Chapter 10
Time Series

10.1 Time Series Data

The main difference between time series data and cross-sectional data is the temporal ordering. To emphasize the proper ordering of the observations, Table 10.1 presents a partial listing of the data on U.S. inflation and unemployment rates from 1948 through 2003. Unlike cross-sectional data, in time series the temporal order in which the observations appear in the data set is very important. In terms of notation, we use the subscript t to denote time and we use it instead of the previous subscript i, i.e., X_t.

Table 10.1 U.S. Inflation and Unemployment Rates, 1965-2011

Year	Inflation	Unemployment
1948	8.1	3.8
1949	-1.2	5.9
1950	1.3	5.3
1951	7.9	3.3
⋮	⋮	⋮
2000	3.4	4.0
2001	2.8	4.7
2002	1.6	5.8
2003	2.3	6.0

A second key difference between time series and cross-sectional data is that in the latter we assume that the sample was randomly drawn from the population. While in time series the variables are also considered random, a variable indexed by time is called a *stochastic process* or a *time series process*. When we collect a time series data set we are one possible outcome or realization of the stochastic process. We can only see a single realization because we cannot go back in time and start the process again.

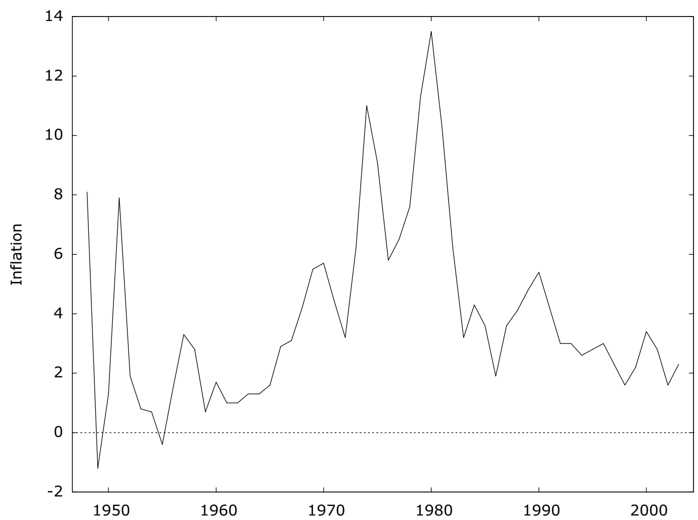

Fig. 10.1 Inflation, 1948-2003.

Graphing the data is particularly important to visualize the dynamics of the variables. Figure 10.1 presents the time series graph of inflation from 1948 through 2003. One can easily identify the periods of high inflation late in the seventies and early eighties. To obtain this graph in Gretl, go to `View` → `Graph specified vars` → `Time series plot` and then select the variables you want to plot against time.

10.2 Time Series Regression Models

10.2.1 Static Models

The simplest *static model* has the form

$$Y_t = \beta_1 + \beta_2 X_t + u_t, \quad t = 1, 2, 3, \ldots, n. \tag{10.1}$$

We call this a static model because we are only modeling a contemporaneous relationship between X_t and Y_t. That is, when a change in X at time t is believed to have an immediate effect on Y: $\Delta Y_t = \beta_2 \Delta X_t$. One example is the static Phillips curve given by:

$$\texttt{inflation}_t = \beta_1 + \beta_2 \texttt{unemployment}_t + u_t. \tag{10.2}$$

where `inflation` is the annual inflation rate, and `unemployment` is the unemployment rate. Estimation in Gretl via OLS is follows the same steps as in the previous chapters. The output for the estimation of Equation 10.1 is:

```
Model 1: OLS, using observations 1948-2003 (T = 56)
Dependent variable: inflation

             coefficient   std. error   t-ratio   p-value
  ---------------------------------------------------------
  const          1.05357       1.54796     0.6806    0.4990
  unemployment   0.502378      0.265562    1.892     0.0639 *

Mean dependent var   3.883929   S.D. dependent var   3.040381
Sum squared resid    476.8157   S.E. of regression   2.971518
R-squared            0.062154   Adjusted R-squared   0.044786
F(1, 54)             3.578726   P-value(F)           0.063892
Log-likelihood      -139.4304   Akaike criterion     282.8607
Schwarz criterion    286.9114   Hannan-Quinn         284.4311
rho                  0.572055   Durbin-Watson        0.801482
```

$$\widehat{\texttt{inflation}} = \underset{(1.5480)}{1.05357} + \underset{(0.26556)}{0.502378}\,\texttt{unemployment}$$

$$T = 56 \quad \bar{R}^2 = 0.0448 \quad F(1,54) = 3.5787 \quad \hat{\sigma} = 2.9715$$

(standard errors in parentheses)

The estimation results indicate that a one point increase in the unemployment rate is linked with a 0.5 increase in the inflation rate. Of course more variables can be included in the model. Notice that we can use this model to predict `inflation` given that we know the values for `unemployment` by simply plugging values for unemployment in the estimated equation. If we do this for the actual unemployment values for 1948-2003 period and graph them, we obtain the fitted values graph. Figure 10.2 plots the actual and the fitted values for inflation.

10.2.2 Finite Distributed Lag Models

The simplest dynamic model is the *finite distributed lag* (FDL) model, where we allow one or more variables to to affect Y_t with a lag. Consider the following example:

$$Y_t = \beta_1 + \beta_2 X_t + \beta_3 X_{t-1} + \beta_4 X_{t-2} + u_t, \tag{10.3}$$

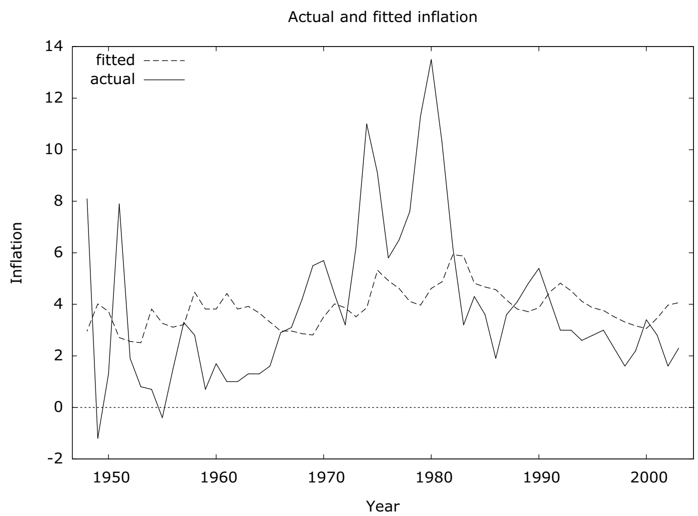

Fig. 10.2 Inflation, 1948-2003. Actual and fitted based on an static model.

where the FDL is of order two. Let's say that we are interested in the effect on Y of a permanent increase in X. Before time t, X equals to a constant c. At time t, X increases permanently to $c+1$. That is, $X_s = c$ for $s < t$ and $X_s = c+1$ for $s \geq t$. Setting the errors to be equal to zero we have:

$$\begin{aligned} Y_{t-1} &= \beta_1 + \beta_2 c + \beta_3 c + \beta_4 c \\ Y_t &= \beta_1 + \beta_2 (c+1) + \beta_3 c + \beta_4 c \\ Y_{t+1} &= \beta_1 + \beta_2 (c+1) + \beta_3 (c+1) + \beta_4 c \\ Y_{t+2} &= \beta_1 + \beta_2 (c+1) + \beta_3 (c+1) + \beta_4 (c+1) \end{aligned} \tag{10.4}$$

and so on. The contemporaneous effect of X on Y is called the *impact multiplier* and in this case this one is given by β_2. However, over time the marginal effect of X on Y is larger. We say that the *long-run multiplier* is the long-run change Y given a permanent increase in X. This one is given by $\beta_2 + \beta_3 + \beta_4$.

Consider the following example in Gretl:

$$\texttt{inf}_t = \beta_1 + \beta_2 \texttt{unem}_t + \beta_3 \texttt{unem}_{t-1} + \beta_4 \texttt{unem}_{t-2} + u_t, \tag{10.5}$$

To estimate this model in Gretl we first need to create the lagged values of `unem`. To do this we have to go to select `unem` and then go to `Add` → `Lags of selected variables` and select the number of lags. An alternative approach is to just include the lags when estimating the model via OLS. That is, when specifying the model in Gretl (`Model` → `Ordinary Least Squares`) there is an icon that allows you to select the lags. Just select two lags for `unem` to obtain:

```
Model 2: OLS, using observations 1950-2003 (T = 54)
Dependent variable: inf

             coefficient   std. error   t-ratio    p-value
  ---------------------------------------------------------
  const       -0.124609     1.68922     -0.07377   0.9415
  unem         0.903211     0.402071     2.246     0.0291  **
  unem_1      -0.856337     0.525700    -1.629     0.1096
  unem_2       0.668123     0.386722     1.728     0.0902  *

Mean dependent var   3.900000   S.D. dependent var   2.961323
Sum squared resid    395.2340   S.E. of regression   2.811526
R-squared            0.149632   Adjusted R-squared   0.098610
F(3, 50)             2.932693   P-value(F)           0.042366
Log-likelihood      -130.3660   Akaike criterion     268.7320
Schwarz criterion    276.6880   Hannan-Quinn         271.8003
rho                  0.661217   Durbin-Watson        0.676987
```

$$\widehat{\texttt{inf}} = \underset{(1.6892)}{-0.124609} + \underset{(0.40207)}{0.903211}\,\texttt{unem} - \underset{(0.52570)}{0.856337}\,\texttt{unem\_1} + \underset{(0.38672)}{0.668123}\,\texttt{unem\_2}$$

$$T = 54 \quad \bar{R}^2 = 0.0986 \quad F(3,50) = 2.9327 \quad \hat{\sigma} = 2.8115$$

(standard errors in parentheses)

A permanent increase in unemployment leads to a contemporaneous increase in inflation of 0.903 (impact multiplier). However, in the long-run the same increase in unemployment leads to a permanent effect on inflation of 0.903 -0.856 + 0.668 = 0.715 (long-run multiplier).

10.2.3 Autoregressive Model

An *autoregresive model* is a simple model where the current values of a variable are related to its past values. The first-order autoregressive model is given by:

$$Y_t = \phi Y_{t-1} + u_t. \tag{10.6}$$

This one is usually denoted by $AR(1)$. A more general model is the pth autoregressive model or $AR(p)$ given by:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + \cdots + \phi_p Y_{t-p} + u_t \tag{10.7}$$

where there are p lags of the variable Y explaining its current value. The estimation of an $AR(p)$ model in Gretl is simple; go to `Model` → `Time series` → `ARIMA` and then select the dependent variable and the *AR* order. Make sure that the *MA* order is zero. For the example above, consider estimating the following model:

$$\mathtt{inf}_t = \phi_1 \mathtt{inf}_{t-1} + \phi_2 \mathtt{inf}_{t-2} + u_t \tag{10.8}$$

The output in Gretl is:

```
Function evaluations: 17
Evaluations of gradient: 8

Model 4: ARMA, using observations 1948-2003 (T = 56)
Estimated using Kalman filter (exact ML)
Dependent variable: inf
Standard errors based on Hessian

             coefficient   std. error     z      p-value
  -------------------------------------------------------
  const        4.02526      0.791433     5.086   3.66e-07 ***
  phi_1        0.815712     0.148739     5.484   4.15e-08 ***
  phi_2       -0.175228     0.152459    -1.149   0.2504

Mean dependent var   3.883929   S.D. dependent var   3.040381
Mean of innovations -0.054104   S.D. of innovations  2.171763
Log-likelihood      -123.2506   Akaike criterion     254.5012
Schwarz criterion    262.6026   Hannan-Quinn         257.6421

                        Real  Imaginary    Modulus  Frequency
  -----------------------------------------------------------
  AR
    Root  1           2.3276    -0.5378     2.3889    -0.0361
    Root  2           2.3276     0.5378     2.3889     0.0361
  -----------------------------------------------------------
```

That yields the following estimated equation:

$$\widehat{\mathtt{inf}}_t = 4.025 + 0.8157\mathtt{inf}_{t-1} - 0.1752\mathtt{inf}_{t-2}. \tag{10.9}$$

where we can see that higher inflation last period has a positive effect on inflation this period. We can use this model to predict the path of `inf` based on its previous values. It Gretl the command to obtain this `Graphs` → `Fitted, actual plot` → `Against time`. The resulting graph is shown in Figure 10.3.

10.2.4 Moving-Average Models

The *moving-average* models express an observed series as a function of the current and lagged unobserved shocks. The simplest moving-average model is the moving-average of order one, or $MA(1)$:

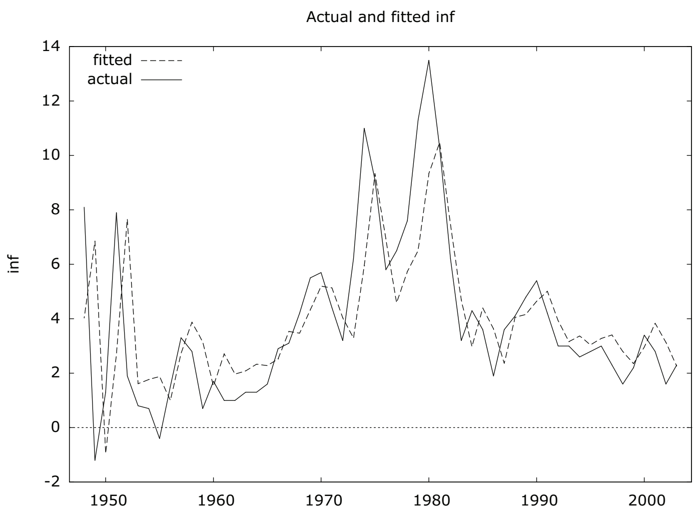

Fig. 10.3 Inflation, 1948-2003. Actual and fitted based on an $AR(2)$ model.

$$Y_t = \theta u_{t-1} + u_t \tag{10.10}$$

A more general moving-average of order q is be written as:

$$Y_t = \theta_1 u_{t-1} + \theta_2 u_{t-2} + \theta_3 u_{t-3} + \cdots + \theta_q u_{t-q} + u_t \tag{10.11}$$

For the example above:

$$\mathtt{inf}_t = \theta_1 u_{t-1} + \theta_2 u_{t-2} + u_t \tag{10.12}$$

the output in Gretl is:

```
Function evaluations: 51
Evaluations of gradient: 19

Model 6: ARMA, using observations 1948-2003 (T = 56)
Estimated using Kalman filter (exact ML)
Dependent variable: inf
Standard errors based on Hessian

             coefficient   std. error     z      p-value
```

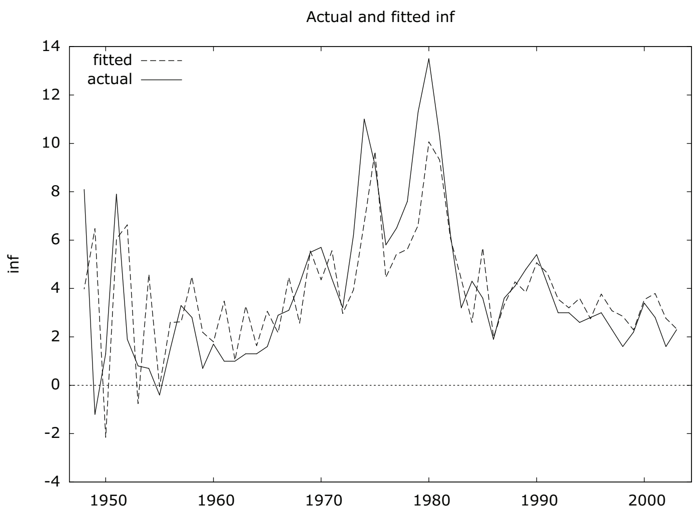

Fig. 10.4 Inflation, 1948-2003. Actual and fitted based on an $MA(2)$ model.

```
    ------------------------------------------------------
    const        3.98267       0.615240    6.473   9.58e-011 ***
    theta_1      1.18549       0.130300    9.098   9.19e-020 ***
    theta_2      0.267922      0.127516    2.101   0.0356    **

  Mean dependent var   3.883929   S.D. dependent var   3.040381
  Mean of innovations -0.041523   S.D. of innovations  1.899580
  Log-likelihood      -116.5030   Akaike criterion     241.0061
  Schwarz criterion    249.1075   Hannan-Quinn         244.1470

                         Real  Imaginary    Modulus  Frequency
    ----------------------------------------------------------
    MA
      Root  1          -1.1343     0.0000     1.1343     0.5000
      Root  2          -3.2904     0.0000     3.2904     0.5000
    ----------------------------------------------------------
```

and the actual and fitted values are presented in Figure 10.4.

10.2.5 Autoregressive Moving Average Models

One can easily combine an $AR(1)$ model and an $MA(1)$ models to obtain an autoregressive moving-average model $ARMA(1,1)$:

$$Y_t = \phi Y_{t-1} + \theta u_{t-1} + u_t \tag{10.13}$$

or a more general $ARMA(p,q)$ model:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \cdots + \theta_q u_{t-q} + u_t \tag{10.14}$$

The output in Gretl for a $ARMA(2,2)$ for inflation is:

```
Function evaluations: 61
Evaluations of gradient: 20

Model 5: ARMA, using observations 1948-2003 (T = 56)
Estimated using Kalman filter (exact ML)
Dependent variable: inf
Standard errors based on Hessian

             coefficient   std. error      z      p-value
  -------------------------------------------------------
  const       3.94843       1.05291      3.750    0.0002  ***
  phi_1       0.828806      0.236639     3.502    0.0005  ***
  phi_2       0.0226838     0.173277     0.1309   0.8958
  theta_1     0.274108      0.197397     1.389    0.1650
  theta_2    -0.587919      0.169467    -3.469    0.0005  ***

Mean dependent var   3.883929   S.D. dependent var   3.040381
Mean of innovations -0.053524   S.D. of innovations  1.831760
Log-likelihood      -114.4824   Akaike criterion     240.9648
Schwarz criterion    253.1169   Hannan-Quinn         245.6761

                        Real  Imaginary    Modulus  Frequency
  -----------------------------------------------------------
  AR
    Root  1           1.1691     0.0000     1.1691     0.0000
    Root  2         -37.7065     0.0000    37.7065     0.5000
  MA
    Root  1          -1.0917     0.0000     1.0917     0.5000
    Root  2           1.5580     0.0000     1.5580     0.0000
  -----------------------------------------------------------
```