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Business-Cycle Phases and Their Transitional Dynamics

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This article examines differences in expansionary and contractionary phases of the business cycle. By extending the nonlinear Markov-switching estimation method of Hamilton to incorporate time-varying probabilities of transitions between the phases, the marginal benefits of the time-varying transition probability Markov-switching model are highlighted. Using this technique, I document the high correlation between the evolution of the phases inferred from the model and traditional reference cycles for monthly output data. Many of the economic variables that determine the time-varying probabilities help to predict turning points. The predictive power of standard leading indicators is evaluated and compared.

KEY WORDS: Leading indicators; Markov-switching model; Maximum likelihood estimation; Monthly industrial production; Time-varying transition probability.

1. INTRODUCTION

This article argues that a Markov-switching model with time-varying transition probabilities (TVTP) can characterize the dynamics of business cycles better than the fixed transition probability (FTP) version and standard linear time series models. TVTP can provide valuable additional information about whether a particular phase has occurred and whether a turning point is imminent by incorporating economic time series from the goods and financial markets that can help identify the phase that the economy is in and that can forecast when the economy may switch phases. With its extra flexibility and nonlinear structure, the TVTP model appears to capture and predict accurately the expansions and contractions of monthly U.S. output data.

The notion that the economy evolves through periods of expansions and recessions is not new. In their seminal work, Burns and Mitchell (1946) made significant headway in documenting recurrent cycles of quantities and prices. Although their research focused on four salient features of the business cycle—a taxonomy of phases, the mean length of the cycle and half-cycles, the amplitude of the fluctuations, and the coherences with other variables—expansions, contractions, and their turning points were the central focus of Burns and Mitchell's data analysis. Their view that output alternates between periods of expansion and contraction of varying durations is consistent with recent empirical research of asymmetric output fluctuations (see Brock and Sayers 1988; Brunner 1992; Neftçi 1984; Sichel 1993).

These inherent asymmetries of business-cycle expansions and contractions challenge previous empirical modeling strategies. Although low-order linear stochastic difference equations can generate patterns that mimic asymmetric time series behavior, a central drawback of these models, from the perspective of this article, is that most linear time series models have a lot of symmetry built in. Unlike in the Markov-switching models, expansions, contractions, and durations play, at best, a small role in accounting for business-cycle behavior. Markov-switching models, in contrast, attempt to model these features. In fact, linear models, like autoregressive integrated moving averages (ARIMA's), vector autoregressive moving averages (VARMA's), and index models, only generate behavior consistent with the business cycle through a small-sample run of either positive or negative shocks and thus are not geared toward estimating these asymmetric business-cycle features. [Business-cycle research strategies with linear models are not mute on the issue of phase identification. The identified phases are not intrinsic to the underlying data-generating process, however. For the best recent examples of this methodology, see Stock and Watson (1993) and Zellner, Hong, and Gulati (1990).]

Recent advances in nonlinear time series techniques allow economic researchers to reconsider the traditional characterization of business-cycle phenomena. Previous research into FTP models by Hamilton (1989) suggested that quarterly real gross national product (GNP) exhibits significant asymmetries arising from the differences in the mean growth rates, in the transition probabilities between the two phases, and in the unconditional expected duration of each phase. More importantly, he documented that both the timing and duration of the phases correspond closely to National Bureau of Economic Research (NBER) contractions and expansions, despite the fact that the structure of this state-dependent model is not geared explicitly toward fitting business-cycle behavior.

Because of its extra flexibility and more intuitive appeal, the TVTP model presented in this article can focus on more features of the business cycle than models with fixed transition probabilities and, as a result, can improve the forecasting ability of these state-dependent models. There are three reasons that time variation may be a significant extension of the FTP model. First, the TVTP model allows the transition probabilities to rise just before a contraction or an expansion begins; an FTP does not. In an FTP model, the transition probabilities are constant before, during, and after turning points. TVTP models, however, have the flexibility to identify systematic variations in the transition probabilities both before and after turning points.

Second, the TVTP model may capture more complex temporal persistence than an FTP model. Both the FTP and TVTP models can distinguish between two sources of business-cycle persistence. One source comes through the autoregressive (AR) parameters. The other source comes through the persistence of the phase over time. The latter source is gauged by the transition probability matrix. Allowing the transition probabilities to vary expands the nature of the persistence that can be identified.

Third, TVTP are intrinsically linked to the notion of time-varying expected durations in the Markov-switching framework. As pointed out by Filardo and Gordon (1993), expected durations can vary across time in the TVTP model. In an FTP model, the expected duration of a phase is constant. This constancy is at odds with both Burns and Mitchell's (1946) view of business cycles and recent studies of postwar business cycles. Diebold, Rudebusch, and Sichel (1993) found that time-varying hazards rates are important in accounting for the duration of postwar contractions, a finding that was verified by Durland and McCurdy (1992) in the context of a semi-Markov-switching model. In related work, Ghysels (1991, 1992a,b) showed that the economic phases are more likely to persist during certain seasons of the year.

The structure of the article is as follows: Section 2 develops the TVTP model, which includes the FTP model as a nested alternative. Section 3 introduces testing issues, and Section 4 presents the empirical results using monthly industrial production as a proxy for aggregate output. I compare the TVTP model with the FTP model and focus on the information content of various leading indicators for business-cycle turning points. The final subsection of Section 4 reviews the forecasting performance of the TVTP model as a business-cycle model.

2. THE TVTP MODEL

The TVTP Markov-switching model of aggregate output growth, y_t, allows for distinct business-cycle phases with state-dependent means and for cyclical dynamics of aggregate output with the lagged predetermined variables. The model assumes that the state of the economy cannot be known with certainty, in the sense that the econometrician can neither observe the state of the economy nor deduce the state indirectly. The states, however, are assumed to be path dependent and to evolve according to a first-order Markov process with TVTP coefficients.

The TVTP model with state-dependent means, predetermined right-side variables, and normally distributed errors yields

$$
\begin{aligned}
y_t &= \mu^0 + \Phi(L)\left(y_{t-1} - \mu^{S_{t-1}}\right) + e_t \quad \text{if state } 0 \\
&= \mu^1 + \Phi(L)\left(y_{t-1} - \mu^{S_{t-1}}\right) + e_t \quad \text{if state } 1, \qquad (1)
\end{aligned}
$$

where $\Phi(L) = \phi_1 + \phi_2 L + \cdots + \phi_r L^{r-1}$ is the lag polynomial, $\mu^{S_t} = \mu_0 + \mu_1 S_t$ is the state-dependent mean, $e_t \sim N(0, \sigma^2)$, and $S_t \in \{0, 1\}$. (This model can easily be extended to include state-dependent AR coefficients, Φ_{S_t}, state-dependent error processes, e_{S_t}, and other dependent variables, $\mathbf{x}_t$.) The two-point stochastic process on S_t can be summarized by the transition matrix

$$
\begin{aligned}
P(S_t = s_t \mid S_{t-1} = s_{t-1}, \mathbf{z}_t) \\
= \Lambda = \begin{bmatrix} q(\mathbf{z}_t) & 1 - p(\mathbf{z}_t) \\ 1 - q(\mathbf{z}_t) & p(\mathbf{z}_t) \end{bmatrix}, \qquad (2)
\end{aligned}
$$

where the history of the economic-indicator variables is $\mathbf{z}_t = \{z_t, z_{t-1}, \ldots\}$.

In this TVTP model, the parameters in Equation (1) and the transition probability parameters in Equation (2) are jointly estimated. The conditional joint density-distribution, f, summarizes the information in the data and explicitly links the transition probabilities to the estimation method and tests. With AR dynamics of order r, the conditional density, f^*, is

$$
\begin{aligned}
&f^*(y_t \mid y_{t-1}, \ldots, y_{-r}, \mathbf{z}_t) \\
&= \sum_{s_t=0}^{1} \cdots \sum_{s_{t-r}=0} f(y_t, S_t = s_t, \\
&\qquad S_{t-1} = s_{t-1}, \ldots, \\
&\qquad S_{t-r} = s_{t-r} \mid y_{t-1}, \ldots, y_{-r}, \mathbf{z}_t) \\
&= \sum_{s_t=0}^{1} \cdots \sum_{s_{t-r}=0}^{1} \hat{f}(y_t \mid S_t = s_t, \ldots, S_{t-r} \\
&\qquad = s_{t-r}, y_{t-1}, \ldots, y_{-r}) \\
&\quad \times P(S_t = s_t \mid S_{t-1} = s_{t-1}, \mathbf{z}_t) \\
&\quad \times P(S_{t-1} = s_{t-1}, \ldots, S_{t-r} \\
&\qquad = s_{t-r} \mid y_{t-1}, \ldots, y_{-r}, \mathbf{z}_{t-1}) \qquad (3)
\end{aligned}
$$

and the log-likelihood function is $L(\theta) = \sum_{t=1}^{T} \ln[f^*(y_t \mid y_{t-1}, \ldots, y_{-r}, \mathbf{z}_t; \theta)]$.

Equation (3) shows exactly how the information in output growth and economic indicators, $\mathbf{z}_t$, affects the model's estimation and inference. Both sources of information enter in two ways, one directly and the other indirectly through the inference of the past states. The information in y_t and its lags directly influences the likelihood through the normal density, $\hat{f}$; the lags of y_t indirectly affect the likelihood through the information they provide about the past states $P(S_{t-1} = s_{t-1}, \ldots, S_{t-r} = s_{t-r} \mid y_{t-1}, \ldots, y_{-r}, \mathbf{z}_{t-1})$. The economic-indicator variables affect the transition probabilities, $P(S_t = s_t \mid S_{t-1} = s_{t-1}, \mathbf{z}_t)$, directly and the distribution of the states, $P(S_{t-1} = s_{t-1}, \ldots, S_{t-r} = s_{t-r} \mid y_{t-1}, \ldots, y_{-r}, \mathbf{z}_{t-1})$ indirectly. In the TVTP model of this article, $\hat{f}$, which is not a function of the variables in $\mathbf{z}_t$, is independent of $\mathbf{z}_t$. Even though the output equation can include $\mathbf{z}_t$ (the nonlinear relationship between the right-side variables of Equation (1) and the economic-indicator variables in the time-varying transition probabilities can guarantee identification), this study sidesteps this feature to emphasize the potential contribution of TVTP to understanding business-cycle dynamics. This specification makes it easier to separate out the marginal contribution of the economic indicators on the inference about the state of the economy.

One empirical hallmark of the Markov-switching model is its inference about the unobserved state of the economy. In the TVTP model, information about the lags of output growth and the information contained in $\mathbf{z}_t$ combine to help identify which state of the economy has occurred. To assess the effect of time variation in the transition probabilities on the inference about the growth state of the economy, the explicit link betwen the transition probabilities, $P(S_t = s_t \mid S_{t-1} = s_{t-1}, \mathbf{z}_t)$, and the inferred probability of the state of the economy, $P(S_t = s_t \mid y_t, \ldots, y_{-r}, \mathbf{z}_t)$, needs to be made. The inferred probability of the state of the economy at time t can be calculated by integrating out the effects of the past states in the joint density-distribution,

$$\begin{aligned} P(S_t = s_t \mid y_t, y_{t-1}, \ldots, y_{-r}, \mathbf{z}_t) &= \sum_{s_{t-1}=0}^{1} \cdots \sum_{s_{t-r}=0}^{1} P(S_t = s_t, \ldots, S_{t-r} \\ &\qquad = s_{t-r} \mid y_t, \ldots, y_{-r}, \mathbf{z}_t) \\ &= \sum_{s_{t-1}=0}^{1} \cdots \sum_{s_{t-r}=0}^{1} \\ &\quad \times \frac{f(y_t, S_t = s_t, \ldots, S_{t-r} = s_{t-r} \mid y_{t-1}, \ldots, y_{-r}, \mathbf{z}_t)}{f^*(y_t \mid y_{t-1}, \ldots, y_{-r}, \mathbf{z}_t)}. \end{aligned} \tag{4}$$

As is clear from Equation (3), the transition probabilities influence the density-distribution, f, and thus directly affect the inferred probabilities through the numerator of the third line of Equation (4). (If the normal density were modeled to include $\mathbf{z}_t$, the influence of the transition probabilities would be obscured.)

The FTP model of Hamilton (1989, 1990) is a nested alternative to the TVTP model. When the economic-indicator variables are not informative about the evolution of the state of the economy, the TVTP model becomes an FTP model.

3. TESTS

3.1 Methodological Issues

With the switching model of Equation (1) and the Markov process on the states, S_t, of Equation (2), the parameters of interest are jointly estimated with maximum likelihood (ML) methods for mixtures of normals. Other algorithms have been suggested for this problem. The EM algorithm was employed by Diebold, Lee, and Weinbach (DLW) (in press) to solve a related estimation problem. For the purposes of this article, however, the ML approach is preferred on the basis of computational ease in a model with AR dynamics. Generally, the EM algorithm is difficult to implement in the presence of AR lags. And as DLW found, the functional form of the TVTP complicates the evaluation in the maximization step. The Gibbs sampler was used by Filardo and Gordon (1993) successfully to model this problem. Although a tractable and attractive strategy for FTP and TVTP, this framework requires tight priors for estimation (see also Albert and Chib 1992; McCulloch and Tsay 1992).

Because the state is unobserved, a technique analogous to a Kalman filter is employed to classify the observations into the two states and to jointly estimate the parameters of the model and the process on the states [see Hamilton (1989) for an outline of the general filter and Filardo (1993) for an outline of the filter for this problem and for restrictions on $\mathbf{z}$ to estimate the model]. With errors that are normally distributed, ML estimates can be calculated by finding the roots of the likelihood equations associated with the log-likelihood function. Kiefer (1978) showed in the case of an iid switching model that a solution to the likelihood equations yields consistent, asymptotically normal, and asymptotically efficient estimators; in addition, the negative of the inverse of the matrix of second partial derivatives of the likelihood function at the true parameter value is a consistent estimate of the asymptotic variance–covariance matrix of the parameter values, given that the second derivative of the likelihood function is nonsingular at the true parameter value. The small-sample properties of the estimators, however, are not well known. As for tests generally, and for the joint tests of the significance of the coefficients of the TVTP specifically, assuming that functions of the restrictions are twice differentiable around the true parameters and the gradient of the functions are of full rank in the neighborhood of the true parameters, standard likelihood-ratio tests of the restrictions are valid.

The rank conditions that justify the preceding testing procedure may be violated for two independent reasons due to the structure of the switching. The first reason stems from the possibility that one data point may represent a separate regime. In this case, the variance parameter may be 0 (i.e., not in the interior of the parameter space), the likelihood is unbounded in theory, and inconsistent estimates may be found in practice. Various suggestions to deal with these problems have been proposed in the literature (see Hamilton 1991; Kiefer 1980; Phillips 1991 for further discussion). In this article, I follow Hamilton's (1989) strategy of modeling the variance across regimes to be the same, an assumption that is consistent with the data.

The second, and perhaps more important, reason involves the natural test of whether the data are best characterized by one state or by two states. Tests of the number of states imply restrictions that make the information matrix of the parameters singular under the null. The regularity conditions of the preceding asymptotically valid test statistics therefore do not hold. Various methods to evaluate the number of states present in the data have been proposed. In the context of Markov-switching models, see Boldin (1990), Garcia (1992), Hamilton (1991), and Hansen (1992). Given that the issue of TVTP is of primary importance in this article, assuming the presence of two states sidesteps this difficult issue and justifies the asymptotic tests.

In searching for a particular functional form of the transition probabilities, any specification that maps the $\mathbf{z}_t$ variables into the unit interval is a valid candidate; the ML estimation, however, will constrain the set of valid candidates. The logistic function, probit function, Cauchy integral, and piecewise continuously differentiable functions are all valid candidates. In this article, I choose to model the transition probabilities in the logistic family of functional forms.

3.2 Parameter Estimates

The switching model in Equation (1) assumes that the state-dependent means characterize the two distinct economic phases. Tests for the existence of two distinct economic phases gauge whether the state-dependent means are statistically different. Because the mean growth rate in state 0 is μ_0 and the mean growth rate in state 1 is $\mu_0 + \mu_1$, the test is whether the difference of the means, μ_1, is different from 0. Furthermore, testing if μ_0 and $\mu_0 + \mu_1$ are negative and positive, respectively, is a necessary condition to determine that the switching models are describing contractions and expansions.

Tests of time variation are joint tests of the appropriateness of the functional form of the time-varying probabilities and the statistical significance of the coefficients on the information variables, $\mathbf{z}_t$. The logistic functional form for the transition probabilities maps the information variables, $\mathbf{z}_t$, into the open interval (0, 1) and thereby guarantees a well-defined log-likelihood function. The parameterization for the tests is

$$p(\mathbf{z}_t) = \frac{\exp\left(\theta_{p0} + \sum_{j=1}^{J_1} \theta_{pj} z_{t-j}\right)}{1 + \exp\left(\theta_{p0} + \sum_{j=1}^{J_1} \theta_{pj} z_{t-j}\right)}$$

$$q(\mathbf{z}_t) = \frac{\exp\left(\theta_{q0} + \sum_{j=1}^{J_2} \theta_{qj} z_{t-j}\right)}{1 + \exp\left(\theta_{p0} + \sum_{j=1}^{J_2} \theta_{qj} z_{t-j}\right)}. \qquad (5)$$

In this specification, the FTP model corresponds to the restriction that $\theta_{pi} = \theta_{qi} = 0$, for $i \neq 0$. This functional form and these restrictions satisfy the necessary conditions to apply the likelihood-ratio test. In particular, under the null hypothesis of no time variation in the transition probabilities, the FTP model is not accepted if $\Psi = 2 \times [L(\theta) - L(\theta_R)]$ exceeds $\chi^2_{J_1+J_2,\alpha}$, where $J_1 + J_2$ is the number of restrictions. The Akaike information criterion (AIC) and the Schwartz criterion (SC) will be employed to assist in choosing the "appropriate" order of the lags of the variables in the TVTP.

The type of "news" contained in the $\mathbf{z}_t$ variables can be inferred from the movements in $p(\mathbf{z}_t)$ and $q(\mathbf{z}_t)$. For example, if p_t increases and q_t decreases when $\mathbf{z}_t$ increases, both the transition probability from the high-growth-rate state to the high-growth-rate state rises and the transition probability from the low-growth-rate state to the high-growth-rate state rises (i.e., $1 - q_t$ increases). Regardless of the economy's state at time t, the probability of being in the high-growth-rate-state at time $t + 1$ increases. In this sense, the "news" in $\mathbf{z}_t$ is good news. In the univariate specification for p_t and q_t, the good-news content of $\mathbf{z}_t$ is measured by θ_{p1} and θ_{q1} having opposite signs.

Time series measures must also be used to assess the marginal contribution of the TVTP model in finding business-cycle dynamics because the statistically significant coefficient values on variables $\mathbf{z}_t$ alone are not sufficient to do so. First, the inferred probabilities of the state are presented and used to verify that cyclical behavior is being captured. Second, the transition probabilities are transformed to reveal their turning-point information.

4. RESULTS

4.1 Data Description

The logarithmic first difference of seasonally adjusted total industrial production (1987 = 100) from the Federal Reserve Board serves as the proxy for the growth rate in aggregate output, y_t. To use the full postwar series, the apparent variance nonstationarity of the series in the first third of the sample (Fig. 1) was dealt with by deflating the pre-1960 observations by the ratio of the subsamples' standard deviations.

The candidate series for the information variables, $\mathbf{z}_t$, are those considered to be useful as business-cycle predictors—the Composite Index of Eleven Leading Indicators (CLI), the CLI's diffusion index (DFI) of the percent of its components rising over one month [smoothed with $S(L) = 1 + 2 \cdot L + 2 \cdot L^2 + L^3$], the Stock and Watson (1989) Experimental Index of Seven Leading Indicators (XLI), the term premium which is the 10-year less the 1-year constant-maturity treasury interest rate (TP), the Standard and Poor's Composite Stock Index (SP), and the Federal Funds Rate (FF). These monthly samples run from January 1948 to August 1992, except for XLI, TP, and FF, which start in January 1960, May 1953, and July 1954, respectively. The CLI and SP are expressed in log growth rates, and the XLI, FF, and TP are level differenced; all series are demeaned so that the constant parameters in the TVTP specification capture the level effects.

Note that the output and economic-indicator series are the revised series and should not be confused with the real-time data that forecasters possess when the numbers first become available. As Diebold and Rudebusch (1987, 1991a,b) noted, revisions to the CLI are especially troublesome, and therefore the revised data should be used with caution. With respect to interpretation, there are two interrelated issues that are being addressed in this article, the nature of business-cycle dynamics and the use of indicator variables to predict the evo-

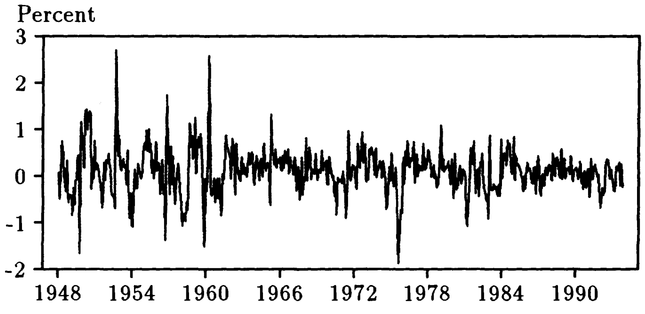

Figure 1. Log Growth Rate of Monthly IP. Dispersion in the first third of the sample appears greater than in the later part. F tests confirm the higher residual variance of autoregressions across the subsamples. To use the whole sample, the early period is deflated by the ratio of the standard deviations.

Table 1. *FTP and TVTP Estimates and Tests of the Statistical Significance of Time Variation*

		TVTP models using the composite leading index				*TVTP models using various series*				
Parameter	*FTP* (1)	(2) $\{cli_{t-1}\}$	(3) $\{cli_{t-1}, cli_{t-2}\}$	(4) $\{cli_{t-1}, dfi_{t-1}\}$	(5) $\{cli_{t-1}, cli_{t-2}, dfi_{t-1}\}$	(6) $\{xli_{t-1}\}$	(7) $\{xli_{t-1}, xli_{t-2}\}$	(8) $\{s\&p_{t-1}\}$	(9) $\{fed_{t-1}\}$	(10) $\{tp_{t-1}, tp_{t-2}\}$
μ_0	−1.712	−.896	−1.288	−.582	−.778	−1.630	−1.710	−1.274	−1.204	−1.858
	(.289)	(.172)	(.228)	(.203)	(.164)	(.297)	(.319)	(.193)	(.160)	(.258)
$\mu_0 + \mu_1$	.330	.461	.371	.494	.455	.362	.355	.426	.473	.314
	(.074)	(.079)	(.063)	(.068)	(.056)	(.078)	(.077)	(.086)	(.079)	(.066)
θ_{q0}	.458	1.293	−3.577	−.493	−2.524	.332	.218	1.182	.858	−.536
	(.329)	(.430)	(.672)	(.596)	(.540)	(.720)	(.823)	(.297)	(.320)	(.985)
θ_{q1}		−1.033	−2.493	−1.208	−2.133	.187	.175	−.143	−1.078	1.344
		(.583)	(.848)	(.628)	(.547)	(.435)	(.423)	(.306)	(.349)	(1.033)
θ_{q2}			−6.699		−5.968		.055			−.170
			(1.012)		(.782)		(.359)			(1.405)
θ_{q3}				−1.784		3.983	4.174	3.854	3.442	4.638
				(.638)		(.467)	(.586)	(.309)	(.349)	(.483)
θ_{p0}	3.906	4.205	3.826	3.371	2.812	−.057	.250	.862	−.486	.782
	(.371)	(.507)	(.550)	(.770)	(.455)	(.492)	(.577)	(.192)	(.295)	(.289)
θ_{p1}		1.654	.726	−1.803	−.870		−.567			−1.127
		(.359)	(.559)	(.811)	(.478)		(.642)			(.373)
θ_{p2}			.167							
			(.502)							
θ_{p3}				3.014	1.631					
				(1.323)	(.527)					
p value*		.002	.000	.001	.000	.934	.931	.047	.055	.028
AIC/SC	129.3/167.7	127.2/174.2	126.7/182.2	127.9/183.5	125.2/180.7	95.2/138.7	98.9/150.3	130.2/177.2	114.6/159.8	115.5/169.4

NOTE: Standard errors are in parentheses. Definitions of variables and sample periods are listed in Section 4.1.
* *p* value for the test of no time variation: $H_0 : \theta_i = 0, i \neq 0$.

lution of the phases. For the former issue, using revised data may be appropriate, especially if the agents of the economy see the true data and the collecting agencies accurately report the data later. As for the latter issue, the results of this article should not be interpreted as a real-time forecasting exercise, an exercise that could prove misleading even if the model is correct but the data are error-ridden. Prediction after a short lag, however, may be appropriate and informative, especially if alternative business-cycle chronologies (e.g., the NBER's chronology) are available after a much longer lag. The inferred probabilities present some evidence in support of this position because the data used to produce the forecasts were available before the NBER called the latest trough.

4.2 Estimates and Interpretation

Both econometric and graphical methods assess the degree to which TVTP models fit the data and characterize business-cycle fluctuations. Table 1 contains the estimates and the tests of FTP and TVTP models. The significance of time variation is captured in the point estimates and tests of the transition probability specification in Equation (1) and Equation (5). The estimates of interest are the state-dependent-growth rates in each state, μ_0 and $\mu_0+\mu_1$, and the coefficients that determine the TVTP, θ_p and θ_q. Heuristic arguments using Figure 2 reveal the relationship between time variation and business-cycle time.

The FTP Model. Column (1) in Table 1 reports the results from an FTP model, which serves as the benchmark model for this study. In contrast to Hamilton (1989), who estimated an FTP model with quarterly real GNP from 1951:Q1 to 1984:Q4, I use monthly data over a longer sample period; my sample is from January 1948 to August 1992. This table presents significant evidence to support the assumption that two distinct growth-rate phases characterize monthly industrial production. The point estimates of the state-dependent means, μ_0 and $\mu_0 + \mu_1$, are statistically different (these inferences are based on the assumption of the existence of two states). Moreover, their magnitudes differ significantly and economically. According to the asymptotic standard errors, the point estimate of the mean growth rate in state 0, μ_0, is significantly negative and the point estimate of the mean growth rate in state 1, $\mu_0 + \mu_1$, is positive. Because the sample dichotomizes into phases that exhibit declining aggregate output and growing aggregate output, each can be labeled as low-growth and high-growth states of the economy.

Even though these estimates suggest that there is significant evidence of low- and high-growth states as in the quarterly data, the monthly FTP model fails to identify as strong a relationship between the growth-rate phases and the "contractions" and "expansions" by the NBER. Figure 2a plots the inferred probability of a low-growth-rate state given the available data, $P(S_t = 0 \mid \cdot)$. When close to 1, this measure provides strong evidence from the data that the economy was in the low-growth state; conversely, when close to 0, there is evidence that the economy was in the high-growth-state phase. Using a criterion that minimizes the probability of misclassification, the inferred probability of a low-growth-rate state can optimally classify the data into two subsamples based on $P(S_t = 0 \mid \cdot)$ being above or below .5. With this criterion, the monthly sample shows some correlation between the model's growth phases and NBER expansions and contractions, but this correlation is weak at best.

The TVTP Model and Economic Performance. The TVTP estimation results in Table 1 verify that the data can

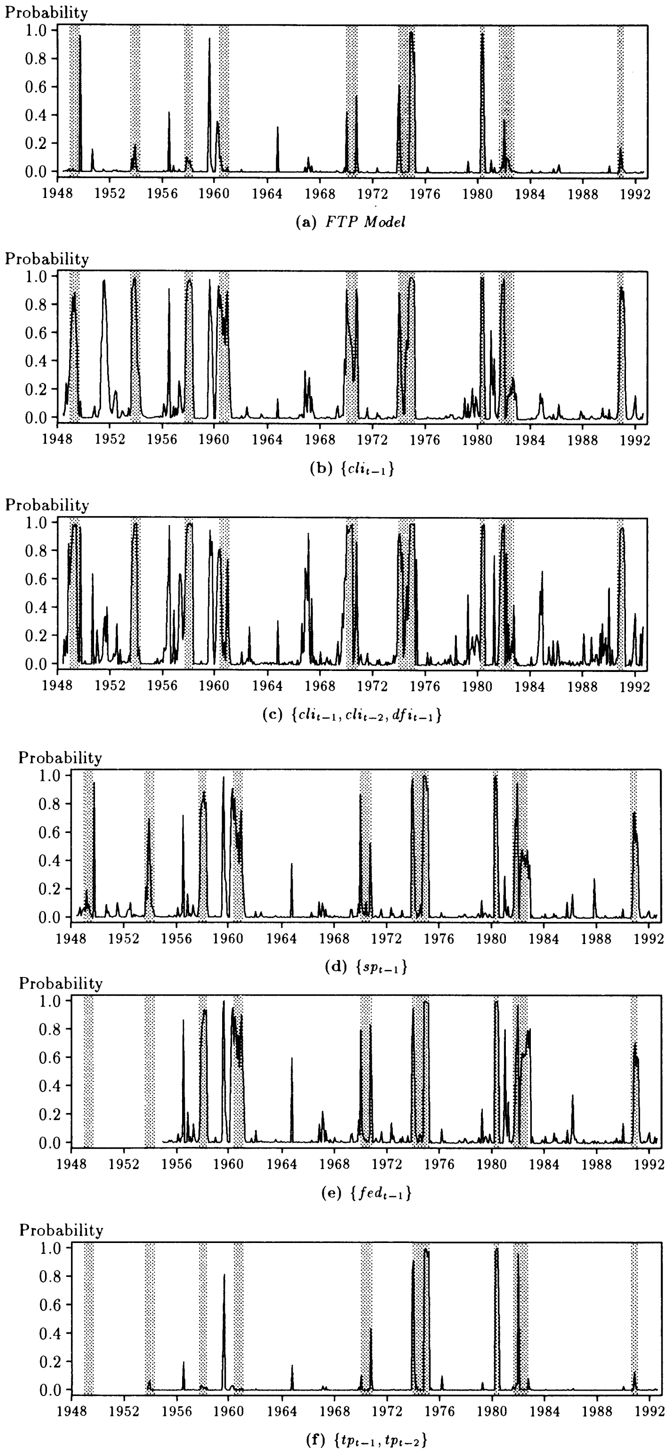

Figure 2. Inferred Probability of the Low Growth Rate State, $P(S_t = 0 \mid y_t, \ldots, y_{-r}, z_t)$, for Various Models: (a) FTP Model; (b) TVTP Model With CLI, $\{cli_{t-1}\}$; (c) TVTP Model With CLI, $(\{cli_{t-1}, cli_{t-2}, dfi_{t-1}\})$; (d) TVTP Model With SP, $\{sp_{t-1}\}$; (e) TVTP Model With FED, $\{fed_{t-1}\}$; (f) TVTP Model With TP, $\{tp_{t-1}, tp_{t-2}\}$.

be classified into high and low growth phases. The estimated means of the two states, μ_0 and $\mu_0 + \mu_1$, measure whether there are two statistically distinct states, and all estimations confirm that μ_0 is significantly negative and $\mu_0 + \mu_1$ is significantly positive. There is evidence that the relatively small sample of contractions in the postwar period affects the precision of the parameters associated with the low-growth state. (The ϕ's and σ are stable across estimations and are available from me on request.)

Columns (2)–(5) in Table 1 list the estimated transition probabilities for the CLI. Examining these CLI results [CLI(2)–CLI(5)] reveals that the TVTP model can significantly improve the fit of the data over the FTP model. In each specification, at least one, and in the case of CLI(5) all, of the point estimates θ_{p1} and θ_{q1} are statistically significant at the 5% level, and all specifications, soundly reject at better than the 1% level the hypothesis that both sets of coefficients are 0. In columns 4 and 5, the transition probabilities include the lagged CLI diffusion index. For these specifications, the TVTP coefficients are jointly statistically significant with p values better than .1%; the AIC and the SC support CLI(5) over CLI(4). Taken together, these four CLI specifications reveal much about the TVTP model generally and the TVTP model's ability to account for U.S. business-cycle behavior.

These TVTP specifications show that CLI information generally represents "good" or "bad" news. This is best illustrated with CLI(2), where p and q are functions of $\{\text{cli}_{t-1}\}$. The TVTP coefficients, θ_q and θ_p, have opposite signs in this specification. Thus the transition probabilities p_t and q_t move in the opposite directions when $z_t = \{\text{cli}_{t-1}\}$ fluctuates. If, for example, z_t is positive and the coefficient estimates θ_p are positive, the probability that the state will be in the high-growth-rate state in the next period increases because both p_t and $1 - q_t$ increase. Such movements in p_t and $1 - q_t$ accord with one's economic intuition of how a useful indicator would fluctuate, and it is in this sense that the CLI information comes in the form of "good" or "bad" news. (When θ_p's and θ_q's do not have opposite signs, the evaluation of the news content is more difficult.)

While satisfying statistical tests for significance, specifications CLI(2) and CLI(5) also go a long way in accounting for the recessionary and expansionary phases of the economy. Figures 2b and 2c plot the inferred probabilities of the state of the economy from these specifications. Comparing these graphs with Figure 2a reveals the relatively high correlation between the inferred probabilities and the NBER chronology. Taking the NBER chronology as given, both TVTP specifications, however, appear to produce several false signals of recessions. Diebold and Rudebusch (1989) examined several scoring measures to examine the relationship between probabilities and binary variables. The quadratic probability score (QPS) can be employed to measure how close, on average, the inferred probabilities and the NBER dates are. With a symmetric loss function (which yields conservative results if missing recession periods are more important than falsely identifying them), this metric confirms the visual impression of the improved fit with the TVTP model: $\text{QPS}_{\text{FTP}} = .324$, but $\text{QPS}_{\text{CLI(2)}} = .200$ and $\text{QPS}_{\text{CLI(5)}} = .237$. A perfect fit would yield a QPS = 0.

Despite the fact that the experimental leading index is constructed to account for the same economic behavior as the CLI, the XLI and CLI appear to contain different information. (Note that the XLI series start is 1960.) Unlike the CLI(2) and CLI(3) results, the TVTP coefficients for the XLI are not as statistically significant. The likelihood-ratio tests of the

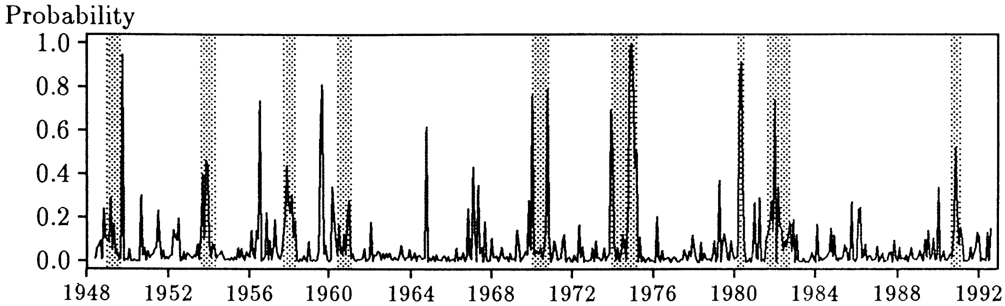

Figure 3. Mean Effect for TVTP. The time-varying transition probability parameters of the $\{cli_{t-1}, cli_{t-2}, dfi_{t-1}\}$ specification are set to 0. This figure does not capture the high correlation between the inferred probabilities and the NBER dates as in Figure 2c. The lack of correlation suggests that the business-cycle dynamics stem mainly from the variation in the transition probabilities rather than from a shift in the means.

zero restrictions are not rejected at the 5% significance level with p values .93 for the $\{\text{xli}_{t-1}\}$ and $\{\text{xli}_{t-1}, \text{xli}_{t-2}\}$ specifications. Moreover, the coefficients are generally statistically insignificant and not of the intuitive sign. As for scoring the inferred probabilities, there appears to be little difference between the FTP model and the XLI(1) and XLI(2) specification: $\text{QPS}_{\text{FTP}} = .289$ whereas $\text{QPS}_{\text{XLI(1)}} = .273$ and $\text{QPS}_{\text{XLI(2)}} = .284$. This result accords with the lack of correlation between the inferred probabilities and NBER dates [a plot of the inferred probabilities for the XLI(1) and XLI(2) specifications would mimic the FTP result in Figure 2a].

Table 1 also reports the estimation results using other candidate indicator variables often suggested as indicators about the future evolution of the economy. Three such candidates are SP (Fama 1990), FF (Bernanke and Blinder 1992), and TP (Harvey 1988), all of which have different starting dates; this must be kept in mind when comparing the results. The estimated coefficients of the TVTP for the results with SP, FF, and TP are statistically significant with the p values of the likelihood-ratio tests ranging from .028 to .055. The SP coefficients are positive and statistically significant for the high-growth-rate transition probability, and insignificant but of the intuitively plausible sign (-) for the low-growth-rate transition probability. FF's coefficients are negative and significant for the low-growth-rate transition probability and only marginally insignificant for the high-growth-rate transition probability coefficient. The sign for θ_p is intuitively plausible, but the negative sign for θ_q is somewhat counterintuitive and may reflect that interest rates tend to fall as recessions begin and tend to rise as expansions begin. The p values of the tests of no time variation for SP and FF are confirmed with the QPS statistics: $\text{QPS}_{\text{SP}} = .252$ (vs. $\text{QPS}_{\text{FTP}} = .324$) and $\text{QPS}_{\text{FED}} = .168$ (vs. .291). Their inferred probabilities in Figures 2d and 2e, however, reveal limited information about the state of the economy. Despite the fact that the transition probability coefficients for TP sum to positive and negative values and jointly reject the test of no time variation at standard confidence levels, the TP inferred probability of the state of the economy in Figure 2f resembles the FTP results in Figure 2a. The scoring statistic confirms this impression with $\text{QPS}_{\text{TP}} = .308$ (vs. $\text{QPS}_{\text{FTP}} = .317$).

Time Variation and Performance of the Economy. In the previous section, the evidence revealed that sensible estimates alone need not guarantee a tight correlation between the inferred probabilities, $P(S_t = 0 \mid \cdot)$, and the business-cycle chronology. This section will show that, once a specification is chosen based on the estimates and inferred probabilities, two further diagnostics should be run to verify the marginal contribution of time variation.

First, the time variation of the transition probabilities may either directly improve the fit of the inferred probabilities or indirectly influence the fit through affecting the estimates of the means, μ_0 and μ_1. As Hamilton (1989) showed, an FTP model with state-dependent means and constant transition probabilities can provide insightful inferences about the state of the economy. A way to control for the state-dependent mean effect on the inferred probabilities is to estimate the TVTP model's parameters and then zero out the parameters on $\mathbf{z}_t$—that is, θ_{pi} and θ_{qi} for $i \neq 0$. (A slightly different experiment would be to use the FTP estimates with the TVTP model's transition probabilities, p_t and q_t.)

I follow this strategy and plot the result in Figure 3. The graph shows the effect on the inferred probabilities of zeroing out the TVTP parameters. Comparing this figure with Figure 2c (inferred probabilities from the TVTP with $z_t = \{\text{cli}_{t-1}, \text{cli}_{t-2}, \text{dfi}_{t-1}\}$) indicates that when the time variation is "turned off," the change in the correlation between the inferred probabilities and the business-cycle chronology dramatically drops. Thus it appears that business-cycle dynamics of this model stem mainly from the variation in the transition probabilities rather than from a shift in the means.

Second, the marginal advantage of TVTP over constant transition probabilities can be assessed by examining the deviations of the transition probabilities from their mean. There are two motivations for plotting this statistic. First, plots of p_t and q_t are difficult to interpret. For example, plotting $p_t = P(S_t = 1 \mid S_{t-1} = 1, \mathbf{z}_t)$ does not show fully the relevant contribution of time variation because the variation is only relevant when $S_{t-1} = 1$. The information in the transition probabilities can be seen more clearly when the transition probability is weighted by the probability $P(S_{t-1} = 1 \mid \cdot)$. Second, since the *marginal* contribution of the transition

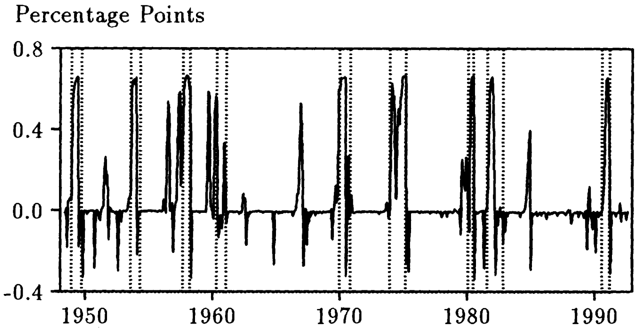

Figure 4. Marginal Contribution of TVTP, q_t.

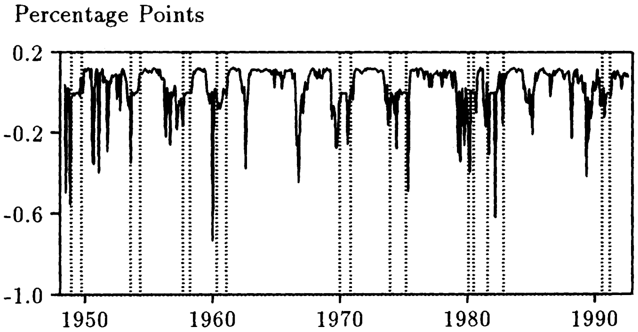

Figure 5. Marginal Contribution of TVTP, p_t.

probabilities is of interest, the mean of the transition probabilities, $\bar{p}$, should be subtracted. Both alterations improve the ability to interpret the variation in p_t. The resulting weighted transition probability series (WTP) for p_t is

$$\begin{aligned} \mathrm{WTP}(p_t) &= [P(S_t = 1 \mid S_{t-1} = 1, \mathbf{z}_t) - \bar{p}] \\ &\quad \times P(S_{t-1} = 1 \mid y_t, \ldots, y_{-r}, \mathbf{z}_t) \\ &= (p_t - \bar{p}) \times P(S_{t-1} = 1) \\ &= p_t \cdot P(S_{t-1} = 1) - \bar{p} \cdot P(S_{t-1} = 1). \qquad (6) \end{aligned}$$

The last line in Equation (6) highlights the benefit of using the TVTP by focusing solely on the variation. This measure will identify when the time variation is important.

Figures 4 and 5 report the WTP for q_t and p_t. The marginal contribution is clear from the deviations from 0. The spikes in the WTP's correspond strongly for q_t and less so for p_t with the turning points of the business cycle denoted by the dotted lines. (The two standard error bands for the WTP's are available from me on request.) These results support the hypothesis that the CLI are providing information about the end of contractions and expansions, a conclusion that is consistent with the aim behind the series' construction (see Hymans 1972; Zarnowitz and Boschan 1975).

Out-of-Sample Forecasting. The out-of-sample forecasting experiment with the TVTP model during the last NBER business cycle confirms the TVTP's marginal benefit over the FTP model and several alternative time series models. The evidence does not unambiguously support the TVTP over all alternatives, but the preponderance of the evidence suggests that during this last NBER business cycle the TVTP model acted as a credible business-cycle model. In addition, the ability of the TVTP model to pick up the business-cycle behavior during the last cycle points out that even though the real time-forecasting potential of the model may be limited (see discussion in Sec. 4.1) such a model may provide a shorter recognition lag than classical business-cycle methods.

In Figure 6, the one-step-ahead output-growth forecasts of five empirical business-cycle models are plotted (solid lines) against the actual realizations for the period of January 1989 to December 1991. The models are an ARIMA (4, 1, 0) model, an FTP model, a TVTP model, a Squashing(4, 4) model (see Granger, Teräsvirta, and Anderson 1993), and a VAR(4, 1, 0) model. The last three models include $\{\mathrm{cli}_{t-1}, \mathrm{dfi}_{t-1}\}$ and their lags. [The Squashing model can be thought of as the natural alternative to the TVTP. Instead of allowing abrupt changes in the means, the means are smooth functions of the economic indicators: $y_t = \alpha_0 + \alpha_1 \cdot h(z_t) + \Phi(L)(y_{t-1} - \alpha_0 - \alpha_1 h(\mathbf{z}_t))$, where $h(\mathbf{z}_t)$ is a logistic function of $\{\mathrm{cli}_{t-1}, \mathrm{cli}_{t-2}, \mathrm{dfi}_{t-1}\}$.]

Comparing the ARIMA and the FTP shows that extra information in the transition probabilities of the TVTP model helps to track the economy during the downturn. In addition, the discrete nature of the state-dependent means helps the TVTP model to pick up the depth of the drop in growth rates better than the Squashing regression and the VAR. In terms of the mean-squared error of the forecasts, the visual patterns are confirmed with the models listed previously measuring .566, .568, .490, .503, and .491, respectively. Although the TVTP closely tracks the economy during the recent downturn, the VAR and Squashing regression models are close seconds. On the whole, the TVTP model appears to be a competitive business-cycle model.

5. CONCLUSION

In this article, I have incorporated TVTP into Hamilton's FTP model to estimate and characterize certain aspects of monthly output. Although the assumption that the business cycle can be meaningfully dichotomized into expansionary and contractionary phases is questioned by many, I find statistically significant evidence that the model supports this two-phase view for the U.S. postwar business cycles. Both statistical and graphical evidence support this finding. The point estimates and their statistical significance show that output growth experiences one phase with a positive growth rate and another with a negative growth rate, the former having higher persistence. Moreover, the persistence of each phase varies across time. The statistical significance of TVTP is confirmed by their relevance and importance in accounting for the evolution of the phases. The ability of certain economic indicators to help account for the TVTP suggests, as Burns and Mitchell (1946) pointed out, that the seeds of the next phase of the economy are found in the present, a feature that many theoretical business-cycle models rarely stress.

In the TVTP specification, the ability of the CLI to perform well as an information variable for the business cycle is an important finding, independent of the time-variation aspect of the model. Relative to some other candidate leading indi-

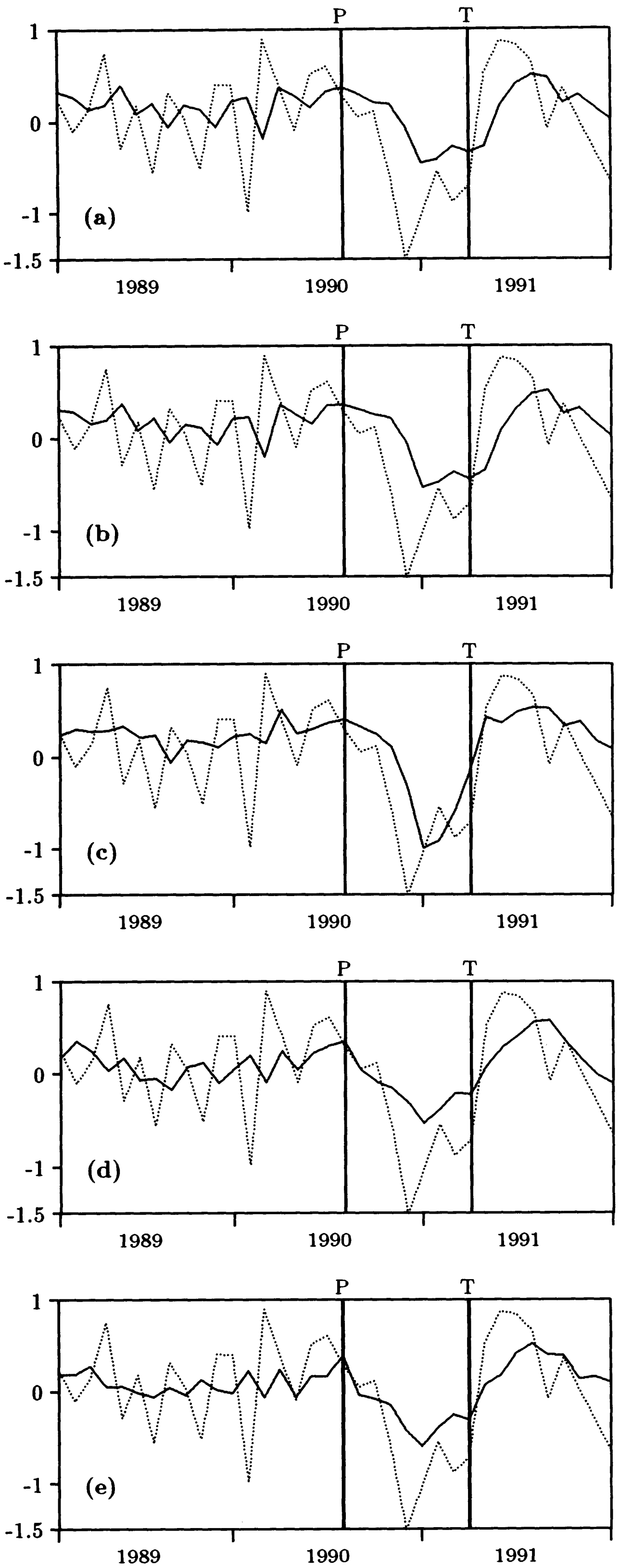

Figure 6. Comparison of Monthly Out-Of-Sample One-Step-Ahead Forecasts of Output Growth Rates for Various Models From January 1989 to December 1991: Forecast,—; Actual, . . . : (a) ARIMA(4, 1, 0) Model; (b) FTP Model; (c) TVTP Model; (d) Squashing Regression Model; (e) VAR(4, 1, 0) Model.

cators, albeit a very limited set of alternatives, they perform well. Moreover, the CLI tends to perform well at the times when the index is touted to have the strongest explanatory power—around turning points. Within this output study of the evolution of business-cycle phases, endless combinations of the leading-indicator series can be tested for their marginal predictive content about expansions and contractions; clearly, theoretical models are needed to suggest the most fruitful avenues of pursuit.

Statistically, this TVTP methodology is quite general and can be used to test a host of economic and noneconomic problems in which the means, variances, and AR parameters are assumed to evolve along in a dichotomous manner. This article has shown that direct estimation of the likelihood function is both feasible and informative.

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