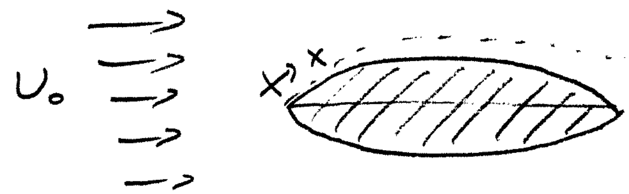
MECE 6373 Viscous Flow II
EXAMPLE PROBLEM

empirical prediction of $x/L)_{lam\,sep}$, $x/L)_{cr}$, and $x/L)_{tr}$ for flow over a surface where

$$U_\infty(x) = U_0\left[1 + 2.0(x/L) - 2.0(x/L)^2\right]$$

$$0 < (x/L) < 1.0$$



$$\text{with} \quad Re_L = \frac{U_0 L}{\nu} = 1 \times 10^6$$

use Thwaites method

$$\lambda(x) = \frac{\theta^2\, dU/dx}{\nu} = \frac{0.45\, dU/dx}{U^6}\int_0^x U^5(x')\,dx'$$

define $x^* = x/L$

$$\lambda(x^*) = \frac{0.45\,\frac{U_0}{L}\left[2.0 - 4.0x^*\right]}{U_0^6\left[1 + 2.0x^* - 2.0x^{*2}\right]^6}$$

$$\times \int_0^{x^*} U_0^5\left[1 + 2.0x^* - 2.0x^{*2}\right]^5 L\, dx^*$$

$$\lambda(x^*) = \frac{0.45\left[2.0 - 4.0x^*\right]}{\left[1 + 2.0x^* - 2.0x^{*2}\right]^6}\int_0^{x^*}\left[1 + 2.0x^* - 2.0x^{*2}\right]^5 dx^*$$

perform integral using RUNGE with

$$Y(1) = \int_0^{x^*} [1 + 2.0x^* - 2.0x^{*2}]^5 dx^*$$

$$F(1) = [1 + 2.0x^* - 2.0x^{*2}]^5$$

$x/L)_{lam\ sep}$ occurs when $\lambda(x^*) = -0.09$ (ind. of Re_L!)

to find $x/L)_{cr}$ use correlation of Wazzan

$$Re_{\delta^*, cr} = \text{func}(H) \quad \text{Fig (5-12)}$$

(use curve fit given in class)

note

$$\theta(x^*) = \left(\frac{\nu\lambda(x^*)}{dU/dx}\right)^{1/2} = \left[\frac{\nu\lambda(x^*)}{U_o/L\ (2.0 - 4.0x^*)}\right]^{1/2}$$

and $\delta^* = H\theta$ so

$$Re_{\delta^*} = \frac{U(x^*)\delta^*}{\nu} = \frac{U_o[1 + 2x^* - 2x^{*2}]}{\nu} H(\lambda) \left[\frac{\nu\lambda(x^*)}{U_o/L\ (2.0 - 4.0x^*)}\right]^{1/2}$$

$$Re_{\delta^*} = \sqrt{Re_L}\ [1 + 2x^* - 2x^{*2}] \left[\frac{\lambda(x^*)}{(2 - 4x^*)}\right]^{1/2} H$$

where $H = \text{func}(z)$ Eq (4.141)

$$z = (0.25 - \lambda)$$

$x/L)_{cr}$ occurs when $Re_{\delta^*} \geq Re_{\delta^*, cr}$

to find $x/L)_{tr}$ use one-step method of Wazzan

$$Re_{x,tr} = func(H) \quad Eq\ (5\text{-}42)$$

where

$$Re_x = \frac{U(x)x}{\nu} = \frac{U_0[1+2x^* - 2x^{*2}]Lx^*}{\nu}$$

$$Re_x = Re_L\, x^* \{1 + 2x^* - 2x^{*2}\}$$

$x/L)_{tr}$ occurs when $Re_x \geq Re_{x,tr}$

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see attached program listing and results

find for $Re_L = 1 \times 10^6$

$x/L)_{lam\ sep}$	$= 0.643$
$x/L)_{cr}$	$= 0.421$
$x/L)_{tr}$	$= 0.564$

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C      RUNGE.FOR  11/15/96 VERSION
C      A FORTRAN PROGRAM WRITTEN BY JOHN BARTON FOR
C      MECH 810 VISCOUS FLOW I
C
       SOLVES UP TO TEN SIMULTANEOUS FIRST ORDER ORDINARY
       DIFFERENTIAL EQUATIONS USING RUNGE-KUTTA MARCHING SCHEME
C      AS OUTLINED IN F.M. WHITE, VISCOUS FLUID FLOW, 2ND ED, 1991, APP C
C
C      X=INDEPENDENT VARIABLE
C      Y(I)=DEPENDENT VARIABLES
C
C      INPUT PARAMETERS:
C              X=BEGINNING VALUE OF INDEPENDENT VARIABLE
C              N=NUMBER OF EQUATIONS (DEPENDENT VARIABLES)
C              XLIM=ENDING VAULE OF INDEPENDENT VARIABLE
C              H=INDEPENDENT VARIABLE STEP SIZE
C              M=INITIALIZATION PARAMETER, SET EQUAL TO ZERO
C              (Y(I),I=1,N) = THE BEGINNING VAULES OF THE N DEPENDENT
C                      VARIABLES  Y(1),Y(2),Y(3)......Y(N)
C
C      OUTPUT PARAMETERS (SENT TO A DATAFILE NAMED OME810.DAT):
C              X,N,XLIM,H,M,(Y(I),I=1,N)  I.E. THE INPUT PARAMETERS
C              X,(Y(I),I=1,N) FOR EACH STEP UP TO XLIM
C
       DIMENSION Y(10),F(10)
       OPEN(UNIT=1,FILE='OME810.DAT')
       WRITE(6,*) 'ENTER X,N,XLIM,H,M,(Y(I),I=1,N)'
       READ(5,*) X,N,XLIM,H,M,(Y(I),I=1,N)
       rel=1.0e6
       WRITE(1,*) X,N,XLIM,H,M,(Y(I),I=1,N)
       WRITE(1,*) ' '
       WRITE(1,*) ' '
c       WRITE(1,*) X,(Y(I),I=1,N)
    8  IF(H*(X-XLIM)) 6,6,7
    6  CALL RUNGE(N,Y,F,X,H,M,K)
       GOTO (10,20),K
C
C      NOTE:  F(1),F(2).....F(N) ARE SYMBOLLIC EXPRESSIONS FOR THE
C      FIRST DERIVITIVES OF Y(1),Y(2)......Y(N)
C
   10  F(1)=(1.+2.*x-2.*x**2)**5
       GOTO 6
   20  continue
c      WRITE(6,*) X,(Y(I),I=1,N)
c      WRITE(1,*) X,(Y(I),I=1,N)
       ust=1.+2.*x-2.*x**2
       ustp=2.-4.*x
       xlam=0.45*ustp*y(1)/ust**6
       z=.25-xlam
       hh=2.+4.14*z-83.5*z**2+854.*z**3-3337.*z**4+4576.*z**5
       if(z.lt.0.0) hh=2.0
       if(hh.lt.2) redc=5.e4
       if((hh.ge.2.).and.(hh.lt.2.4)) redc=5.e4*(8.82e-4)**(hh-2.0)
       if((hh.ge.2.4).and.(hh.lt.2.6)) redc=3.e3*(1.56e-4)**(hh-2.4)
       if((hh.ge.2.6).and.(hh.le.3.0)) redc=520.*(1.62e-2)**(hh-2.6)
       if(hh.gt.3.0) redc=100.
       redst=hh*sqrt(rel)*ust*(xlam/ustp)**0.5
       rextr=10.**(-40.4557+64.8066*hh-26.7538*hh**2+3.3819*hh**3)
       rex=rel*x*ust
       write(6,123) x,xlam,hh,redst,redc,rex,rextr
       write(1,123) x,xlam,hh,redst,redc,rex,rextr
  123  format(1x,f6.3,1x,f6.3,1x,f6.3,1x,f7.0,1x,f7.0,1pe12.2,1x,e12.2)
       GOTO 8
    7  STOP
       END
       SUBROUTINE RUNGE(N,Y,F,X,H,M,K)
       DIMENSION Y(10),F(10),Q(10)
       M=M+1
       GOTO (1,4,5,3,7),M
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```
    1  DO 2 I=1,N
    2  Q(I)=0.0
       A=0.5
       GOTO 9
    3  A=1.707107
       IF YOU NEED MORE ACCURACY, USE
C      A=1.7071067811865475244
    4  X=X+0.5*H
    5  DO 6 I=1,N
       Y(I)=Y(I)+A*(F(I)*H-Q(I))
    6  Q(I)=2.0*A*H*F(I)+(1.0-3.0*A)*Q(I)
       A=0.2928932
C      IF YOU NEED MORE ACCURACY, SET
C      A=0.29289321881345247 56
       GOTO 9
    7  DO 8 I=1,N
    8  Y(I)=Y(I)+H*F(I)/6.0-Q(I)/3.0
       M=0
       K=2
       GOTO 10
    9  K=1
   10  RETURN
       END
```

0.000000E+00 1 6.500000E-01 1.000000E-02 0
0.000000E+00

x^*	λ	H	Re_{δ^*}	$Re_{\delta^*,cr}$	Re_x	$Re_{x,tr}$
.010	.008	2.567	170.	692.	1.02E+04	6.49E+06
.020	.015	2.547	235.	829.	2.08E+04	8.50E+06
.030	.021	2.530	283.	961.	3.17E+04	1.06E+07
.040	.026	2.516	321.	1086.	4.31E+04	1.27E+07
.050	.030	2.504	354.	1204.	5.47E+04	1.48E+07
.060	.033	2.494	382.	1315.	6.68E+04	1.68E+07
.070	.036	2.486	408.	1416.	7.91E+04	1.87E+07
.080	.039	2.478	432.	1508.	9.18E+04	2.04E+07
.090	.041	2.472	454.	1590.	1.05E+05	2.21E+07
.100	.043	2.467	475.	1663.	1.18E+05	2.35E+07
.110	.044	2.463	495.	1726.	1.32E+05	2.48E+07
.120	.045	2.460	514.	1780.	1.45E+05	2.59E+07
.130	.046	2.457	532.	1824.	1.59E+05	2.69E+07
.140	.047	2.455	550.	1861.	1.74E+05	2.76E+07
.150	.047	2.453	567.	1889.	1.88E+05	2.82E+07
.160	.048	2.452	584.	1909.	2.03E+05	2.86E+07
.170	.048	2.451	600.	1923.	2.18E+05	2.89E+07
.180	.048	2.450	616.	1930.	2.33E+05	2.91E+07
.190	.048	2.450	632.	1931.	2.48E+05	2.91E+07
.200	.048	2.451	648.	1926.	2.64E+05	2.90E+07
.210	.048	2.451	664.	1916.	2.80E+05	2.88E+07
.220	.048	2.452	680.	1901.	2.96E+05	2.85E+07
.230	.047	2.453	695.	1882.	3.11E+05	2.81E+07
.240	.047	2.455	711.	1859.	3.28E+05	2.76E+07
.250	.046	2.456	727.	1832.	3.44E+05	2.70E+07
.260	.046	2.458	742.	1801.	3.60E+05	2.64E+07
.270	.045	2.460	758.	1767.	3.76E+05	2.57E+07
.280	.044	2.463	774.	1730.	3.93E+05	2.49E+07
.290	.043	2.465	790.	1691.	4.09E+05	2.41E+07
.300	.042	2.468	806.	1649.	4.26E+05	2.32E+07
.310	.041	2.471	822.	1605.	4.43E+05	2.24E+07
.320	.040	2.475	838.	1559.	4.59E+05	2.14E+07
.330	.039	2.478	855.	1510.	4.76E+05	2.05E+07
.340	.038	2.482	871.	1461.	4.93E+05	1.95E+07
.350	.036	2.486	888.	1409.	5.09E+05	1.85E+07
.360	.035	2.491	905.	1357.	5.26E+05	1.75E+07
.370	.033	2.495	923.	1303.	5.42E+05	1.65E+07
.380	.031	2.500	940.	1248.	5.59E+05	1.55E+07
.390	.030	2.505	958.	1192.	5.76E+05	1.45E+07
.400	.028	2.511	976.	1136.	5.92E+05	1.35E+07
.410	.026	2.517	995.	1079.	6.08E+05	1.26E+07
.420	.023	2.523	1014.	1021.	6.25E+05	1.16E+07
.430	.021	2.530	1033.	963.	6.41E+05	1.06E+07
.440	.019	2.537	1053.	905.	6.57E+05	9.68E+06
.450	.016	2.544	1074.	846.	6.73E+05	8.76E+06
.460	.013	2.553	1095.	787.	6.89E+05	7.87E+06
.470	.010	2.562	1117.	728.	7.04E+05	7.01E+06
.480	.007	2.571	1139.	669.	7.20E+05	6.17E+06
.490	.004	2.582	1162.	609.	7.35E+05	5.37E+06
.500	.000	2.594	1187.	550.	7.50E+05	4.60E+06
.510	-.004	2.607	1212.	506.	7.65E+05	3.87E+06
.520	-.008	2.622	1239.	476.	7.80E+05	3.18E+06
.530	-.012	2.639	1267.	444.	7.94E+05	2.54E+06
.540	-.017	2.658	1298.	409.	8.08E+05	1.96E+06
.550	-.022	2.682	1330.	372.	8.22E+05	1.44E+06
.560	-.027	2.709	1366.	331.	8.36E+05	9.99E+05
.570	-.033	2.743	1407.	288.	8.49E+05	6.42E+05
.580	-.039	2.785	1452.	242.	8.63E+05	3.74E+05
.590	-.046	2.838	1504.	195.	8.75E+05	1.94E+05
.600	-.053	2.905	1566.	148.	8.88E+05	8.68E+04
.610	-.061	2.992	1640.	103.	9.00E+05	3.35E+04
.620	-.069	3.106	1732.	100.	9.12E+05	1.17E+04
.630	-.078	3.257	1848.	100.	9.24E+05	4.58E+03
.640	-.087	3.458	1996.	100.	9.35E+05	3.73E+03
.650	-.098	3.730	2192.	100.	9.46E+05	3.57E+04