Basic Thermodynamic Formulas (Exam Equation Sheet)

Control Mass (no mass flow across system boundaries)

Conservation of mass: $m = constant$

Conservation of energy (1st Law): $\begin{cases} Q - W & = \Delta E \\ & = \Delta U + \Delta KE + \Delta PE \\ & = m\left\{\Delta u + \frac{v_2^2 - v_1^2}{2} + g(z_2 - z_1)\right\} \end{cases}$

Entropy Balance (2nd Law): $\Delta S = \int \left(\frac{\delta Q}{T}\right)_b + \sigma$

Control Volume (mass flow across system boundaries)

Conservation of mass: $\frac{dm_{CV}}{dt} = \sum \dot{m}_i - \sum \dot{m}_e$; where $\dot{m} = \frac{A\overline{V}}{v}$ is the mass flow rate

Conservation of energy (1st Law): $\frac{dE_{CV}}{dt} = \dot{Q} - \dot{W} + \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e\right)$

Entropy Balance (2nd Law): $\frac{dS_{CV}}{dt} = \sum \frac{\dot{Q}_k}{T_k} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{\sigma}_{CV}$

Heat Transfer and Work Relationships

Conduction: $\dot{Q} = -kA\frac{dT}{dx}$

Convection: $\dot{Q} = hA(T_s - T_\infty)$

Radiation: $\dot{Q} = \varepsilon\sigma A(T_s^4 - T_{sur}^4)$

Heat transfer for an internally reversible process: $Q_{int.rev} = \int T\, dS$

Boundary work: $W_b = \int PdV$

Polytropic process: $\left\{\begin{array}{c} PV^n = Const \\ n = 1, W_b = P_1V_1 ln\frac{V_2}{V_1} \\ n \neq 1, W_b = \frac{P_2V_2 - P_1V_1}{1-n} \end{array}\right\}$

Polytropic process for **Ideal Gas**: $\left\{\begin{array}{c} \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(n-1)/n} \\ \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} \end{array}\right\}$

Reversible steady flow work: $w_{rev} = -\int vdP + \frac{v_1^2 - v_2^2}{2} + g(z_1 - z_2)$

Ideal gas, no change in kinetic and potential energy

Isentropic: $w_{rev} = \frac{kR(T_1 - T_2)}{k-1} = \frac{kRT_1}{k-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{\left(\frac{k-1}{k}\right)}\right]$

Isothermal: $w_{rev} = RTln\left(\frac{P_1}{P_2}\right)$

Polytropic: $w_{rev} = \frac{nR(T_1 - T_2)}{n-1} = \frac{nRT_1}{n-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{\left(\frac{n-1}{n}\right)}\right]$

Thermal Efficiency of Cycles $\left\{\boldsymbol{\eta_{th} = \frac{desired\ output}{required\ input}}\right\}$ $\left\{\boldsymbol{\dot{Q}_{cycle} = \dot{W}_{cycle}}\right\}$ $\left\{\boldsymbol{\frac{\dot{Q}_H}{\dot{Q}_L} = \frac{T_H}{T_L}\ reversible\ cycles\ only}\right\}$

Heat engine: $\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} \leq 1 - \frac{T_L}{T_H}$

Refrigeration: $COP_R = \beta = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{\dot{Q}_L}{\dot{Q}_H - \dot{Q}_L} \leq \frac{T_L}{T_H - T_L}$

Heat pump: $COP_{HP} = \gamma = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L} \leq \frac{T_H}{T_H - T_L}$

Isentropic (Adiabatic) Efficiencies:

Turbine: $\eta_T = \frac{actual\ work}{isentropic\ work} = \frac{h_i - h_e}{h_i - h_{e,s}}$

Compressor: $\eta_C = \frac{isentropic\ work}{actual\ work} = \frac{h_{e,s} - h_i}{h_e - h_i}$

Nozzle: $\eta_N = \frac{actual\ KE\ at\ exit}{isentropic\ KE\ at\ exit} = \frac{V_e^2/2}{V_{e,s}^2/2}$

Specific Heat Relationships $\left\{C_v = \left.\frac{\delta u}{\delta T}\right)_v\ ,\ C_p = \left.\frac{\delta h}{\delta T}\right)_P\right\}$

Incompressible Substance (solids and liquids)

Use saturation tables: $v \cong v_{f@T}$; $u \cong u_{f@T}$; $h \cong h_{f@T} + v_{f@T}(P - P_{sat})$; $s \cong s_{f@T}$

OR Variable specific heat: $\left\{\begin{matrix} \Delta u = \int C(T)dT \\ \Delta h = \int C(T)dT + v\Delta P \\ \Delta s = \int \frac{C}{T} dT \end{matrix}\right\}$

Constant specific heat: $\left\{\begin{matrix} \Delta u = C\Delta T \\ \Delta h = C\Delta T + v\Delta P \\ \Delta s = C_{avg} ln\frac{T_2}{T_1} \end{matrix}\right\}$

Ideal Gases $\left\{Pv = RT, C_p = C_v + R,\ k = \frac{C_p}{C_v}\right\}$ Use ideal gas tables $u = \frac{\bar{u}}{M}$; M = Molar mass (Table A-1)

OR Variable specific heats: $\left\{\begin{matrix} \Delta u = \int C_v(T)\,dT \\ \Delta h = \int C_p(T)\,dT \\ \Delta s = s_2^o - s_1^o - Rln\frac{P_2}{P_1} \end{matrix}\right\}$

Isentropic process: $\left\{\begin{matrix} \frac{P_2}{P_1} = exp\left(\frac{s_2^o - s_1^o}{R}\right) = \frac{P_{r2}}{P_{r1}} \\ \frac{v_2}{v_1} = \frac{T_2 P_1}{T_1 P_2} = \frac{v_{r2}}{v_{r1}} \end{matrix}\right\}$

Constant specific heats: $\left\{\begin{matrix} \Delta u = C_{v,avg}\ \Delta T \\ \Delta h = C_{p,avg}\ \Delta T \\ \Delta s = C_v ln\frac{T_2}{T_1} + Rln\frac{v_2}{v_1} \\ \Delta s = C_p ln\frac{T_2}{T_1} - Rln\frac{P_2}{P_1} \end{matrix}\right\}$

Isentropic process: $\left\{\begin{matrix} \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{(k-1)} \\ \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\left(\frac{k-1}{k}\right)} \\ \frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^k \end{matrix}\right\}$

Other (saturated liquid/vapor mixtures, superheated vapors, e.g. water and refrigerants) → Use Tables A2 - A18

Useful Relations: $h = u + Pv$; $v = v_f + xv_{fg}$; $u = u_f + xu_{fg}$; $h = h_f + xh_{fg}$; $s = s_f + xs_{fg}$;

$$quality = x = \frac{m_g}{m_{total}}\ ;\ m_{total} = m_f + m_g$$