MECE 6372 / MECE 6373

Summary of Complex Variable Theory

Viscous Flow I & II

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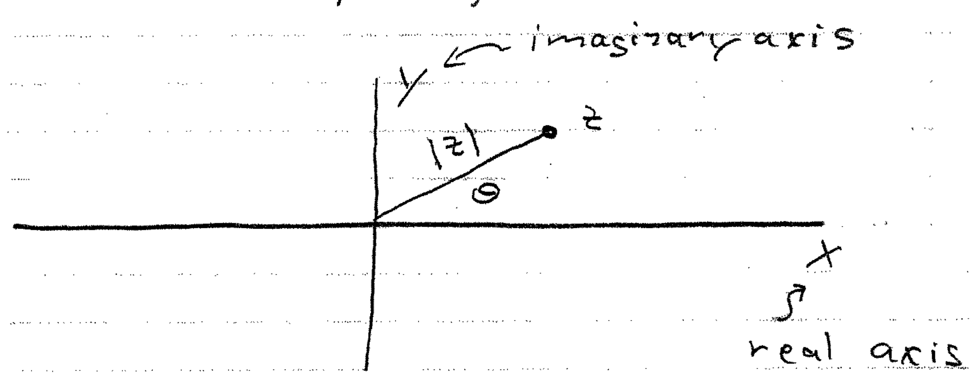
- imaginary unit, j : defined such that $j^2 = -1$
- complex variable, z : an ordered pair of real variables (x, y) such that $z = x + jy$
- complex conjugate of z : $z^* = x - jy$
- $Re[z] = x$ and $Im[z] = y$
- equality: $z_1 = z_2$ if $x_1 = x_2$ and $y_1 = y_2$
- addition: $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
- subtraction: $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$
- multiplication: $z_1 z_2 = (x_1 + jy_1)(x_2 + jy_2)$

$$= (x_1 x_2 - y_1 y_2) + j(x_1 y_2 + x_2 y_1)$$

- division: $z_1 / z_2 = z_1 z_2^* / z_2 z_2^*$

$$= \frac{(x_1 x_2 + y_1 y_2) + j(-x_1 y_2 + x_2 y_1)}{(x_2^2 + y_2^2)}$$

- any complex variable can be represented as a point on the complex plane



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- polar form $z = |z| (\cos\theta + i\sin\theta)$

where $|z| = \text{modulus } z = (x^2+y^2)^{1/2}$

and $\theta = \text{argument } z = \tan^{-1}\frac{y}{x} \pm \pi$ if $x < 0$

principal value $-\pi < \theta < \pi$

- Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$

$\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$ $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

- exponential polar form $z = |z| e^{i\theta}$

where $|z| = (zz^*)^{1/2}$

- conjugate: $z^* = |z| e^{-i\theta}$

- multiplication: $z_1 z_2 = |z_1||z_2| e^{i(\theta_1+\theta_2)}$

- division: $z_1/z_2 = \dfrac{|z_1|}{|z_2|} e^{i(\theta_1-\theta_2)}$

- conjugate of a sum: $(z_1+z_2)^* = z_1^* + z_2^*$

- conjugate of a product: $(z_1 z_2)^* = z_1^* z_2^*$

- conjugate of a quotient: $(z_1/z_2)^* = z_1^*/z_2^*$

- a complex function $f(z)$ is analytic in region R if the derivative of $f(z)$ exists for each point in R

- series expressions of common complex functions

$$e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \cdots \frac{z^n}{n!} + \cdots$$

$$\sin(z) = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \cdots$$

$$\cos(z) = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + \cdots$$

- derivatives of common complex functions

$$\frac{d[e^{az}]}{dz} = ae^{az}$$

$$\frac{d[\sin(az)]}{dz} = a\cos(az)$$

$$\frac{d[\cos(az)]}{dz} = -a\sin(az)$$

$$\frac{d[z^n]}{dz} = nz^{n-1}$$