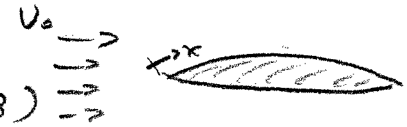
VISCOUS FLOW II
EXAMPLE PROBLEM

empirical prediction of $\frac{x_{sep}^{lam}}{L}$, $\frac{x_{crit}}{L}$, $\frac{x_{tr}}{L}$ for flow over a surface where $U_0 L/\nu = 4\times10^6$ and

$$U_\infty(x) = U_0\left[1 + 2.0\left(\frac{x}{L}\right) - 2.0\left(\frac{x}{L}\right)^2\right]$$

$(U_\infty)_{max} = 1.5$ at $x/L = 0.5$)

use a combination of Thwaites method and Wazzan's correlation



from Eqs. (4-132) and (4-138)

$$\lambda = \frac{\theta^2 U_\infty'}{\nu} = \frac{U_0[+2.0/L - 4.0\,x/L^2]}{\nu}\,\frac{0.45\,\nu}{U_0^6[1+2.0(x/L)-2.0(x/L)^2]^6}$$

$$\times \int_0^x U_0^5\left[1+2.0\left(\frac{x'}{L}\right)-2.0\left(\frac{x'}{L}\right)^2\right]^5 dx'$$

change variable, $\xi = x/L$, $dx = L\,d\xi$

$$\lambda = \frac{[+2.0-4.0(x/L)]\left(\frac{1}{L}\right)(0.45)\,L}{[1-2.0(x/L)+2.0(x/L)^2]^6}\int_0^{(x/L)}\left[1+2.0\xi-2.0(\xi)^2\right]^5 d\xi$$

march along in $\xi = (x/L)$ until $\lambda = -0.090$ for laminar separation point (solve integral numerically, use simple BASIC computer program)

find $(x/L)_{sep}^{lam} = 0.6425$ (ind. of Re)

to find $x/L)_{crit}$ use correlation of Wazzan, Fig. 5-12

for given λ, H can be found using Eq (4-141)

and

$$\theta = \left(\frac{\nu\lambda}{U_\infty'}\right)^{1/2} = \left[\frac{\nu\lambda}{U_0/L\,(2.0-4.0(x/L))}\right]^{1/2}$$

$$\left(\frac{\theta}{L}\right) = \left[\frac{\lambda}{(2.0 - 4.0(x/L))}\right]^{1/2} \Big/ \sqrt{\frac{U_0 L}{\nu}}$$

then

$$\frac{\delta^*}{L} = \left(\frac{\theta}{L}\right) H$$

and

$$Re_{\delta^*} = \frac{U_\infty \delta^*}{\nu} = \frac{U_0\left[1 + 2.0\left(\frac{x}{L}\right) - 2.0\left(\frac{x}{L}\right)^2\right] L H}{\nu} \times \left[\frac{\lambda}{(2.0 - 4.0(x/L))}\right]^{1/2} \sqrt{\frac{\nu}{U_0 L}}$$

so

$$Re_{\delta^*} = \sqrt{\frac{U_0 L}{\nu}} \left[1 + 2.0\left(\frac{x}{L}\right) - 2.0\left(\frac{x}{L}\right)^2\right] H \left[\frac{\lambda}{(2.0 - 4.0(x/L))}\right]^{1/2}$$

march along in (x/L) until $Re_{\delta^*} = Re_{\delta^*,crit}$ from Fig. 5-12

find at $x/L)_{crit} = 0.30$	$Re_{\delta^*} = 1629$
$H = 2.47$	$\cong Re_{\delta^*,crit} \approx 1600$

to find transition point to turbulence, use one-step method of Wazzan, Eq (5-42)

note

$$Re_x = \frac{Ux}{\nu} = \frac{U_0\left[1 + 2.0(x/L) - 2.0(x/L)^2\right] L\,(x/L)}{\nu}$$

$$= \left(\frac{U_0 L}{\nu}\right)\left[1 + 2.0(x/L) - 2.0(x/L)^2\right](x/L)$$

march along in (x/L) until $Re_x = Re_{x,tr}$ as given by Eq (5-42) find $x/L)_{tr} = 0.52$

(note: since $x/L)_{tr} < x/L)_{sep}^{lam}$, separation will be delayed, find $x/L)_{sep}^{turb}$ for turbulent in Chap 6)

x/L	λ	H	Re_{δ^*}	$Re_{\delta^* crit}$ (Fig 5.12)	Re_x	$Re_{x,tr}$
0.1	0.0443	2.46	967	≈2000	9.72×10^5	2.51×10^7
0.2	0.0497	2.45	1315	≈2000	1.06×10^6	3.07×10^7
0.3	0.0433	2.47	1629	≈1800	1.70×10^6	2.42×10^7
0.4	0.0281	2.51	1970	≈1100	2.37×10^6	1.38×10^7
0.5	0.000	2.59	2390	≈500	3.00×10^6	4.60×10^6
0.6	-0.0536	2.91	3153	≈200	3.55×10^6	8.16×10^4

x/L	Re_θ	$2.9\, Re_x^{0.4}$	$1.174\left[1 + \frac{22,400}{Re_x}\right] Re_x^{0.46}$
0.1	393	539	501
0.2	538	744	707
0.3	661	902	875
0.4	785	1028	1014
0.5	921	1130	1128
0.6	1083	1209	1218

(separation)

if use one-step method of Michel

$$Re_\theta = Re_{\delta^*} / H$$

and $Re_{\theta,tr} = 2.9\, Re_x^{0.4}$

or correlation of Cebeci and Smith

with $Re_{\theta,tr} = 1.174 \left[1 + \left(\frac{22{,}400}{Re_x} \right) \right] Re_x^{0.46}$ Eq (5.41)

find no transition up to laminar sep. point

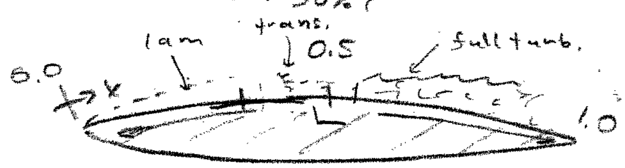
EXAMPLE PROBLEM (Continued)

$$U_\infty(x) = U_0\left[1 + 2.0(x/L) - 2.0(x/L)^2\right]$$

$$Re_L = \frac{U_0 L}{\nu} = 4\times10^6$$

$$x/L)_{crit} = 0.30$$ ← note correction

$$x/L)_{tr,50\%T} = 0.52$$



what if 1% T ?, find $x/L)_{tr,1\%T} = ?$

since not Falkner-Skan, use Dunham cor.

$$Re_{\theta,tr} = (0.27 + 0.73e^{-80T})\left[550 + \frac{680}{1 + 100T - 21\lambda_{tr}}\right]$$

$$= 0.598\left[550 + \frac{680}{2 - 21\lambda_{tr}}\right]$$

as before, using Thwaites method

$$\frac{\theta}{L} = \frac{1}{\sqrt{Re_L}}\left[\frac{\lambda}{[2.0 + 4.0(x/L)]}\right]^{1/2}$$

$$Re_\theta = \frac{U_\infty L(\theta/L)}{\nu} = \sqrt{Re_L}\left[\frac{\lambda}{[2.0 - 4.0(x/L)]}\right]^{1/2}$$

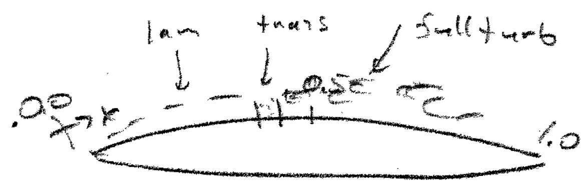
$$\times\left[1 + 2.0(x/L) - 2.0(x/L)^2\right]$$

corrected values

(x/L)	λ	Re_{δ^*}	Re_θ	$Re_{\theta,tr}$
0.1	0.0943	967	393	709
0.2	0.0997	1315	538	755
0.3	0.0433	1629	661	702
0.4	0.0281	1970	785	617
0.5	0.000	2390	921	532
0.6	-0.0536	3153	1083	459

find then

$$x/L)_{tr,\,1\%T} \approx 0.32 < x/L)_{tr,\,0\%T} = 0.52$$



```
10 PRINT "enter dxi,reo"
20 INPUT DXI,REO
30 NXI=1!/DXI
32 XI=0!
34 ITG=0!
40 FOR N = 1 TO NXI
50 XI=XI+DXI
60 ITG=ITG+(1!+2!*XI-2!*XI*XI)^5*DXI
70 LAM=(+2!-4!*XI)*.45*ITG/(1!+2!*XI-2!*XI*XI)^6
75 Z=.25-LAM
80 H=2!+4.14*Z-83.5*Z*Z+854*Z^3-3337*Z^4+4576*Z^5
90 RED=SQR(REO)*(1!-2!*XI+4!*XI*XI)*H*SQR(LAM/(2!-4!*XI))
100 REX=REO*(1!+2!*XI-2!*XI*XI)*XI
110 LREXT=-40.4557+64.8066*H-26.7538*H*H+3.3819*H^3
120 REXT=10^(LREXT)
130 PRINT XI,LAM,H,RED,REX,REXT
140 NEXT N
150 END
```