Exam 1

Econ 8375 - Econometrics II

October 26, 2011

1. *(3 points)* Solve the following first-order difference equation

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t. \tag{1}$$

   You can either solve it iteratively of using the lag operator L.

2. *(6 points)* Given the following AR(1) process

$$y_t = \phi y_{t-1} + \varepsilon_t, \tag{2}$$

   where $\{\varepsilon_t\}$ is a white-noise process with variance σ^2.

   (a) Derive its moving average representation (using the lag operator L or by substituting backwards).

   (b) Derive the unconditional mean and variance.

   (c) Obtain the two-step-ahead forecast.

3. *(10 points)* Given the process

$$y_t = \text{ARMA}(p,q) + \varepsilon_t \tag{3}$$

   (a) Explain a way to test for GARCH errors based on the Ljung-Box Q-statistic.

   (b) Explain the LM test for ARCH errors developed in McLeod and Li.

   (c) Explain how to model the conditional variance to depend on variable x_t. When is this type of model useful?

   (d) Explain how to modify the mean equation to depend on the conditional variance. When is this type of model useful?

(e) Explain how to modify the conditional variance to allow for an asymmetric effect of x_t on the conditional variance. When is this type of model useful?

4. *(6 points)* When jointly modeling two variables in a two-equation model with shocks ε_{1t} and ε_{2t}, you define the conditional variance-covariance matrix as
$$H = \begin{bmatrix} h_{11t} & h_{21t} \\ h_{12t} & h_{22t} \end{bmatrix},$$
where $\varepsilon_{1t} = v_{1t}(h_{11t})^{1/2}$, $\varepsilon_{2t} = v_{2t}(h_{22t})^{1/2}$, var$(v_{1t}) =$ var$(v_{2t}) = 1$, and h_{ijt} are just the time t conditional variance of the shock i if $i = j$, and it is the conditional covariance if $i \neq j$.

   (a) Write down the conditional variances of the multivariate GARCH(1,1) process when you allow the volatility terms to interact with each other.

   (b) What is the problem when estimating such a model?

   (c) What is the typical restriction imposed to estimate such a model?

5. *(3 points)* Describe *one* of the following procedures

   (a) Box-Jenkins Model Selection.

   (b) Zivot and Andrews unit root test.

6. *(3 points)* Consider the following random walk plus drift process
$$y_t = y_0 + a_0 t + \sum_{i=1}^{t} \varepsilon_i. \tag{4}$$
Show that by either *differencing* or *detrending* you can transform the process into a stationary sequence.

7. *(4 point)* From the figure below, identify the different processes:

   (a) White noise: ε_t

   (b) MA(1): $y_t = \varepsilon_t + 0.80\varepsilon_{t-1}$

   (c) AR(1): $y_t = +0.9y_{t-1} + \varepsilon_t$

   (d) AR(1): $y_t = -0.9y_{t-1} + \varepsilon_t$

   (e) ARCH: $\varepsilon_t = v_t(1 + 0.8\varepsilon_{t-1})^{1/2}$

(f) Conditional variance: $h_t = 1 + 0.65\varepsilon_{t-1}^2$

(g) Random walk: $y_t = y_{t-1} + \varepsilon_t$

(h) Stationary trend: $y_t = 30 - 0.3t + \varepsilon_t$

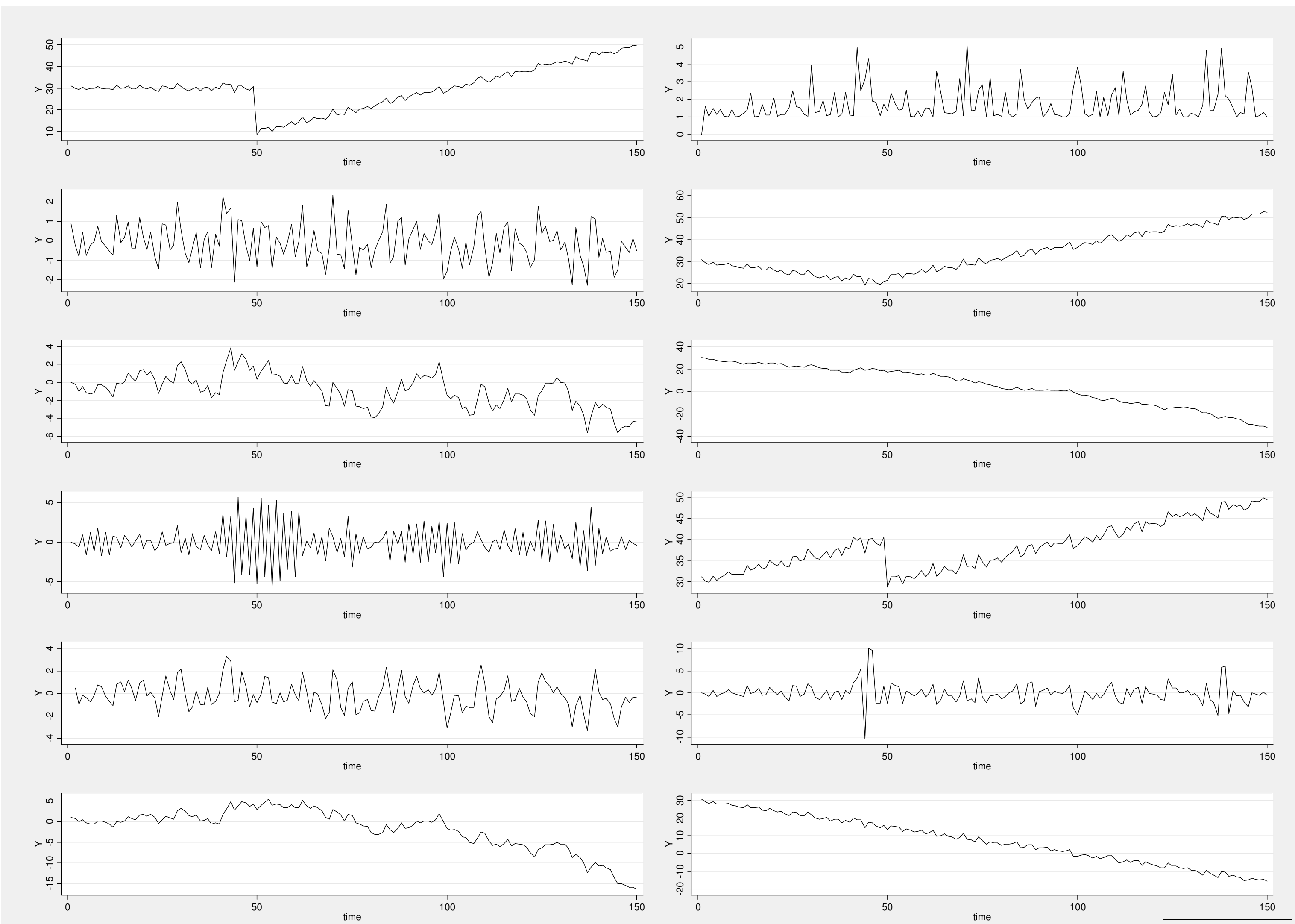
(i) Random walk plus drift: $y_t = y_{t-1} - 0.3 + \varepsilon_t$ with $y_0 = 30$

(j) Model (A): $y_t = \mu_1 + \beta t + (\mu_2 - \mu_1)D_L + \varepsilon_t$

(k) Model (B): $y_t = \mu + \beta_1 t + (\beta_2 - \beta_1)D_L^* + \varepsilon_t$

(l) Model (C): $y_t = \mu + \beta_1 t + (\mu_2 - \mu_1)D_L + (\beta_2 - \beta_1)D_L^* + \varepsilon_t$

where $D_L = 1$ if $t > \tau$ and zero otherwise, and $D_L^* = t - \tau$ if $t > \tau$ and 0 otherwise. What is the value of τ?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