Math for Intro to DL

Review in 5 min

Summation

Summation is denoted by using **Σ** notation, where **Σ** is an enlarged capital Greek letter sigma. For example, the sum of the first `n` natural integers can denoted as

$$\sum_{i=1}^{n} i = 1 + 2 + \ldots + n$$

Product

Pi Notation, or Product Notation, is used in mathematics to indicate repeated multiplication. For example,

$$\prod_{i=1}^{n} i = 1 \times 2 \times \ldots \times n$$

Linear Function

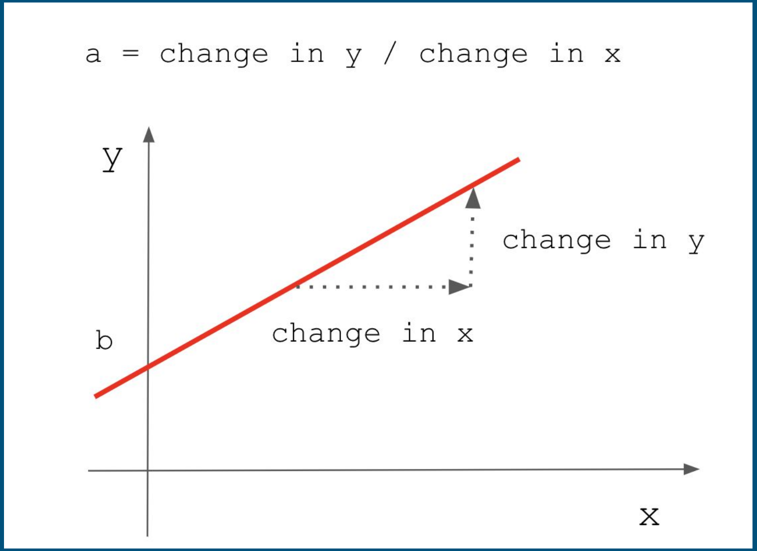
A function is a mathematical concept that describes the relationship between two sets. When there are variables **x** and **y**, if **x** changes, **y** indicates what rule changes. Usually, a function is expressed as y=f(x) using the function f and the variable **x**.

A linear function is a case in which **y** is expressed as a linear expression with respect to **x**. For example, it can be expressed as the following function expression.

y=ax+b (a not = 0)

If **x** is a linear form, a must be nonzero for **x** to remain linear.

In the linear function formula y=ax+b, a is the slope and **b** is the **y-intercept**. The slope refers to the degree of inclination, and the slope **a** of the graph is determined according to how much the **y** value increases when the **x** value in the figure increases. The **y-intercept** is the point at which the graph intersects the **y-axis**. In the figure, the **y-intercept** that intersects the **y-axis** is **b**.



Linear Function

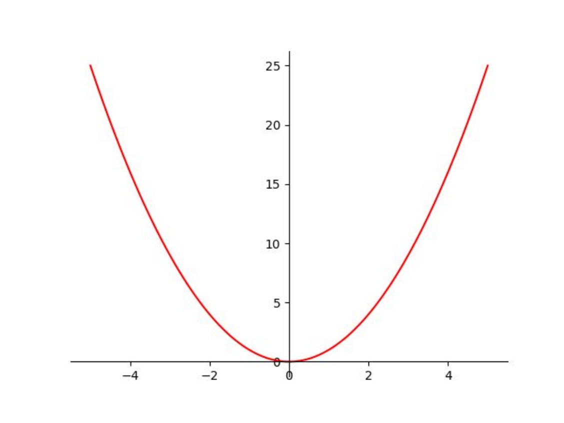
Here are some real-life applications of the linear function.

- A movie streaming service charges a monthly fee of $4.50 and an additional fee of $0.35 for every movie downloaded. Now, the total monthly fee is represented by the linear function $f(x) = 0.35x + 4.50$, where x is the number of movies downloaded in a month.
- A t-shirt company charges a one-time fee of $50 and $7 per T-shirt to print logos on T-shirts. So, the total fee is expressed by the linear function $f(x) = 7x + 50$, where x is the number of t-shirts.

Quadratic Function

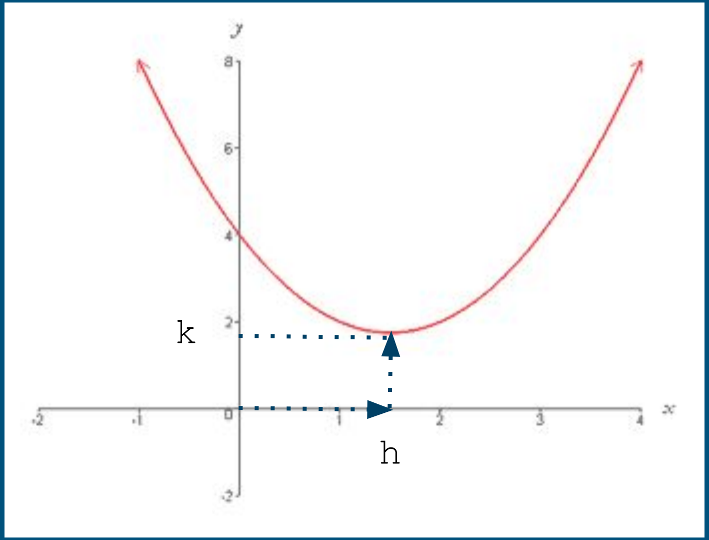
A **quadratic function** is a polynomial function with one or more variables in which the highest exponent of the variable is two. Since the highest degree term in a quadratic function is of the second degree, therefore it is also called the polynomial of degree 2. A quadratic function has a minimum of one term which is of the second degree. It is an algebraic function.

The parent quadratic function is of the form **$f(x) = x^2$** and it connects the points whose coordinates are of the form (number, number^2). Transformations can be applied on this function on which it typically looks of the form **$f(x) = a(x - h)^2 + k$** and further it can be converted into the form **$f(x) = ax^2 + bx + c$**. Let us study each of these in detail in the upcoming sections.



Quadratic Function

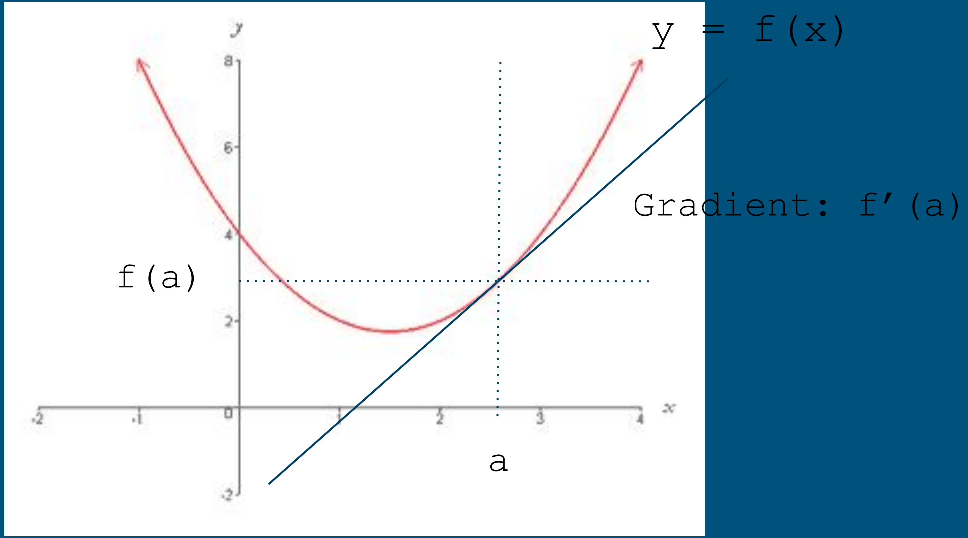
If you move the graph in parallel by h in the x-axis direction and k in the y-axis direction, it is as follows.



$$y = a(x - \mathbf{h})^2 + \mathbf{k}$$

Ordinary differential, Instantaneous rate of change, and Gradient

Derivative, in mathematics, is the rate of change of a function with respect to a variable. Derivatives are fundamental to the solution of problems in calculus and differential equations. The derivative tells us the rate of change of one quantity compared to another at a particular instant or point (so we call it "instantaneous rate of change")



Instantaneous speed

The derivative predicts change. Ok, how do we measure speed (change in distance)?

Officer: Do you know how fast you were going?

Driver: I have no idea.

Officer: 95 miles per hour.

Driver: But I haven't been driving for an hour!

We clearly don't need a "full hour" to measure your speed. We can take a before-and-after measurement (over 1 second, let's say) and get your instantaneous speed. If you moved 140 feet in one second, you're going ~95mph. Simple, right?

Partial Differentiation

Differentiating a function of multiple variables with respect to only one variable is partial differentiation.

Instead of differentiating for all variables, **we differentiate only one variable** we want and treat all other variables as constants.

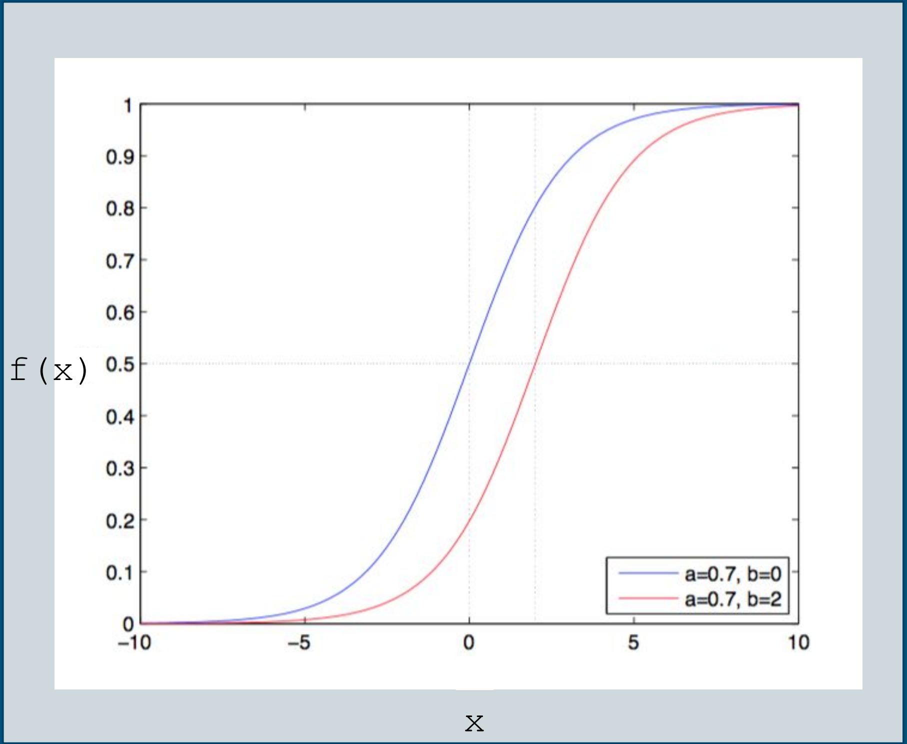
For example, if you only want to differentiate on x, write the equation as follows.

$$f(x, y) = x^2 + xy + 2$$

$$\frac{\partial f}{\partial x} = 2x + y$$

Sigmoid Function

$$f(x) = \frac{1}{1+e^{-(ax+b)}}$$



a is a coefficient to adjust the slope

b is to adjust the position of the center

Logarithm

To understand the logarithm, let's start with the exponent.

If **a** is raised to the power of **x** and it’s equal to **b**, the expression is

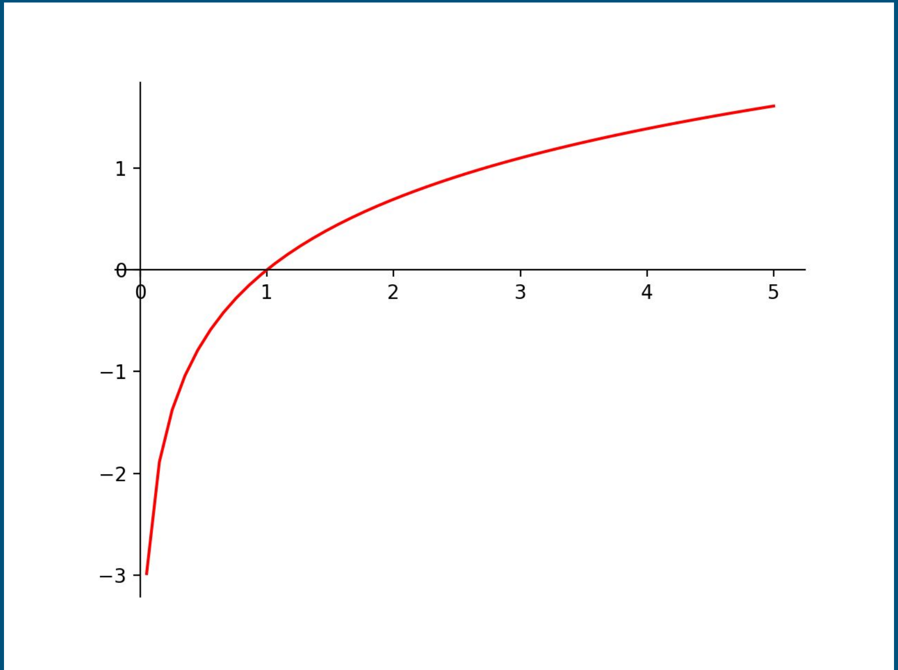
$$a^x = b$$

Let's say you know **a** and **b** but don't know **x**. In that case, you can use log to get it.

$$log_a b = x$$

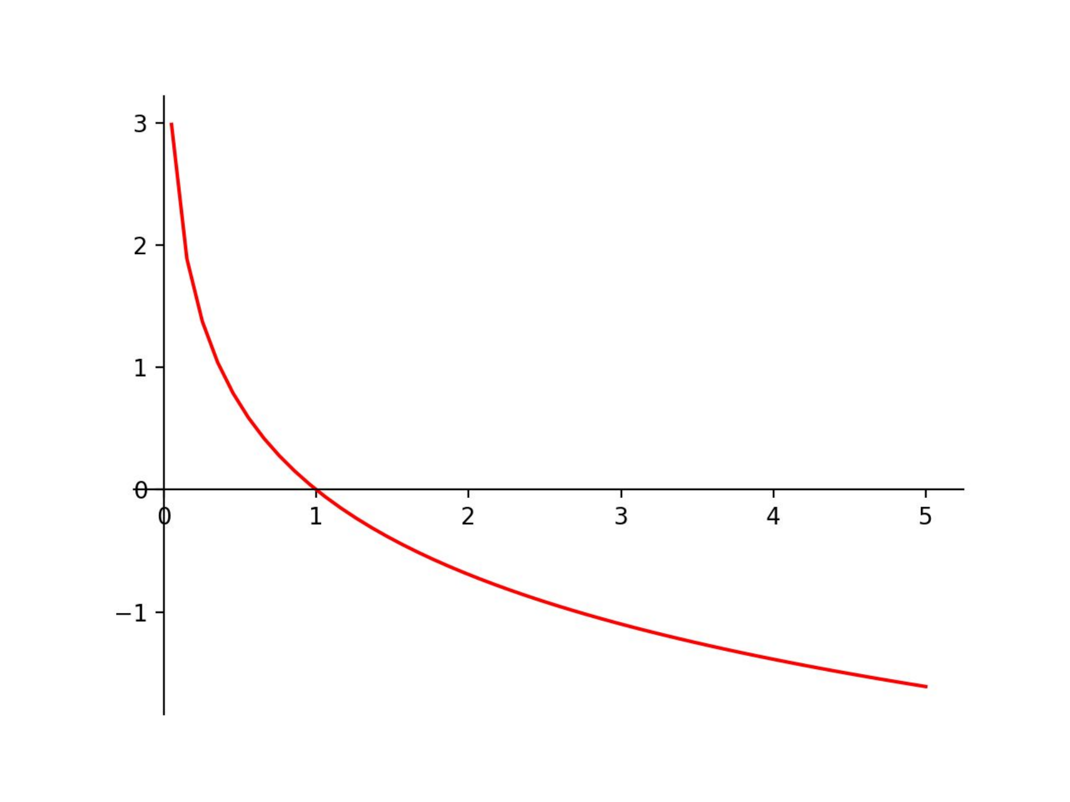
Log function (natural log)

$$y = \log x$$



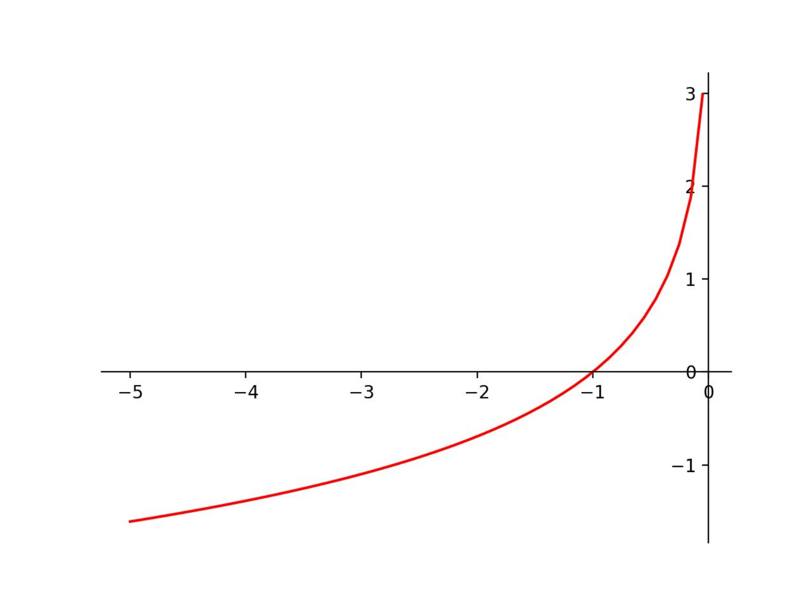
Log function

$$y = -\log x$$



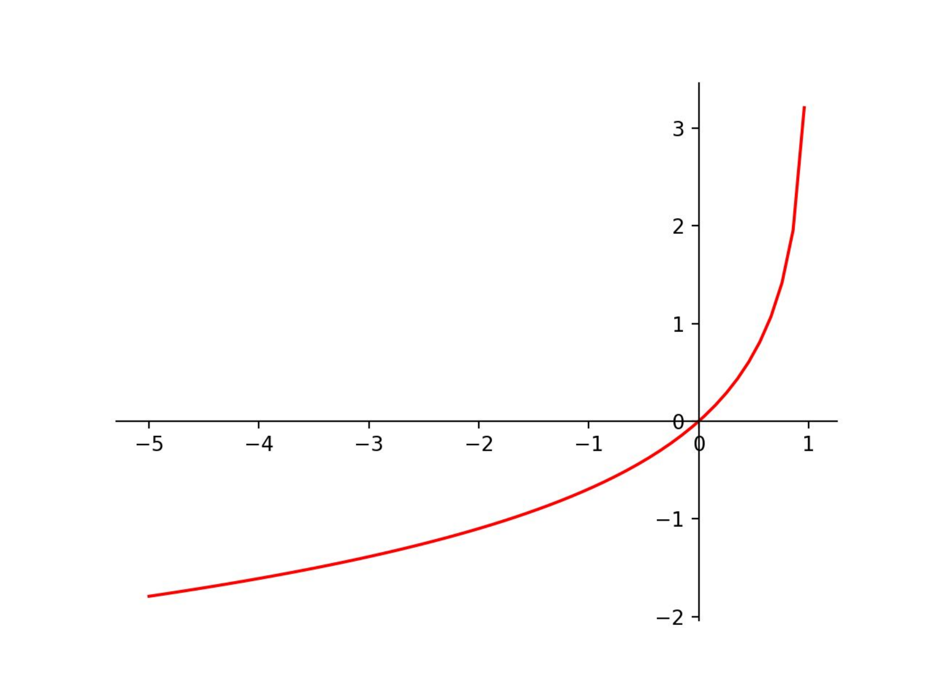
Log function

$$y = -\log(-x)$$

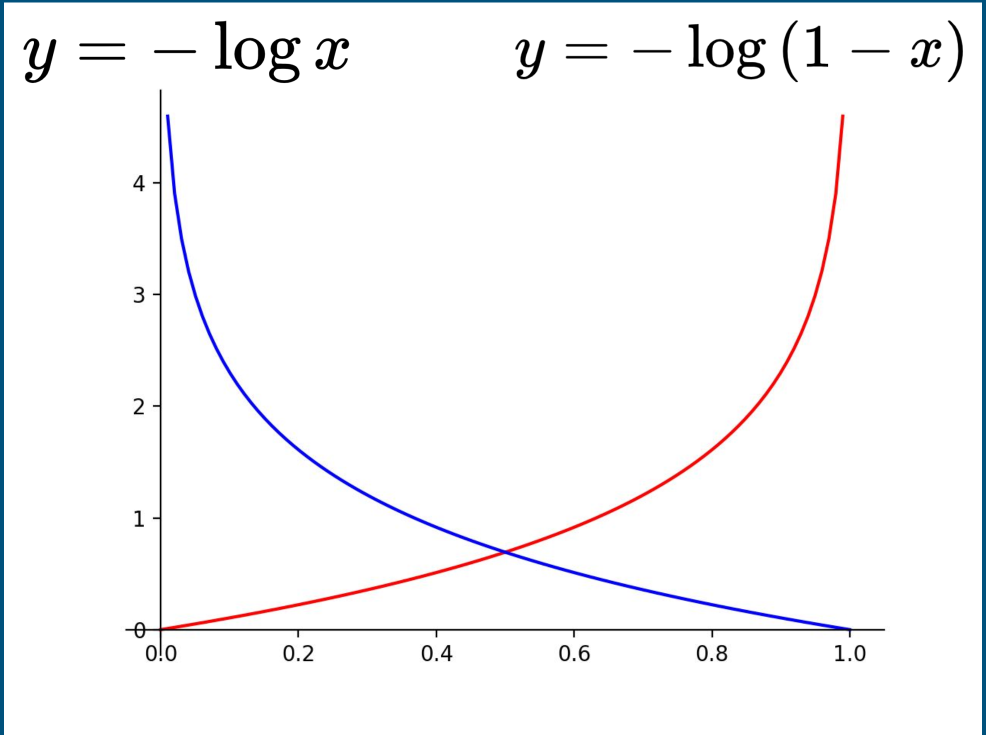


Log function

$$y = -\log(1 - x)$$



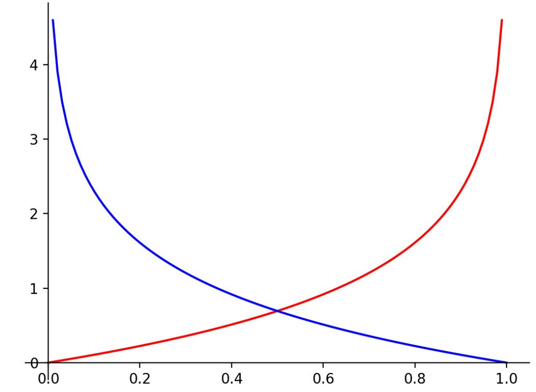
Log functions



Lab 6

Using matplotlib package in python, please plot two log functions below. Submit source code and output image file.

$y = -\log x \qquad y = -\log(1-x)$



Hint

```
import matplotlib.pyplot as plt

import numpy as np

np.seterr(divide = 'ignore')

x = np.linspace(0,1)

y1 = -np.log(x)

y2 = -np.log(1-x)

plt.plot(x, y1)

plt.plot(x, y2)

plt.show()
```