they perceive as a failure, even a small one. Whenever intonations of my voice led them to a wrong direction, students took it as a violation of some gentlemen's rules. It looks like some American students cannot afford the natural human love for intellectual challenges because of the pressures of grades and formal records.

If the teacher's recommendations do not lead them straight to the right answer, they perceive it as the teacher's fault, not as a pedagogical device. But with this attitude one cannot develop intellectual independence.

Some seem to think that they should be perfect from the very beginning, and if they are not, this is a fatal failure, like an incurable disease. They seem to feel obliged to give the right answer as quickly as cowboys shoot in Westerns, and if they miss, they just feel themselves to be losers and have no ways to deliberately and systematically develop themselves.

Officially, certain prerequisites are requested for every course. I wanted to check students' actual prerequisites and found that many of them could not solve simple, almost arithmetical, problems.

I included in my courses a problem which I had solved in middle school:

Tom and Dick can do a job in 2 hours. Tom and Harry can do the same job in 3 hours. Dick and Harry can do the same job in 4 hours. How long will it take for all the three of them to do this job?

This problem can be solved by elementary algebra and a few arithmetical calculations. Most of my students could not solve it. One of them wrote the following system of equations:

$$T + D = 2, \quad T + H = 3, \quad D + H = 4.$$

got a bad grade and asked me why. I asked in return, which parameters she meant by T , D and H —time or something else. She said that she meant no parameters, just Tom, Dick and Harry. I replied: 'This is illiterate.' A Russian student would grasp the chance to learn something new, but the American took this as a fatal failure, left the room with tears in her eyes and dropped from my course. I regret this even now, but what else could I say?

This case is typical in the sense that many students avoid discussing their mistakes, it looks like a useless pain for them. If you learn for competence, which is valuable for you as such, you can benefit from your mistakes. But if you learn for grades, and your self-esteem completely depends on external evaluations, it is plain masochism to keep in mind lost opportunities.

Another problem which I included in my courses:

It took 5 days for Huckleberry Finn to go down stream on the Mississippi by ship and 7 days to return by the same ship. How long will it take for him to make the same way down stream by raft?

The answer can be got by a calculation like $2(1/5 - 1/7) = 35$ days. Only a few solved this problem although they had plenty of time and wanted to get points promised for it.

At the last lecture of my business calculus course I gave a problem:

When 1000 pounds of cucumbers were brought to the shop they contained 99% of water. But while they were kept unsold, some water evaporated, and the percentage of water dropped to 98%. How many pounds do they weigh now?

The students grabbed their calculators, but seemed not to know what to calculate. After a while, one produced a complicated and wrong answer. And it was pretended that these students had learned to solve differential equations! Of course, they had not! All they had learned was to follow a few recipes without thinking—a bright start for their careers!

Well, let us admit that most people can manage without being able to solve differential equations. But why did the students waste their time? The syllabus, the textbook, all the course design aped those for future professionals, but with one 'small' change: applying recipes instead of solving problems. But this change annihilated the whole enterprise.

Thus, students lost several months, but had not learned to solve any problems at all, because to solve problems means to think productively, that is to produce ideas which are not given in advance. And this is what they were completely deprived of.

All the problems given above should be solved in high school or even middle school. Solving problems like these and writing down the solutions is a valuable experience of productive formal thinking which is hardly avoidable for every man or woman in the modern civilization. All normal teenagers have brains mature enough to solve such problems, and those who solve them at 14, really can learn calculus at 18. But most of my students seemed to have no such experience. What had they done throughout their many years of schooling?

It seems that to a great extent they had filled in boxes, that is to say, chosen the right answer among several ready-made ones. Multiple-choice tests are convenient because their results are easy to process. This seems to be the main reason why such tests are so often used. Perhaps such tests give valuable information to educators, but they grossly limit students' initiative, fragment their activity and deprive them of self-organizational experience.

Suppose that you are an average student. If you write solutions, even wrong ones, you can analyze them and learn something from your mistakes. But if you just put tallies into boxes, you don't remember why you made the choice; so you cannot analyze your mistakes and cannot benefit from them. All your hope is that your conditional reflexes will gradually improve, but you can not control this process, like an animal in a problem box.

Well, better late than never, that is why I gave the problems mentioned above to my students. But I could not give more than just a few problems of this sort, because I had to follow the syllabus.

ON TEACHING OF MATHEMATICS IN A LIBERAL ARTS SETTING

Nowadays, throughout the world, every youngster is assigned to learn some 'mathematics', but most of those who are in charge of this huge enterprise, can not explain in reasonable terms what is all this for and what is meant by 'mathematics' in this context. What is the purpose of mathematical education for those many who will not become professional mathematicians? This is an enormously important question, but too comprehensive to discuss here in detail. Let us at least understand that it has no straightforward utilitarian answer. Very little of mathematics is used by most people in their work or other activities. Managers and lawyers, social workers and policemen, drivers and farmers, politicians and officials, doctors and nurses, cooks and barbers, writers and artists, sportspeople, businesspeople, salespeople and showpeople, do not solve quadratic equations, do not use set theory, the theory of numbers, functions or algorithms, analytical or projective geometry, and do not differentiate or integrate.

Please, do not think that I am AGAINST teaching mathematics. I am FOR it. What I want to emphasize is that a teacher should never expect that students will have a chance to apply recipes literally. If you teach nothing but recipes, you teach nothing. This is especially true when teaching such an abstract subject as mathematics: it makes sense only when it is teaching one to think, to learn and to solve problems. When this takes place, teaching mathematics may be enormously useful for everybody.¹⁰ Here (as elsewhere in this paper) I do not pretend that my opinions are original. A lot has been said in the same vein, for example: 'In mathematics 'know-how' is the ability to solve problems and it is much more important than mere possession of information.'¹¹

But thinking and solving non-trivial problems are conspicuous by their absence in many 'developmental' courses. (Nobody knows what these courses actually develop.) Many courses of mathematics in liberal arts settings are made up by the following simple rule: take the professional course, keep the shell and eliminate the kernel. That is, keep the pretensions, terms, even some formulations, but eliminate everything that needs thinking. At first sight it may seem easy to avoid this, because there are lots of problems in various textbooks; solving these problems would certainly benefit students much more than business calculus which is neither business nor calculus. But this won't do because of the

¹⁰One small example of a successful solution of a practical problem. Once my daughter (who was 12) needed a dictionary, and we went to a bookstore. She chose one but could not find the number of words in it, printed in the first or last pages. Then she chose a page that looked typical, counted the number of words in it, looked at the number of the last page, rounded both numbers to the first digit and mentally multiplied them. Thus in a few seconds she obtained an adequate estimate of what she needed. I was delighted. This may be called 'mathematical common sense'. What a contrast with most of my business-calculus students who were helpless without their calculators and without a detailed instruction of what to do and in which order!

¹¹George Pólya, 'On The Curriculum For Prospective High School Teachers.'

market pressures. Suppose that some author writes a textbook with problems that need thinking for their solution and some college gives a course using this book. Students who take this course will modestly think instead of learning recipes with bombastic labels. The college will have to admit that its students simply learn to solve some mathematical problems and thereby just become more intelligent. Which parents will send their offspring to it? Which firm will hire the students? What will they boast of?

To survive against competition every university and every college has to pretend that it gives something modern, advanced and immediately marketable. But is it possible to give advanced courses to students who are ignorant in elementary mathematics? Of course not. What to do? Very simple! Emasculate the course by excluding everything non-trivial, reduce the students' task to applying ready-made recipes without understanding—and you will survive and succeed. Your pretensions that you teach something advanced will allow the students to pretend that they are educated, and this will allow the firms and departments that hire them to pretend that they hire educated people. But at some point this chain of pretensions will have to break.

EFFECTIVENESS . . . TO DO WHAT?

The American ability to get things done has become proverbial. The question is what should be done. I have no panacea, but I invite at least to see the problem. Many seem not to see any problem at all. I tried to figure out what political leaders of this country think about the quality of education and concluded that they think nothing about it. They speak of giving everyone an opportunity to obtain an education, but they say nothing about the quality of that education.

Well, some people really are concerned about education. But their arguments fall short of the problem if they boil down to the warning: Learn better or Big Bad Russian or Japanese or Western European will come and eat you! This may impress political or military leaders. But pragmatism falls flat with those who actually have to learn, that is students, because only selfless intrinsic motivation (based on natural curiosity) can provide the lavish creative spirit which is essential for really successful studies. In creative activities (and learning certainly is among these) only those succeed who love what they do. Would Newton discover his laws if he had learned for grades as a student? Of course, not. He learned because he was enormously curious, because he loved to learn and to think, not because of some bureaucratic stick or carrot. I know what you want to tell me: that most university students are not Newtons. But they were not born this way. Everybody knows that children are naturally curious. And curiosity is just another name for desire to learn. Based on all my experience I believe that most children are 'Newtons', at least in spontaneous curiosity and love for puzzles. Nature provides a fertile soil for development, and it is our responsibility to cultivate it, to develop nature into culture, to transform the

natural curiosity and love for challenges into mature love for regular intellectual work. Only spiritual, that is intrinsic motivation can provide impulses strong enough to support the hard intellectual work that is necessary for really fruitful development. Students who really understand what their best interests are, learn what they like and care about their spiritual growth much more than about such bureaucratic trifles as grades.¹²

Now many Americans say: 'We have won the Cold War.' This is wrong. The Soviet rulers certainly lost the Cold War, but this does not yet mean that Americans won. The Soviet bureaucrats lost because they lived in the lunatic world of 'advantages of the Soviet system', 'Soviet type of democracy', 'building of Communism', 'enthusiasm of Soviet people' and other slogans of their own propaganda. Lack of realism, fear of any independent opinion, enormous discrepancy between reality and official versions undermined the Soviet rule. Much can and will be said about the collapse of Soviets, but I am sure that the dominance of bureaucratic fictions at the expense of reality certainly played a major role.

Regretfully, all the same can be said about some part of American education. There are people among students, their parents, teachers and officials who do not understand what education is about. They anchor their aspirations and priorities to the bureaucratic form rather than to the substance of culture. Let me repeat that there is nothing special about Americans in this respect. There are lots of countries where average education is worse than American (Russia, for example).

I do not agree with those who condemn the so-called 'American practicality'. To be really practical for a student means to use every opportunity to learn more.¹³ Students who learn for grades only think that they are practical; in fact they are small-minded. It is very unpractical to waste one's young years (and parents' money) to get grades without understanding. And it is unwise to discourage the teacher from doing his best. Not only a scientist, but also an engineer, an official and a businessman will have a very small way in his business if he can only apply ready-made recipes. He may survive for a while under the protection of his country's strength and wealth, but in return he will contribute to her problems rather than solutions.

Indeed, there is a rule BY the people in America, but not always FOR the people. People command to the intellectuals, in a sovereign way, something like the following: 'Give certificates of competence to our offspring without any delay! And don't waste taxpayers' money by teaching students too much! And don't you dare to discriminate against ignorant ones!' Intellectuals obey implic-

¹²One of my students at BU, Biao Wang, solved all the 'extra-credit' problems, although he got an A without it. When I asked him why, he shrugged his shoulders and replied casually: 'Just for pleasure'. What a magnificent contrast with the small-minded fuss about getting grades with minimal learning!

¹³Of course, children of intellectuals never complain about a teacher who violates their 'constitutional' right of ignorance, because their parents have explained to them what are their best interests.

itly and give out bombastic graduation papers with an open hand. Everybody is glad: Scientists return to their research having paid as little efforts and attention to teaching as possible. Bunches of youngsters get impressive certificates which are the most marketable results of their studies. Parents have realized their dream to 'educate' their children. Some of the richest and smartest parents are also glad: they find special ways for their children to get REAL education, so their future is assured. But what about the future of others? Is it assured as well?

Those who learn for grades expect to succeed in their business. TODAY they are right insofar as almost every American who has a degree, however ignorant, can live better than even competent people in much poorer countries around the world. A person with a diploma should not fail to find a job in his or her field of competence: this is a common belief in this country. But this cannot last long in the situation when 'competence' and a diploma tautologically mean each other. The advantages enjoyed by Americans are the results of real competence and real efforts of previous generations, whose heritage is now getting devaluated as a result of the bureaucratic character of the educational system. And someday ignorant people with degrees and diplomas may want power according to their papers rather than real competence. We Russians, have some experience of this sort, and it is not quite unique. In all countries (including America) activists of ignorance try to dictate their will to universities, and sometimes they succeed, at the expense of those who really want to learn.

Which part of American education really develops students' competence and which comes to pretentious trivialities, like 'business calculus'? I don't yet know. And I don't know who knows. I am learning about it by experience and it will take a long time to learn. But it is clear to me right now that the winners in the modern world will be those countries which will really teach their students to think and to solve problems. I sincerely wish America to be among these.