



Date stamping bubbles in Real Estate Investment Trusts



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ABSTRACT

We test for the existence of single and multiple bubble periods in four Real Estate Investment Trust (REIT) indices using the Supremum Augmented Dickey–Fuller (SADF) and the Generalized SADF. These methods allow us to estimate the beginning and the end of bubble periods. Our results provide statistically significant evidence of speculative bubbles in the REIT index and its three components: Equity, Mortgage and Hybrid REITs. These results may be valuable for real estate financial managers and for investors in REITs.

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1. Introduction

Academics have suggested and employed various time series methods to capture speculative bubbles in asset prices such as integration and cointegration tests (Diba & Grossman, 1988a; Diba & Grossman, 1988b), variance bound tests (LeRoy & Porter, 1981; Shiller, 1981), specification tests (West, 1988) as well as Chow and CUSUM-type tests (Homm & Breitung, 2012). The new recursive flexible window right-tailed ADF testing procedure introduced in Phillips, Wu, and Yu (2011) and further enhanced in Phillips, Shi, and Yu (2015) outperforms preceding methods in detecting and date-stamping bubbles and can serve as a real-time warning signal to monitor the dynamics of asset prices.

In this paper, we employ the Phillips et al. (2015)'s novel Generalized Supremum Augmented Dickey–Fuller (GSADF) to test for the existence of speculative bubbles and to identify the origination and the collapse of bubbles in various Real Estate Investment Trust (REIT) indices.¹ Specifically, we search for explosive

autoregressive behavior in inflation-adjusted REIT indices from January 1980 through September 2013. We also explain the conditions under which empirical evidence of explosive behavior can be interpreted as a bubble in the price of the underlying financial asset.

The literature on testing for speculative bubbles in REITs is limited and the results are mixed. Jirasakuldech, Campbell, and Knight (2006) use unit root and co-integration tests to find that Equity REITs are not affected by rational bubbles. Waters and Payne (2007) use the Residuals-Augmented Dickey–Fuller (RADF) and find no periodically collapsing bubble in total REIT index and Equity REIT index, negative periodically collapsing bubble in Mortgage REIT index and inconclusive results for Hybrid REIT index. Moreover, Payne and Waters (2007) use both Momentum Threshold Autoregressive (MTAR) and RADF to find mixed results for Equity REIT index. Anderson, Brooks, and Tsolacos (2011) use regime switching processes (Evans, 1991; Van Norden & Schaller, 1999) to directly test for the presence of speculative bubbles in REITs. Although they find some evidence of negative bubbles (most notably in mortgage REIT index), the authors could not observe speculative bubbles in Equity, Mortgage and Hybrid REITs.

There exists important work on the link between REITs, stocks and real estate markets as well as on speculative bubbles in real

Commission (SEC) which amount to a collective market capitalization of \$719 billion (www.nareit.com).

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¹ With the exception of the period during the 2007–2008 financial crisis, REITs had an upward trend in both number of firms and market capitalization. As of January 31, 2014, 204 publicly-traded REITs were registered with the Securities and Exchange

Table 1
Descriptive statistics.

	Mean	Median	Std. dev.	Min	Max	HHI	Con. ratio (%)
Total REIT index	208.608	141.252	186.007	16.976	719.965	217.649	18.495
Equity REIT index	211.596	140.305	195.704	16.861	756.551	283.533	21.595
Mortgage REIT index	215.222	157.819	148.439	35.761	612.798	1944.359	67.511
Hybrid REIT index	214.127	121.387	210.257	16.199	940.127	4120.406	81.388
Real REIT index	1.129	0.853	0.777	0.212	3.107	–	–
Real equity REIT index	1.136	0.848	0.822	0.210	3.265	–	–
Real mortgage REIT index	1.249	0.982	0.645	0.428	3.123	–	–
Real hybrid REIT index	1.146	0.749	0.864	0.202	4.057	–	–

Notes: The monthly REIT value-weighted indices are obtained from CRSP/Ziman Real Estate Data Series. The database provides stock prices for individual REITs trading on the NASDAQ, New York Stock Exchange (NYSE) and NYSE MKT (formerly known as the American Stock Exchange). The Consumer Price Index (CPI) is obtained from the Federal Reserve Bank of St. Louis. We obtain the real REIT indices by dividing the REIT monthly value-weighted index by CPI to adjust for the inflation over the sample period. Our sample spans from January 1980 to September 2013 with the total number of observations being 405. HHI is the sample mean Herfindahl–Hirschman Index, calculated as the sum of the squares of the market share percentages of all the individual components in an index using beginning-of-period market capitalizations. It can range from 0 to 10,000. The sample mean concentration ratio (Con. ratio) is calculated as the ratio of the market value of the largest four securities in the portfolio versus the market value of the entire portfolio computed using the beginning-of-period market capitalizations. It can range from 0% to 100%. Both HHI and Con. ratio are obtained from the CRSP/Ziman Real Estate Data Series.

estate prices. Goodman and Thibodeau (2008) aim at disentangling the roles of economic fundamentals and speculation on the high house appreciation rates during 2000–2005, while Mikhed and Zemčik (2009) detect bubbles using panel data on price–rent ratios for the 1975–2006 period. Moreover, Himmelberg, Mayer, and Sinai (2005) explain how to assess whether there is a bubble and what underlying factors support housing demand, while Damianov and Escobari (2015) examine the dynamics of price segments during the housing bubble. In a related study, Hendershott, Hendershott, and Ward (2003) summarize some evidence on price movements to present arguments for and against the existence of irrational bubbles.²

On the links between REITs, stocks and real estate markets, it is important to keep in mind that REIT is not a pure real estate asset. Glascock, Lu, and So (2000) use cointegration and vector error correction models to show that starting in the early 1990s REITs behave more like stocks and less like bonds. In addition, Clayton and MacKinnon (2003) examine the link between REITs, financial assets and real estate returns. They show that during the 1970s and 1980s the behavior of REITs was closer to large cap stocks but during the 1990s REITs were more strongly related to real-estate related factors and small cap stocks. Nneji, Brooks, and Ward (2013) present a multivariate bubble model to evaluate whether the stock and real estate bubbles spill over into REITs. Their results suggest a transmission of speculative bubbles from real estate into REITs. Although these studies suggest the existence of links between REITs, stocks, bonds and real estate markets, our empirical approach to study bubbles does not allow us to draw inferences on any potential connection.

Our results show evidence of four statistically significant speculative bubbles in the inflation-adjusted REIT value-weighted index (August to November of 1990; February to April of 1993; October 1996 to April 1998; November 2003 to June 2007). When extending the analysis to three components of the total REIT index, we find evidence of speculative bubbles in all three indices. For Equity REIT index, which is the major component of the total REIT index, the results show that the bubble periods are identical to those of the total index. For Mortgage REIT index, we find evidence of four periodic bubbles (January to May 1983; September 1996 to November 1997; May to August 2001; May 2003 to April 2004) while for Hybrid REITs index we find evidence of three periodic bubbles (November 1996 to February 1998; November 2002 to April 2004; August 2006 and collapses May 2007).

In sum, the contribution of this study is twofold. First, it empirically tests for the existence of speculative bubbles in total REIT index as well as its three components (Equity REITs, Mortgage REITs and Hybrid REITs). Second, it estimates the beginning and the end of bubble periods in REITs with the GSADF methodology that allows for the existence of multiple bubbles in a single series.

The remainder of this paper proceeds as follows. Section 2 describes the sample data. Section 3 presents the empirical approach employed to detect and date-stamp periodic bubbles in REITs. Section 4 discusses the results while Section 5 concludes.

2. Data

Real Estate Investment Trusts are dividend-paying stocks that mainly invest in real estate. The REIT index is comprised of three components: Equity REITs, Mortgage REITs and Hybrid REITs. The Equity REIT index includes securities backed by the value of real estate assets (e.g., shopping malls, office buildings or apartments) and generates revenues mainly from their properties' rent. The Mortgage REIT index is backed by residential and commercial mortgage obligations and mortgage-backed securities. Since Mortgage REITs do not own real estate assets, their main source of revenues is the interest earned on the mortgage loans. The Hybrid REIT index is a combination of Equity and Mortgage REITs.

The monthly REIT value-weighted index is obtained from CRSP/Ziman Real Estate Data Series. The database provides stock prices for individual REITs trading on the NASDAQ, New York Stock Exchange (NYSE) and NYSE MKT (formerly known as the American Stock Exchange). The Consumer Price Index (CPI) is obtained from the Federal Reserve Bank of St. Louis. We divide the REIT monthly value-weighted index by CPI to adjust for the inflation over the sample period. The sample covers the period from January 1980 to September 2013 comprising 405 monthly observations.

Table 1 reports the descriptive statistics along with two measures of concentration based on market capitalization; the sample mean Herfindahl–Hirschman Index (HHI) and the sample mean concentration ratio (Con. Ratio).³ Both measured were obtained from CRSP/Ziman Real Estate Data Series. The HHI can range from 0 to 10,000 while the concentration ratio goes from 0% to 100%. Higher values indicate a higher level of concentration. The Total

³ The HHI is calculated as the sum of the squares of the market share percentages of all the individual components in an index. The concentration ratio is calculated as the ratio of the market value of the largest four securities in the portfolio versus the market value of the entire portfolio. Both concentration measures are based on the beginning-of-period market capitalizations.

² For a survey on housing bubbles see Mayer (2011).

REIT index is the less concentrated index with a HHI of 217.649 and concentration ratio of 18.495%. The Equity REIT index has very similar values. On the other hand, the Mortgage REIT index may be classified as moderately concentrated with a HHI of 1944.359 and a concentration ratio of 67.511%. The highest levels of concentration appear in the Hybrid REIT index where the largest four securities have a share of about 81.40% of the entire portfolio.

Fig. 1 shows the four real (inflation-adjusted) REIT value-weighted indices (Total REIT Index, Equity REIT Index, Mortgage REIT Index and Hybrid REIT Index) from January 1980 to September 2013. It is notable that since January 1980, REITs prices have been increasing for most of the years. It is at the turn of the century where REIT index experience a large run up that peaks in early 2007 and collapses shortly after. In fact, between 2007 and 2009, REITs prices dropped to the levels observed a decade earlier.

The Equity and Hybrid REIT indices have a relatively closer price movement to the total REIT index than Mortgage REIT index. As illustrated in Fig. 1, the time series variation in all three indices between early 1995 and late 1999 is consistent with the existence of a speculative bubble. During this period, the Mortgage REIT index appears to have experience the largest bubble from all four series in the figure. Moreover, there is evidence of a second bubble period that starts in the early 2000. During this apparent second bubble, the Mortgage REIT index appreciates faster and reaches its peak relatively earlier than the other indices, and also collapse earlier and faster than the Equity and Hybrid REIT indices. Lastly, in the post crisis period, Equity and Hybrid REITs have a sharper recovery compare to Mortgage REITs. In 2013, the Equity REIT index reached and the Hybrid REIT index surpassed their respective pre-crisis peaks. The Mortgage REIT index, however, has approximately reached levels of only about half of its peak reached around 2004 and 2005.

3. Empirical strategy

To test for explosive behavior and date stamp the beginning and the end of the bubble periods, we start with the following Augmented Dickey–Fuller (ADF) regression equation:

$$\Delta y_t = a_{r_1, r_2} + \beta_{r_1, r_2} y_{t-1} + \sum_{i=1}^k \phi_{r_1, r_2}^i \Delta y_{t-i} + \varepsilon_t \tag{1}$$

where, y_t is the corresponding Real REIT index, Δy_t denotes first differences, and the error term is assumed to follow a normal distribution, i.e., $\varepsilon \sim iidN(0, \sigma_{\varepsilon}^2)$. The subscripts r_1 and r_2 are the fractions of the total sample size that specify the starting and ending points of a subsample period.

We are interested in testing the unit root null hypothesis against the alternative of mildly explosive behavior in y_t . The corresponding test statistics is $ADF_{r_1}^{r_2} = \hat{\beta}_{r_1, r_2} / s.e.(\hat{\beta}_{r_1, r_2})$. Notice that ADF_0^1 is the well known standard ADF test statistic. To detect episodes of explosive behavior, Phillips et al. (2011) propose a recursive procedure on the estimation of $ADF_{r_1}^{r_2}$ using different subsample of data. The test statistic is defined as the supremum value of the $ADF_0^{r_2}$ as defined by

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_0^{r_2} \tag{2}$$

The idea is that when the SADF statistic exceeds the right tale critical value, the unit root null hypothesis is rejected in favor of explosive behavior. Using simulations, Homm and Breitung (2012) find that the SADF has greater power that the methods in Bhargava (1986), the modified Busetti and Taylor (2004) and the modified Kim (2000). Moreover, Phillips et al. (2015) argue that this procedure can detect exuberance that may arise from a variety of sources including mildly explosive behavior that may be induced by changing fundamentals such as the time varying discount factor. Appendix A follows Phillips et al. (2015) to explain some conditions under which empirical evidence of explosive behavior may be used to conclude the existence of bubbles.

While SADF performs well for a single boom and bust in a series, it may not consistently identify multiple bubbles. Phillips et al. (2015) propose the Generalized SADF (GSADF) to deal with multiple episodes of boom and bust. The GSADF methodology is designed to use a rolling and recursive sample, covering a larger number of subsamples than the SADF by allowing both, the ending point (r_2) and the initial point (r_1) to change. The GSADF statistic is given by,

$$GSADF(r_0) = \sup_{r_1 \in [0, r_2 - r_0], r_2 \in [r_0, 1]} ADF_{r_1}^{r_2} \tag{3}$$

Rejection of the null in favor of the explosive alternative hypothesis requires that $GSADF(r_0)$ is greater than the right tail critical value.

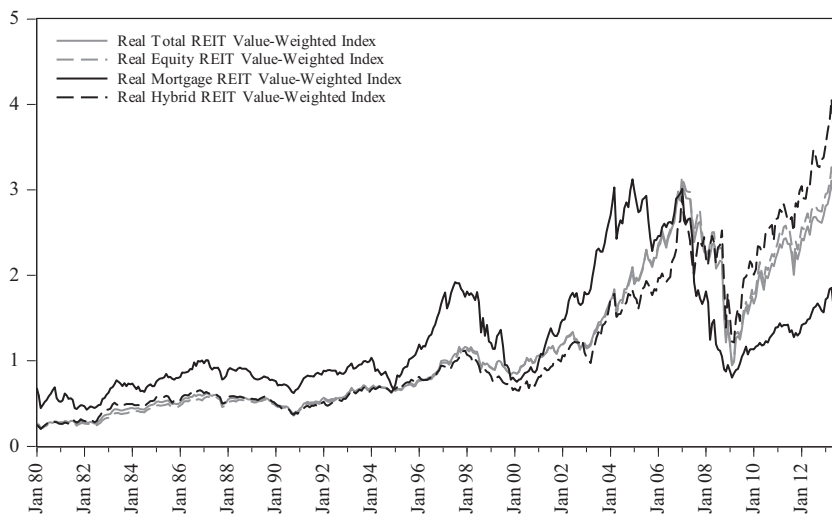


Fig. 1. Real (Inflation-Adjusted) REITs—Jan 1980 to Sep 2013. Notes: This figure depicts real (inflation-adjusted) REIT value-weighted index and its three components: Equity, Mortgage and Hybrid REITs. The real REIT indices are obtained by dividing the REIT monthly value-weighted indices (obtained from CRSP/Ziman Real Estate Data Series) by the Consumer Price Index (CPI, obtained from the Federal Reserve Bank of St. Louis). The sample spans from January 1980 to September 2013 with the total number of observations being 405.

Table 2
The SADF test and the GSADF test statistics.

	Real REIT index test statistic				Finite sample critical values		
	Total REIT	Equity REIT	Mortgage REIT	Hybrid REIT	90%	95%	99%
SADF	4.936	5.267	2.986	4.778	1.406	1.659	2.217
GSADF	4.936	5.267	2.986	4.778	3.264	3.796	5.293

Notes: The real REIT indices are obtained by dividing the REIT monthly value-weighted indices (obtained from CRSP/Ziman Real Estate Data Series) by the Consumer Price Index (CPI, obtained from the Federal Reserve Bank of St. Louis). The sample spans from January 1980 to September 2013 with the total number of observations being 405. SADF is Supremum Augmented Dickey–Fuller proposed by Phillips et al. (2011), and GSADF is Generalized SADF methodology proposed by Phillips et al. (2015). Critical values of both tests are obtained from Monte Carlo simulations with 2000 replications. The smallest window in the recursive procedures has 8 observations.

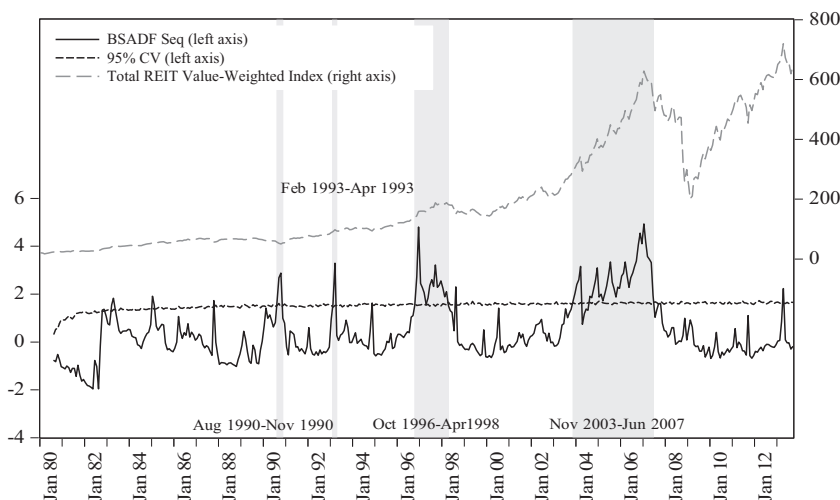


Fig. 2. GSADF. Bubble periods in the Real REIT Value-Weighted Index. Notes: The real REIT index is obtained by dividing the REIT monthly value-weighted index (obtained from CRSP/Ziman Real Estate Data Series) by the Consumer Price Index (CPI, obtained from the Federal Reserve Bank of St. Louis). The sample spans from January 1980 to September 2013 with the total number of observations being 405. The Backward Supremum Augmented Dickey–Fuller (BSADF) follows Phillips et al. (2015) with the 95% critical values coming from Monte Carlo simulations with 2000 replications (the sample size is 405 and the smallest window has 8 observations).

To date stamp the beginning and the end of the bubble Phillips et al. (2015) suggest obtaining the Backward Sup ADF (BSADF) statistic,

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} SADF_{r_1}^2 \tag{4}$$

to then use the first observation in which the BSADF exceeds its critical value as the beginning of the bubble,

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \{r_2 : BSADF_{r_2}(r_0) > scv_{r_2}^\alpha\} \tag{5}$$

and the first observation after $\hat{r}_e + 3/T$ in which BSADF falls below its critical value as the end of the corresponding bubble episode,⁴

$$\hat{r}_f = \inf_{r_2 \in [\hat{r}_e + 3/T, 1]} \{r_2 : BSADF_{r_2}(r_0) < scv_{r_2}^\alpha\} \tag{6}$$

where, $scv_{r_2}^\alpha$ denotes the $100(1 - \alpha)\%$ critical value of the SADF based on $\lfloor r_2 T \rfloor$ observations and at a significance level α .⁵ The distributions of the statistics $GSADF(r_0)$ and $BSADF_{r_2}(r_0)$ in Eqs. (3) and (4) are non-standard, hence we will use Monte Carlo simulations to obtain the critical values.

4. Results

We obtain the SADF and GSADF statistics as described in Eqs. (2) and (3) for the real (inflation-adjusted) REIT value-weighted index and its three components: Equity REITs, Mortgage REITs and Hybrid REITs. Table 2 reports the estimation results, the SADF and

GSADF test statistics, as well as the 90%, 95% and 99% critical values for the two tests that we obtained using Monte Carlo simulations with 2000 replications (sample size is 405 and the smallest window is 8% or 2% of the sample). For the total REIT index, both SADF and GSADF exceed their 1% and 5% right-tail critical value, respectively (SADF: 4.936 > 2.217 and GSADF: 4.936 > 3.796). These results remain robust for the Equity REIT index (SADF: 5.267 > 2.217 and GSADF: 5.267 > 3.796) and Hybrid REIT index (SADF: 4.778 > 2.217 and GSADF: 4.778 > 3.796). For Mortgage REIT index, while SADF exceed its 1% right-tail critical value (SADF: 2.986 > 2.217), GASDF is below its right-tail critical values. Overall, the SADF test statistics provide statistically significant evidence of explosive behavior in all four inflation-adjusted REIT indices. Moreover, the GSADF provide statistically significant evidence for the existence of multiple bubbles in the total REIT index as well as in the Equity REIT and Hybrid REIT indices. Note that the rolling windows used in by the SADF in Eq. (2) are a subsample of the windows used by the GSADF in Eq. (3). This explains why the SADF and GSADF statistics are both the same in our REIT indices.

Fig. 2 plots the recursive BSADF statistics against their corresponding 95% critical value sequences to identify four bubble episodes in the REITs index; the first two are relatively short, one from August to November of 1990 and another one from February to April of 1993. The third bubble begins in October 1996 and bursts in April 1998. In 1998, the rise in inflation-adjusted real estate prices was greater than 10% per year in most West Coast cities (Shiller, 2007). The most recent bubble is the longest-lasting one, as BSADF estimates its origination in November of 2003 and its termination in June of 2007.

Figs. 3–5 follow the same procedures, as described in Eqs. (1) through (6), to identify episodes of speculative bubbles in the

⁴ We use $3/T$ as we identify bubble that lasts at least three months.
⁵ $\lfloor \cdot \rfloor$ is the floor function that gives the integer part of the argument.

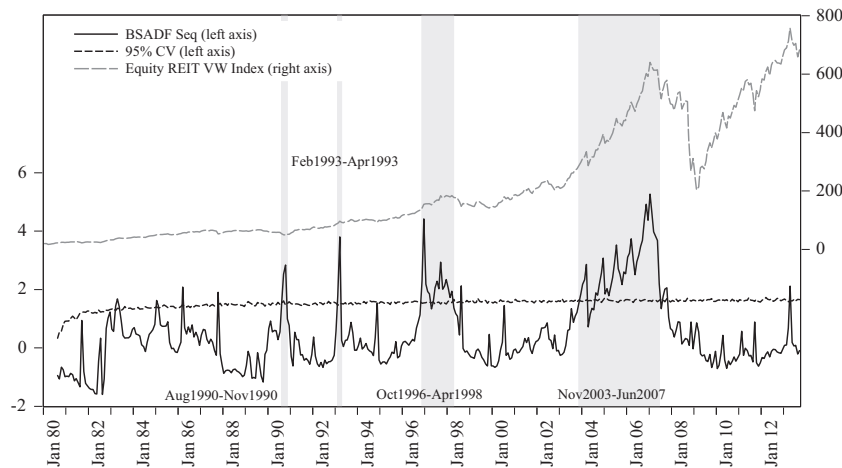


Fig. 3. GSADF. Bubble periods in the Real Equity REIT Value-Weighted Index. *Notes:* The real Equity REIT index is obtained by dividing the Equity REIT monthly value-weighted index (obtained from CRSP/Ziman Real Estate Data Series) by the Consumer Price Index (CPI, obtained from the Federal Reserve Bank of St. Louis). The sample spans from January 1980 to September 2013 with the total number of observations being 405. The Backward Supremum Augmented Dickey–Fuller (BSADF) follows Phillips et al. (2015) with the 95% critical values coming from Monte Carlo simulations with 2000 replications (the sample size is 405 and the smallest window has 8 observations).

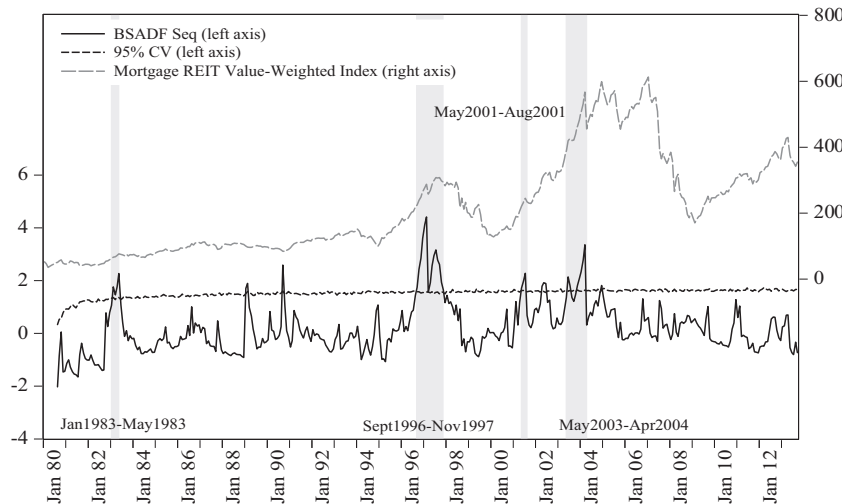


Fig. 4. GSADF. Bubble periods in the Real Mortgage REIT Value-Weighted Index. *Notes:* The real Mortgage REIT index is obtained by dividing the Mortgage REIT monthly value-weighted index (obtained from CRSP/Ziman Real Estate Data Series) by the Consumer Price Index (CPI, obtained from the Federal Reserve Bank of St. Louis). The sample spans from January 1980 to September 2013 with the total number of observations being 405. The Backward Supremum Augmented Dickey–Fuller (BSADF) follows Phillips et al. (2015) with the 95% critical values coming from Monte Carlo simulations with 2000 replications (the sample size is 405 and the smallest window has 8 observations).

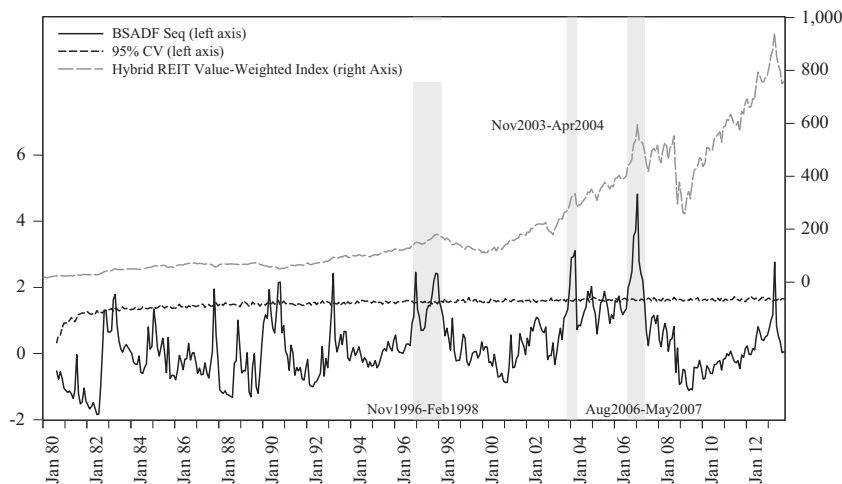


Fig. 5. Bubble periods in the Real Hybrid REIT Value-Weighted Index. *Notes:* The real Hybrid REIT index is obtained by dividing the Hybrid REIT monthly value-weighted index (obtained from CRSP/Ziman Real Estate Data Series) by the Consumer Price Index (CPI, obtained from the Federal Reserve Bank of St. Louis). The sample spans from January 1980 to September 2013 with the total number of observations being 405. The Backward Supremum Augmented Dickey–Fuller (BSADF) follows Phillips et al. (2015) with the 95% critical values coming from Monte Carlo simulations with 2000 replications (the sample size is 405 and the smallest window has 8 observations).

Equity, Mortgage and Hybrid REITs, respectively. Since the Equity REIT index is the major component of the total index, the four bubble periods identified in the Equity REIT index are identical to bubbles observed in the total index (Fig. 3). Mortgage REIT index shows four periodic bubbles (Fig. 4); the first one is relatively short, lasting from January to May 1983. The second one is the largest and lasts longer than a year from September 1996 to November 1997. The third bubble is relatively short, from May to August 2001, while the last bubble begins in May 2003 and lasts until April 2004. Hybrid REIT index exhibits three periodic bubbles (Fig. 5); the first one begins in November 1996 and lasts until February 1998. The second and the shortest bubble lasts from November 2002 to April 2004, while the last bubble begins in August 2006 and collapses May 2007.

5. Conclusion

This paper is the first study to employ the new recursive flexible window right-tailed ADF testing procedure introduced in Phillips et al. (2011) and further enhanced in Phillips et al. (2015) to empirically detect and date-stamp the origination and the collapse of speculative bubbles in REITs. Each speculative bubble originates when the Backward Supremum Augmented Dickey–Fuller (BSADF) statistic exceeds its corresponding 95% critical value, and ends when its BSADF falls below that critical value.

Using data from January 1980 to September 2013, we first examine the Inflation-Adjusted REIT value-weighted index to find evidence of four speculative bubbles (August to November of 1990; February to April of 1993; October 1996 to April 1998; November 2003 to June 2007). We further extend the analysis to three different components of the total REIT index separately and find speculative bubbles in all three indices. For the Equity REIT index the results show evidence of identical bubble periods to the total index. For the Mortgage REIT index, we find evidence of four periodic bubbles (January to May 1983; September 1996 to November 1997; May to August 2001; May 2003 to April 2004) and for the Hybrid REIT index, the data find evidence of three periodic bubbles (November 1996 to February 1998; November 2002 to April 2004; August 2006 and collapses May 2007).

Our results may be valuable for real estate financial managers and for investors in REITs as Phillips et al. (2015) argue that the approach is anticipative as an early warning alert system. However, it is important to keep in mind that even when working with real time data, the methods assess if a bubble exists but do not help to predict a collapse.

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Appendix A.

One key benefit of using the SADF and the GSADF to test for explosive behavior and interpret this as existence of bubbles is that we do not need to observe fundamentals. However, one drawback is that empirical evidence of explosive behavior may not necessarily imply the existence of bubbles. For example, if the income stream is growing unexpectedly faster than previously the methods may lead to mistakenly conclude that there is a bubble. To formalize this idea, define a bubble B_t as the difference between the after-dividend price P_t of an asset and the market fundamental P_t^f , i.e.,

$B_t = P_t - P_t^f$. Then the market fundamentals simply follow the asset pricing equation:

$$P_t^f = \sum_{i=0}^{\infty} \left(\frac{1}{1+r_f} \right)^i E_t(D_{t+i} + U_{t+i}) \quad (6)$$

where, r_f is the risk-free interest rate, D_t is the payoff or dividend received from the asset, and U_t represents the unobserved fundamentals. Under no bubbles, the degree of stationarity of P_t is entirely determined by the degree of stationarity of P_t^f . That is, following Eq. (6) it would depend on the character of the dividend series and the unobserved fundamentals. For example, if the dividend series is integrated of order one and the fundamentals are either stationary or integrated of order one, then the asset price is at most integrated of order one. If the bubble series satisfy the submartingale property $E_t(B_{t+1}) = (1+r_f)B_t$, asset prices will be explosive in the presence of bubbles. Hence, if the dividend series is stationary after differencing and the unobserved fundamentals are at most integrated of order one, then empirical evidence of explosive behavior, as obtained with the SADF and GSADF, may be used to conclude the existence of bubbles.

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