# Computer Handout 11: Forecasting with Trend, Seasonal, and Cyclical Component

**Diego Escobari**

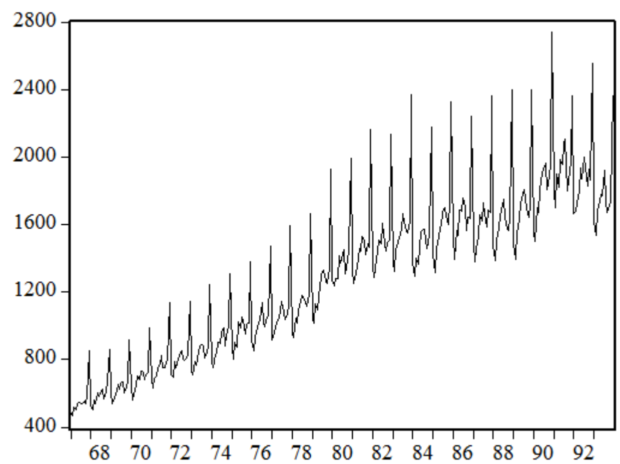
**Econ 3342**

This Computer Handout 11 will cover an example on how to forecast a model with trend, seasonal component, and cyclical component.

The variable we will use is monthly U.S. liquor sales from January 1968 until December 1993. For the sample:

smpl 1967m1 1993m12

The time series graph of the data is:

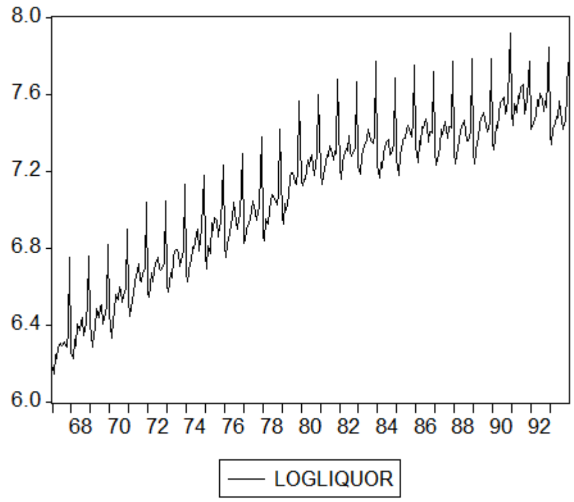



Because the variance seems to be larger for larger values of sales, we will work with the variable in logs:

smpl 1967m1 1998m12

genr logliquor = log(liquor)

The time series graph of logliquor is:




The series shows a strong seasonal component with sales being higher in December. As a first step, let's estimate the model with a quadratic trend:

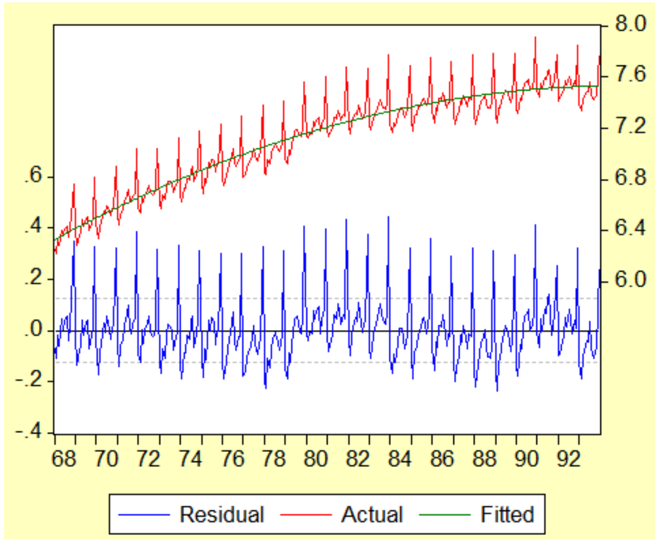
```
smpl 1968m1 1993m12

ls logliquor c @trend @trend^2
```

Dependent Variable: LOGLIQUOR
Method: Least Squares
Date: 11/15/10 Time: 00:16
Sample: 1968M01 1993M12
Included observations: 312

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| C | 6.237356 | 0.024496 | 254.6267 | 0.0000 |
| @TREND | 0.007690 | 0.000336 | 22.91552 | 0.0000 |
| @TREND^2 | -1.14E-05 | 9.74E-07 | -11.72695 | 0.0000 |

| | | | |
|---|---|---|---|
| R-squared | 0.892394 | Mean dependent var | 7.112383 |
| Adjusted R-squared | 0.891698 | S.D. dependent var | 0.379308 |
| S.E. of regression | 0.124828 | Akaike info criterion | -1.314196 |
| Sum squared resid | 4.814823 | Schwarz criterion | -1.278206 |
| Log likelihood | 208.0146 | F-statistic | 1281.296 |
| Durbin-Watson stat | 1.752858 | Prob(F-statistic) | 0.000000 |

with the following in-sample forecast, forecast errors:




The seasonal component (and any potential cyclical component) is still in the error term. Let's look at the autocorrelation and the partial autocorrelation function for various values of the displacement:

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | 0.117 | 0.117 | 4.3158 | 0.038 |
| | | 2 | -0.149 | -0.165 | 11.365 | 0.003 |
| | | 3 | -0.106 | -0.069 | 14.943 | 0.002 |
| | | 4 | -0.014 | -0.017 | 15.007 | 0.005 |
| | | 5 | 0.142 | 0.125 | 21.449 | 0.001 |
| | | 6 | 0.041 | -0.004 | 21.979 | 0.001 |
| | | 7 | 0.134 | 0.175 | 27.708 | 0.000 |
| | | 8 | -0.029 | -0.046 | 27.975 | 0.000 |
| | | 9 | -0.136 | -0.080 | 33.944 | 0.000 |
| | | 10 | -0.205 | -0.206 | 47.611 | 0.000 |
| | | 11 | 0.056 | 0.080 | 48.632 | 0.000 |
| | | 12 | 0.888 | 0.879 | 306.26 | 0.000 |
| | | 13 | 0.055 | -0.507 | 307.25 | 0.000 |
| | | 14 | -0.187 | -0.159 | 318.79 | 0.000 |
| | | 15 | -0.159 | -0.144 | 327.17 | 0.000 |
| | | 16 | -0.059 | -0.002 | 328.32 | 0.000 |
| | | 17 | 0.091 | -0.118 | 331.05 | 0.000 |
| | | 18 | -0.010 | -0.055 | 331.08 | 0.000 |
| | | 19 | 0.086 | -0.032 | 333.57 | 0.000 |
| | | 20 | -0.066 | 0.028 | 335.03 | 0.000 |
| | | 21 | -0.170 | 0.044 | 344.71 | 0.000 |
| | | 22 | -0.231 | 0.180 | 362.74 | 0.000 |
| | | 23 | 0.028 | 0.016 | 363.00 | 0.000 |
| | | 24 | 0.811 | -0.014 | 586.50 | 0.000 |
| | | 25 | 0.013 | -0.128 | 586.56 | 0.000 |
| | | 26 | -0.221 | -0.136 | 603.26 | 0.000 |
| | | 27 | -0.196 | -0.017 | 616.51 | 0.000 |
| | | 28 | -0.092 | -0.079 | 619.42 | 0.000 |
| | | 29 | 0.045 | -0.094 | 620.13 | 0.000 |
| | | 30 | -0.043 | 0.045 | 620.77 | 0.000 |
| | | 31 | 0.057 | 0.041 | 621.89 | 0.000 |
| | | 32 | -0.095 | -0.002 | 625.07 | 0.000 |
| | | 33 | -0.195 | 0.026 | 638.38 | 0.000 |
| | | 34 | -0.240 | 0.088 | 658.74 | 0.000 |

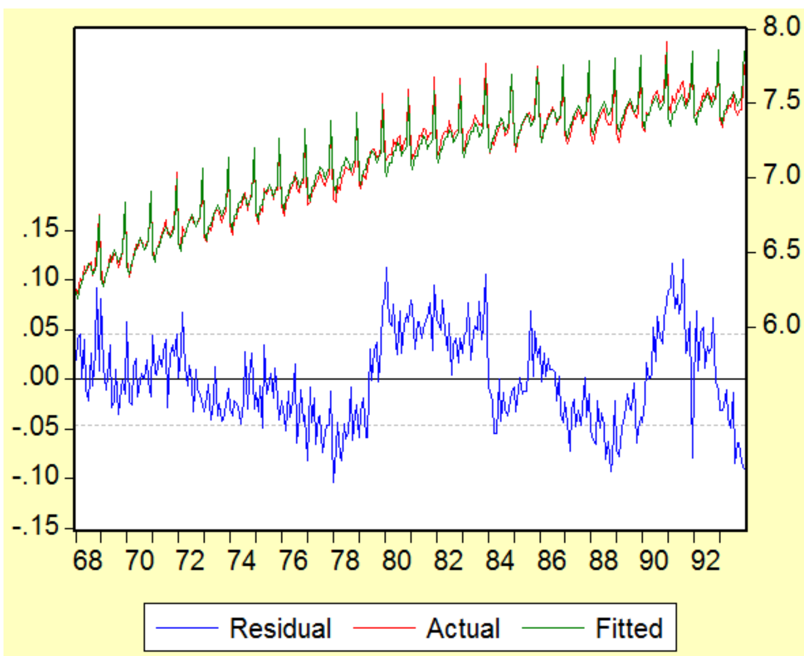
Notice the seasonal displacements at 12 and 24 and some evidence of cyclical dynamics. If we estimate the model with the monthly dummies we have:

Dependent Variable: LOGLIQUOR
Method: Least Squares
Date: 11/15/10 Time: 01:48
Sample: 1968M01 1993M12
Included observations: 312

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| @TREND | 0.007656 | 0.000123 | 62.35882 | 0.0000 |
| @TREND^2 | -1.14E-05 | 3.56E-07 | -32.06823 | 0.0000 |
| D1 | 6.147456 | 0.012340 | 498.1699 | 0.0000 |
| D2 | 6.088653 | 0.012353 | 492.8890 | 0.0000 |
| D3 | 6.174127 | 0.012366 | 499.3008 | 0.0000 |
| D4 | 6.175220 | 0.012378 | 498.8970 | 0.0000 |
| D5 | 6.246086 | 0.012390 | 504.1398 | 0.0000 |
| D6 | 6.250387 | 0.012401 | 504.0194 | 0.0000 |
| D7 | 6.295979 | 0.012412 | 507.2402 | 0.0000 |
| D8 | 6.268043 | 0.012423 | 504.5509 | 0.0000 |
| D9 | 6.203832 | 0.012433 | 498.9630 | 0.0000 |
| D10 | 6.229197 | 0.012444 | 500.5968 | 0.0000 |
| D11 | 6.259770 | 0.012453 | 502.6602 | 0.0000 |
| D12 | 6.580068 | 0.012463 | 527.9819 | 0.0000 |

| | | | |
|---|---|---|---|
| R-squared | 0.986111 | Mean dependent var | 7.112383 |
| Adjusted R-squared | 0.985505 | S.D. dependent var | 0.379308 |
| S.E. of regression | 0.045666 | Akaike info criterion | -3.291086 |
| Sum squared resid | 0.621448 | Schwarz criterion | -3.123131 |
| Log likelihood | 527.4094 | Durbin-Watson stat | 0.586187 |

The graph with the in-sample forecasting errors is:




and the correlogram:

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | 0.700 | 0.700 | 154.34 | 0.000 |
| | | 2 | 0.686 | 0.383 | 302.86 | 0.000 |
| | | 3 | 0.725 | 0.369 | 469.36 | 0.000 |
| | | 4 | 0.569 | -0.141 | 572.36 | 0.000 |
| | | 5 | 0.569 | 0.017 | 675.58 | 0.000 |
| | | 6 | 0.577 | 0.093 | 782.19 | 0.000 |
| | | 7 | 0.460 | -0.078 | 850.06 | 0.000 |
| | | 8 | 0.480 | 0.043 | 924.38 | 0.000 |
| | | 9 | 0.466 | 0.030 | 994.46 | 0.000 |
| | | 10 | 0.327 | -0.188 | 1029.1 | 0.000 |
| | | 11 | 0.364 | 0.019 | 1072.1 | 0.000 |
| | | 12 | 0.355 | 0.089 | 1113.3 | 0.000 |
| | | 13 | 0.225 | -0.119 | 1129.9 | 0.000 |
| | | 14 | 0.291 | 0.065 | 1157.8 | 0.000 |
| | | 15 | 0.211 | -0.119 | 1172.4 | 0.000 |
| | | 16 | 0.138 | -0.031 | 1178.7 | 0.000 |
| | | 17 | 0.195 | 0.053 | 1191.4 | 0.000 |
| | | 18 | 0.114 | -0.027 | 1195.7 | 0.000 |
| | | 19 | 0.055 | -0.063 | 1196.7 | 0.000 |
| | | 20 | 0.134 | 0.089 | 1202.7 | 0.000 |
| | | 21 | 0.062 | 0.018 | 1204.0 | 0.000 |
| | | 22 | -0.006 | -0.115 | 1204.0 | 0.000 |
| | | 23 | 0.084 | 0.086 | 1206.4 | 0.000 |
| | | 24 | -0.039 | -0.124 | 1206.9 | 0.000 |
| | | 25 | -0.063 | -0.055 | 1208.3 | 0.000 |
| | | 26 | -0.016 | -0.022 | 1208.4 | 0.000 |
| | | 27 | -0.143 | -0.075 | 1215.4 | 0.000 |
| | | 28 | -0.135 | -0.047 | 1221.7 | 0.000 |
| | | 29 | -0.124 | -0.048 | 1227.0 | 0.000 |
| | | 30 | -0.189 | 0.086 | 1239.5 | 0.000 |
| | | 31 | -0.178 | -0.017 | 1250.5 | 0.000 |
| | | 32 | -0.139 | 0.073 | 1257.3 | 0.000 |

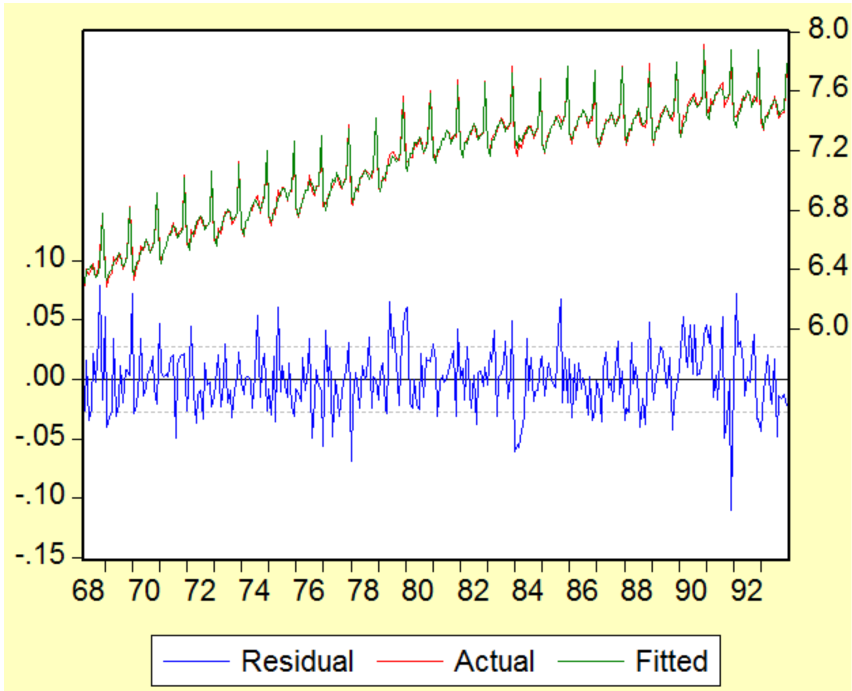
The seasonality disappeared, but there is still a strong cyclical component. With an AR(3) model for the cycle we have:

Dependent Variable: LOGLIQUOR
Method: Least Squares
Date: 11/16/10 Time: 18:01
Sample: 1968M01 1993M12
Included observations: 312
Convergence achieved after 5 iterations

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| @TREND | 0.008606 | 0.000981 | 8.769192 | 0.0000 |
| @TREND^2 | -1.41E-05 | 2.53E-06 | -5.556632 | 0.0000 |
| D1 | 6.073054 | 0.083910 | 72.37571 | 0.0000 |
| D2 | 6.013822 | 0.083931 | 71.65232 | 0.0000 |
| D3 | 6.099208 | 0.083936 | 72.66516 | 0.0000 |
| D4 | 6.101522 | 0.083923 | 72.70386 | 0.0000 |
| D5 | 6.172528 | 0.083935 | 73.53965 | 0.0000 |
| D6 | 6.177129 | 0.083936 | 73.59368 | 0.0000 |
| D7 | 6.223323 | 0.083928 | 74.15083 | 0.0000 |
| D8 | 6.195681 | 0.083931 | 73.81864 | 0.0000 |
| D9 | 6.131818 | 0.083929 | 73.05989 | 0.0000 |
| D10 | 6.157592 | 0.083923 | 73.37197 | 0.0000 |
| D11 | 6.188480 | 0.083921 | 73.74181 | 0.0000 |
| D12 | 6.509106 | 0.083916 | 77.56681 | 0.0000 |
| AR(1) | 0.268805 | 0.052909 | 5.080488 | 0.0000 |
| AR(2) | 0.239688 | 0.053697 | 4.463723 | 0.0000 |
| AR(3) | 0.395880 | 0.053109 | 7.454150 | 0.0000 |

| | | | |
|---|---|---|---|
| R-squared | 0.995069 | Mean dependent var | 7.112383 |
| Adjusted R-squared | 0.994802 | S.D. dependent var | 0.379308 |
| S.E. of regression | 0.027347 | Akaike info criterion | -4.307442 |
| Sum squared resid | 0.220625 | Schwarz criterion | -4.103496 |
| Log likelihood | 688.9610 | Durbin-Watson stat | 1.886119 |

| | | | |
|---|---|---|---|
| Inverted AR Roots | .95 | -.34+.55i | -.34-.55i |

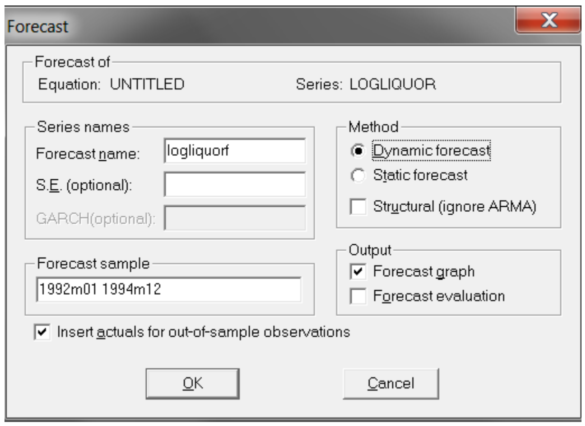


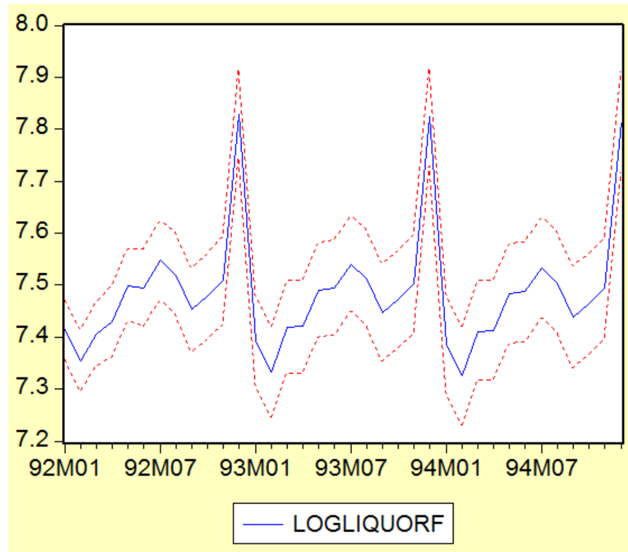

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | 0.056 | 0.056 | 0.9779 | 0.323 |
| | | 2 | 0.037 | 0.034 | 1.4194 | 0.492 |
| | | 3 | 0.024 | 0.020 | 1.6032 | 0.659 |
| | | 4 | -0.084 | -0.088 | 3.8256 | 0.430 |
| | | 5 | -0.007 | 0.001 | 3.8415 | 0.572 |
| | | 6 | 0.065 | 0.072 | 5.1985 | 0.519 |
| | | 7 | -0.041 | -0.044 | 5.7288 | 0.572 |
| | | 8 | 0.069 | 0.063 | 7.2828 | 0.506 |
| | | 9 | 0.080 | 0.074 | 9.3527 | 0.405 |
| | | 10 | -0.163 | -0.169 | 18.019 | 0.055 |
| | | 11 | -0.009 | -0.005 | 18.045 | 0.081 |
| | | 12 | 0.145 | 0.175 | 24.938 | 0.015 |
| | | 13 | -0.074 | -0.078 | 26.750 | 0.013 |
| | | 14 | 0.149 | 0.113 | 34.034 | 0.002 |
| | | 15 | -0.039 | -0.060 | 34.532 | 0.003 |
| | | 16 | -0.089 | -0.058 | 37.126 | 0.002 |
| | | 17 | 0.058 | 0.048 | 38.262 | 0.002 |
| | | 18 | -0.062 | -0.050 | 39.556 | 0.002 |
| | | 19 | -0.110 | -0.074 | 43.604 | 0.001 |
| | | 20 | 0.100 | 0.056 | 46.935 | 0.001 |
| | | 21 | 0.039 | 0.042 | 47.440 | 0.001 |
| | | 22 | -0.122 | -0.114 | 52.501 | 0.000 |
| | | 23 | 0.146 | 0.130 | 59.729 | 0.000 |
| | | 24 | -0.072 | -0.040 | 61.487 | 0.000 |

To obtain the forecast we need first to modify the sample, this is to be able to include the forecasted values:

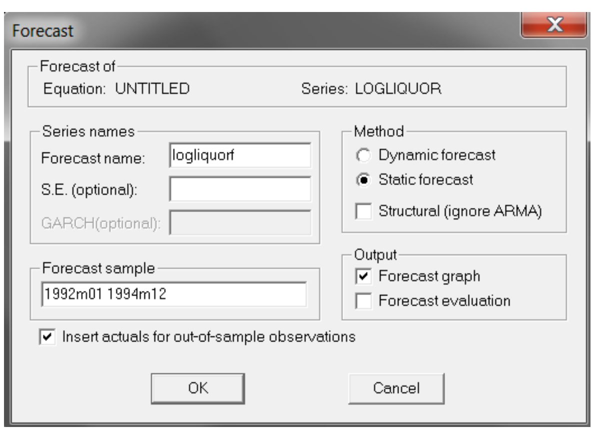
```
smpl 1968m1 1994m12
```

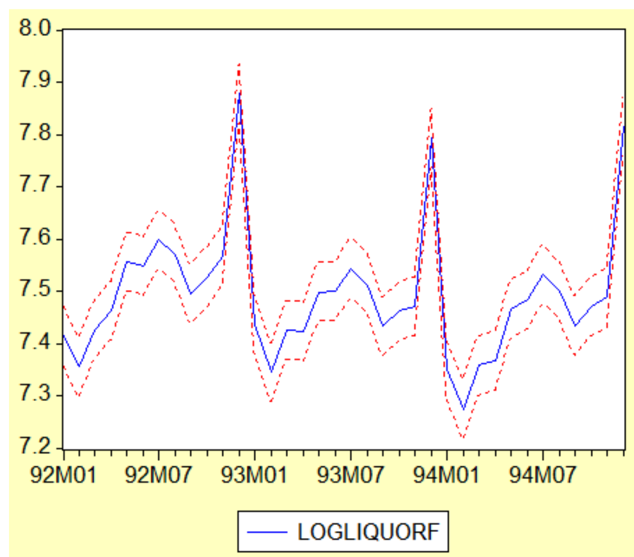
1) The dynamic forecast:






2) The static forecast:



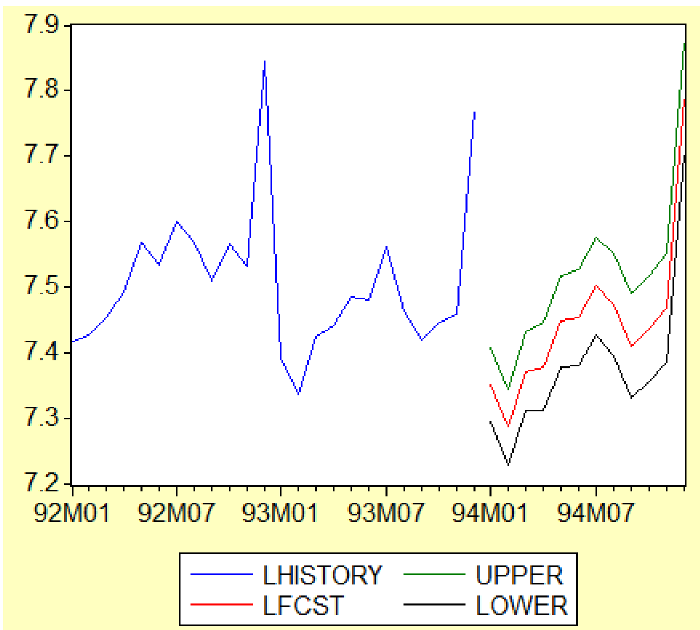



The difference between the dynamic and the static forecast is that the *dynamic forecast* uses previously forecasted values of the lagged dependent variables in forming forecasts of the current value. The *static forecast* calculates the sequence of one-step ahead forecasts, using actual, rather than forecasted values for lagged dependent variables, if available.

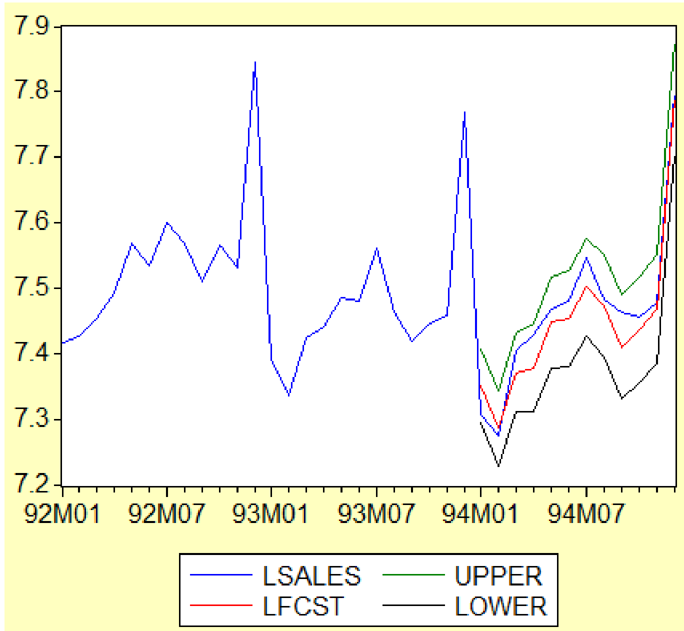
A step-by-step approach to obtain the static forecast is:

```
smpl 1966:1 1993:12

genr lhistory=logliquor

smpl 1994:1 1998:12
```

```
forecast yhat se

genr lfcst=yhat

genr fcst=@exp(yhat)

genr upper = yhat + 1.96*se

genr lower = yhat - 1.96*se

smpl 1992:1 1994:12

group group01 lhistory lfcst upper lower
```




```
group group02 logliquor lfcst upper lower
```




For the details in some of the steps, please refer to the previous Computer Handout.

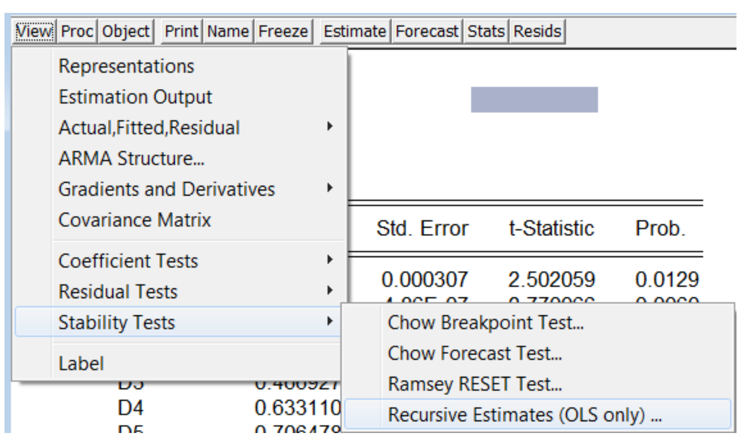
**Recursive Estimation Procedures**

Recursive estimation procedures can only be estimated when the model was estimated by ordinary least squares. Usual estimation of ARMA models use nonlinear least squares procedures that are not compatible with recursive estimation procedures.

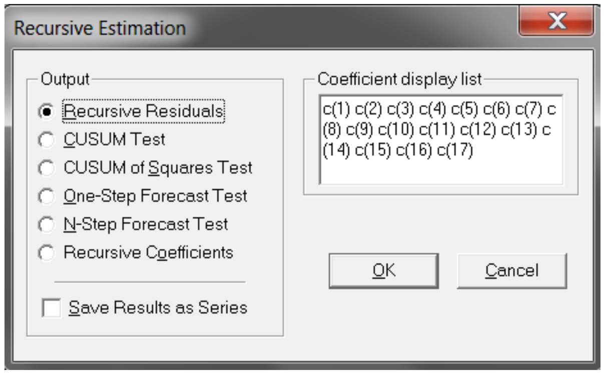
To be able to work with our model, estimate it again, but use the lag operators rather than the AR(1) notation, that is:

ls logliquor @trend @trend^2 d1 d2 d3 d4 d5 d6 d7 d8 d9 d10 d11 d12 logliquor(-1) logliquor(-2) logliquor(-3)

Go to "View," then "Stability Tests," and finally "Recursive Estimates."

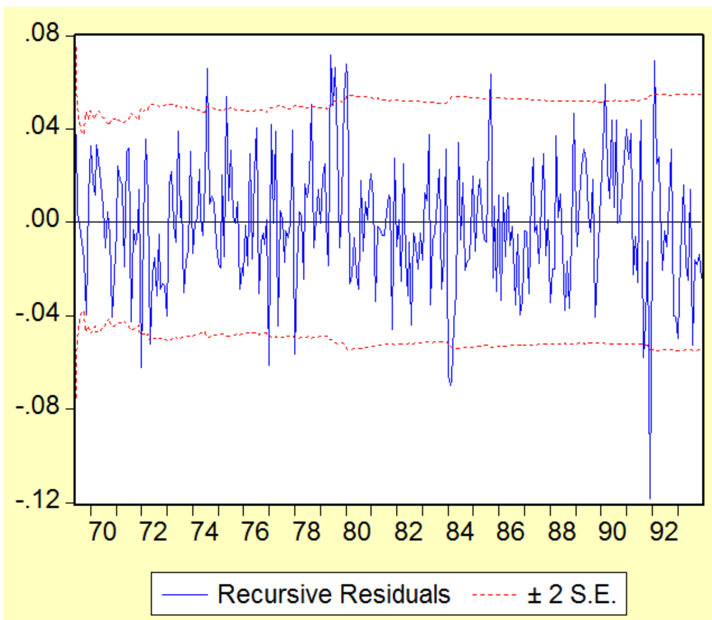



You will obtain the following menu:

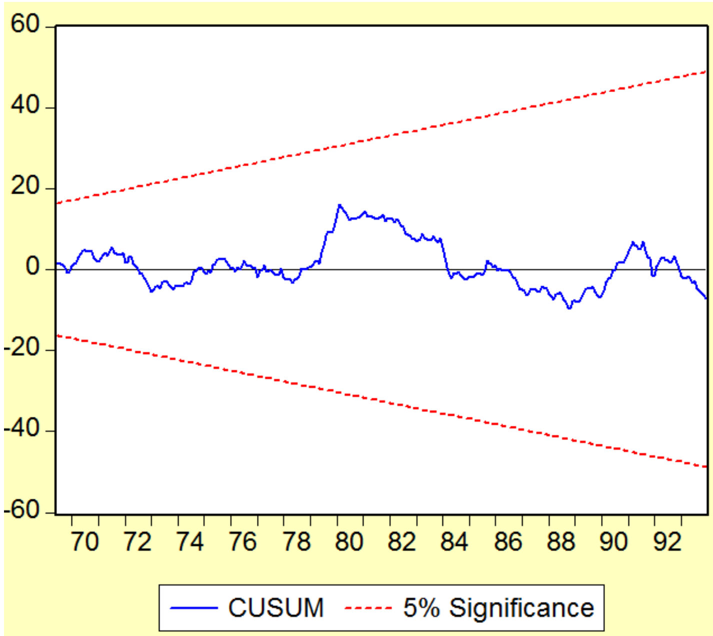



The options we discussed in class are (1) Recursive residuals, (2) CUSUM test, and (3) Recursive coefficients.

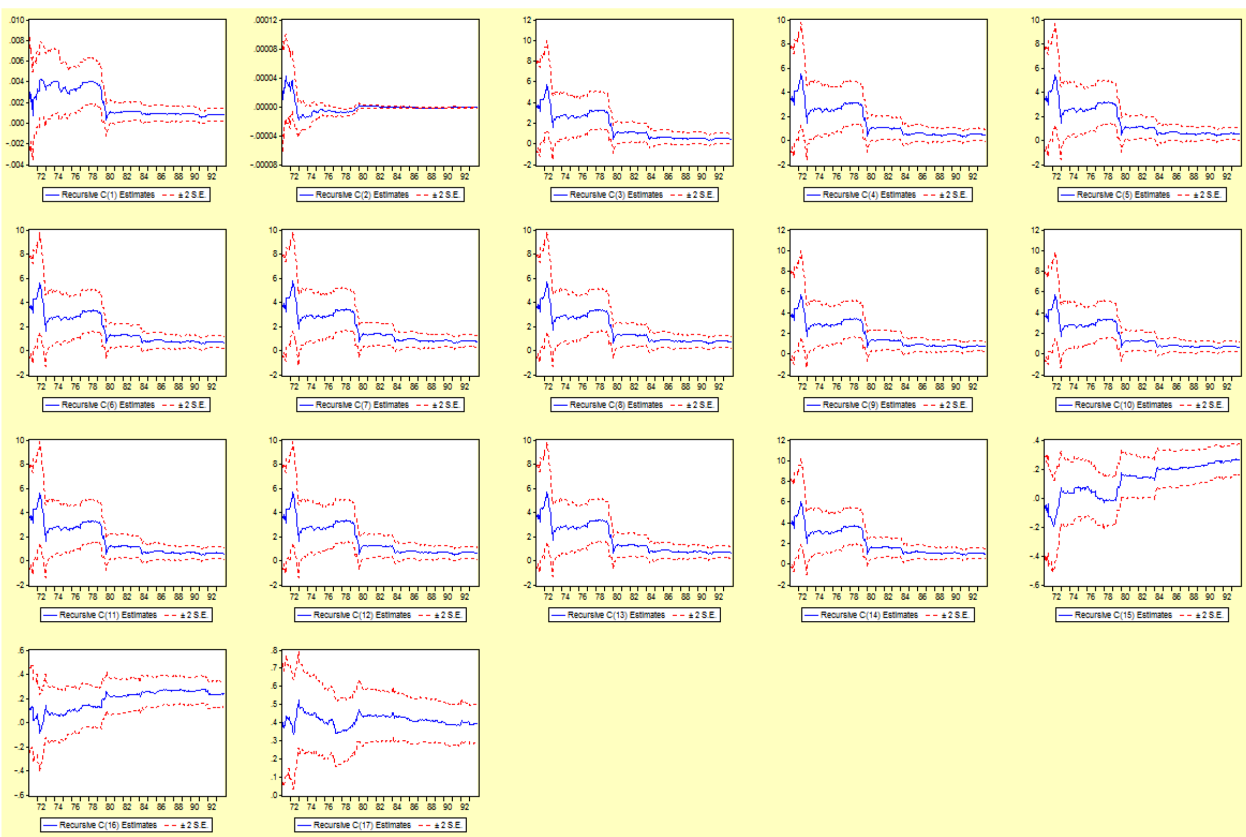
(1) Recursive residuals: (Figure 10.17)




(2) CUSUM test: (Figure 10.19)




(3) Recursive coefficients: (Figure 10.18)



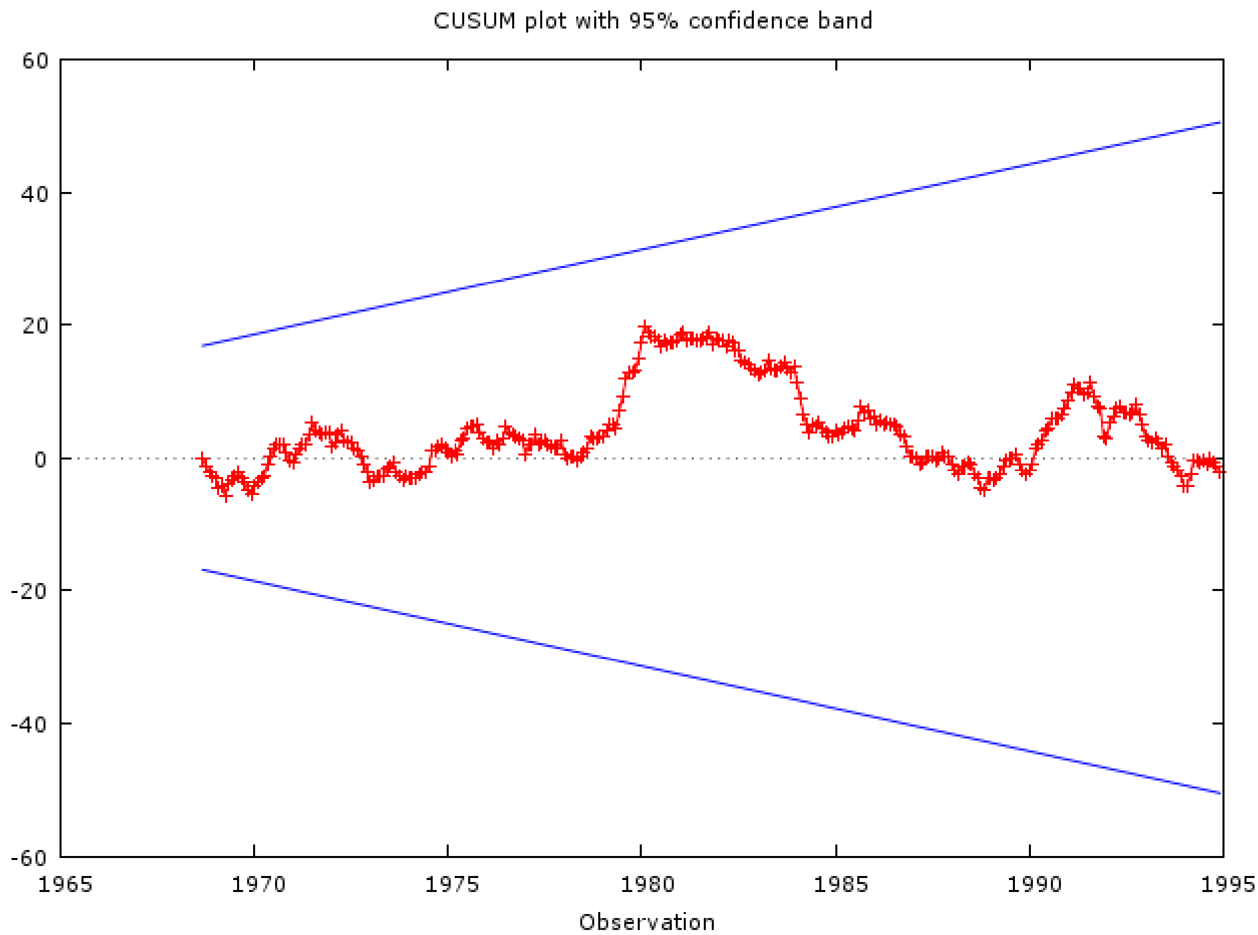
**Gretl**

Using the drop-down menus go to “add” and create a “Time trend” and a squared time trend. To estimate the model via OLS, just go to “Model” then “Ordinary Least Squared.” To create the AR(3) part go to the end of the window and click on the “lags” icon. A new window will open and at the bottom part you will be asked if you want to include lags of the dependent variable. Accept and select 3 lags.

You will be getting the following model:

```
Model 2: OLS, using observations 1967:04-1994:12 (T = 333)
Dependent variable: l_LIQUOR

                 coefficient    std. error    t-ratio    p-value
  ---------------------------------------------------------------
  D1             0.501434      0.240507       2.085    0.0379     **
  D10            0.667905      0.235938       2.831    0.0049     ***
  D11            0.717712      0.235114       3.053    0.0025     ***
  D2             0.485542      0.239436       2.028    0.0434     **
  D3             0.562791      0.237776       2.367    0.0185     **
  D4             0.721367      0.231471       3.116    0.0020     ***
  D5             0.792641      0.231825       3.419    0.0007     ***
  D6             0.744328      0.233823       3.183    0.0016     ***
  D7             0.771219      0.234742       3.285    0.0011     ***
  D8             0.702785      0.236234       2.975    0.0032     ***
  D9             0.639344      0.236466       2.704    0.0072     ***
  time           0.000862912   0.000301173    2.865    0.0044     ***
  sq_time       -1.45633e-06   4.79531e-07   -3.037    0.0026     ***
  D12            1.04880       0.235026       4.462    1.13e-05   ***
  l_LIQUOR_1     0.285118      0.0519226      5.491    8.21e-08   ***
  l_LIQUOR_2     0.221151      0.0529005      4.181    3.77e-05   ***
  l_LIQUOR_3     0.383793      0.0519233      7.392    1.31e-012  ***

Mean dependent var   7.104375   S.D. dependent var   0.395350
Sum squared resid    0.244941   S.E. of regression   0.027841
R-squared            0.995280   Adjusted R-squared   0.995041
F(16, 316)           4164.401   P-value(F)           0.000000
Log-likelihood       728.7711   Akaike criterion    -1423.542
Schwarz criterion   -1358.804   Hannan-Quinn        -1397.727
rho                  0.062639   Durbin's h           3.523214

Log-likelihood for LIQUOR = -1636.99

CUSUM test for parameter stability -
  Null hypothesis: no change in parameters
  Test statistic: Harvey-Collier t(315) = -0.120527
  with p-value = P(t(315) > -0.120527) = 0.904143
```

I was not able to find the recursive residuals and the parameter stability graphs, but I did find the CUSUM test. From the model you just estimated, go to “Tests” and then “CUSUM test” to obtain:


CUSUM plot with 95% confidence band
60
40
20
0
-20
-40
-60
1965
1970
1975
1980
1985
1990
1995
Observation