C

Mathematical Formulas

Mathematical Constants

$\pi = 3.14159\ldots \qquad e = 2.71828\ldots \qquad 2\pi \text{ radians} = 360 \text{ degrees}$

$1 \text{ radian} = \dfrac{180}{\pi} \text{ degrees} = 57.2958^\circ \qquad 1 \text{ degree} = \dfrac{\pi}{180} \text{ radians} = 0.0174533 \text{ rad}$

Conversions: Multiply degrees by $\dfrac{\pi}{180}$ to obtain radians

Multiply radians by $\dfrac{180}{\pi}$ to obtain degrees

Exponents

$A^n A^m = A^{n+m} \qquad \dfrac{A^m}{A^n} = A^{m-n} \qquad (A^m)^n = A^{mn} \qquad A^{-m} = \dfrac{1}{A^m}$

$(AB)^n = A^n B^n \qquad \left(\dfrac{A}{B}\right)^n = \dfrac{A^n}{B^n} \qquad A^{m/n} = \sqrt[n]{A^m} \qquad A^0 = 1 \quad (A \neq 0)$

Logarithms

$\log \equiv$ common logarithm (logarithm to the base 10) $\qquad 10^x = y \qquad \log y = x$

$\ln \equiv$ natural logarithm (logarithm to the base e) $\qquad e^x = y \qquad \ln y = x$

$e^{\ln A} = A \qquad 10^{\log A} = A \qquad \ln e^A = A \qquad \log 10^A = A$

$\log AB = \log A + \log B \qquad \log \dfrac{A}{B} = \log A - \log B \qquad \log \dfrac{1}{A} = -\log A$

$\log A^n = n \log A \qquad \log 1 = \ln 1 = 0 \qquad \log 10 = 1 \qquad \ln e = 1$

$\ln A = (\ln 10)(\log A) = 2.30259 \log A \qquad \log A = (\log e)(\ln A) = 0.434294 \ln A$

Trigonometric Functions

$\tan x = \dfrac{\sin x}{\cos x} \qquad \cot x = \dfrac{\cos x}{\sin x} \qquad \sec x = \dfrac{1}{\cos x} \qquad \csc x = \dfrac{1}{\sin x}$

$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad \cot^2 x + 1 = \csc^2 x$

$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \qquad \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\sin 2x = 2 \sin x \cos x \qquad \cos 2x = \cos^2 x - \sin^2 x \qquad \tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$

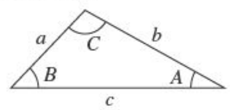
$\tan x = \dfrac{1 - \cos 2x}{\sin 2x} = \dfrac{\sin 2x}{1 + \cos 2x}$

$\sin^2 x = \dfrac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \dfrac{1}{2}(1 + \cos 2x)$

For any triangle with sides a, b, c and opposite angles A, B, C:

Law of sines $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Law of cosines $\quad c^2 = a^2 + b^2 - 2ab \cos C$



Quadratic Equation and Quadratic Formula

$ax^2 + bx + c = 0 \qquad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Infinite Series

$\dfrac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots \qquad (-1 < x < 1)$

$\sqrt{1+x} = 1 + \dfrac{x}{2} - \dfrac{x^2}{8} + \dfrac{x^3}{16} - \ldots \qquad (-1 < x < 1)$

$\dfrac{1}{\sqrt{1+x}} = 1 - \dfrac{x}{2} + \dfrac{3x^2}{8} - \dfrac{5x^3}{16} + \ldots \qquad (-1 < x < 1)$

$e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots \qquad (-\infty < x < \infty)$

$\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \ldots \qquad (-\infty < x < \infty)$

$\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \ldots \qquad (-\infty < x < \infty)$

Note: If x is very small compared to 1, only the first few terms in the series are needed.

Derivatives

$$\frac{d}{dx}(ax) = a \qquad \frac{d}{dx}(x^n) = nx^{n-1} \qquad \frac{d}{dx}(au) = a\frac{du}{dx}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} \qquad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v(du/dx) - u(dv/dx)}{v^2}$$

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx} \qquad \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \qquad \frac{du}{dx} = \frac{1}{dx/du}$$

$$\frac{d}{dx}(\sin u) = \cos u\frac{du}{dx} \qquad \frac{d}{dx}(\cos u) = -\sin u\frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = \sec^2 u\frac{du}{dx} \qquad \frac{d}{dx}(\cot u) = -\csc^2 u\frac{du}{dx}$$

$$\frac{d}{dx}(\sec u) = \sec u \tan u\frac{du}{dx} \qquad \frac{d}{dx}(\csc u) = -\csc u \cot u\frac{du}{dx}$$

$$\frac{d}{dx}(\arctan u) = \frac{1}{1+u^2}\frac{du}{dx} \qquad \frac{d}{dx}(\log u) = \frac{\log e}{u}\frac{du}{dx} \qquad \frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}(a^u) = a^u \ln a\frac{du}{dx} \qquad \frac{d}{dx}(e^u) = e^u\frac{du}{dx}$$

Indefinite Integrals

Note: A constant must be added to the result of every integration

$$\int a\,dx = ax \qquad \int u\,dv = uv - \int v\,du \quad \text{(integration by parts)}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1) \qquad \int \frac{dx}{x} = \ln|x| \quad (x \neq 0)$$

$$\int \frac{dx}{x^n} = \frac{x^{1-n}}{1-n} \quad (n \neq 1) \qquad \int (a+bx)^n\,dx = \frac{(a+bx)^{n+1}}{b(n+1)} \quad (n \neq -1)$$

$$\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx) \qquad \int \frac{dx}{(a+bx)^2} = -\frac{1}{b(a+bx)}$$

$$\int \frac{dx}{(a+bx)^n} = -\frac{1}{(n-1)(b)(a+bx)^{n-1}} \quad (n \neq 1)$$

$$\int \frac{dx}{a^2+b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} \quad (x \text{ in radians}) \quad (a > 0, b > 0)$$

$$\int \frac{dx}{a^2-b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a+bx}{a-bx}\right) \quad (x \text{ in radians}) \quad (a > 0, b > 0)$$

$$\int \frac{x\,dx}{a+bx} = \frac{1}{b^2}[bx - a\ln(a+bx)] \qquad \int \frac{x\,dx}{(a+bx)^2} = \frac{1}{b^2}\left[\frac{a}{a+bx} + \ln(a+bx)\right]$$

$$\int \frac{x\,dx}{(a+bx)^3} = -\frac{a+2bx}{2b^2(a+bx)^2} \qquad \int \frac{x\,dx}{(a+bx)^4} = -\frac{a+3bx}{6b^2(a+bx)^3}$$

$$\int \frac{x^2 dx}{a + bx} = \frac{1}{2b^3} [(a + bx)(-3a + bx) + 2a^2 \ln(a + bx)]$$

$$\int \frac{x^2 dx}{(a + bx)^2} = \frac{1}{b^3} \left[\frac{bx(2a + bx)}{a + bx} - 2a \ln (a + bx) \right]$$

$$\int \frac{x^2 dx}{(a + bx)^3} = \frac{1}{b^3} \left[\frac{a(3a + 4bx)}{2(a + bx)^2} + \ln (a + bx) \right]$$

$$\int \frac{x^2 dx}{(a + bx)^4} = -\frac{a^2 + 3abx + 3b^2x^2}{3b^3(a + bx)^3}$$

$$\int \sin ax \, dx = -\frac{\cos ax}{a} \qquad \int \cos ax \, dx = \frac{\sin ax}{a}$$

$$\int \tan ax \, dx = \frac{1}{a} \ln (\sec ax) \qquad \int \cot ax \, dx = \frac{1}{a} \ln (\sin ax)$$

$$\int \sec ax \, dx = \frac{1}{a} \ln (\sec ax + \tan ax) \qquad \int \csc ax \, dx = \frac{1}{a} \ln (\csc ax - \cot ax)$$

$$\int \sin^2 ax \, dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \qquad \int \cos^2 ax \, dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \quad (x \text{ in radians})$$

$$\int x \sin ax \, dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} \quad (x \text{ in radians})$$

$$\int x \cos ax \, dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a} \quad (x \text{ in radians})$$

$$\int e^{ax} \, dx = \frac{e^{ax}}{a} \qquad \int xe^{ax} \, dx = \frac{e^{ax}}{a^2} (ax - 1) \qquad \int \ln ax \, dx = x(\ln ax - 1)$$

$$\int \frac{dx}{1 + \sin ax} = -\frac{1}{a} \tan \left(\frac{\pi}{4} - \frac{ax}{2} \right) \qquad \int \sqrt{a + bx} \, dx = \frac{2}{3b} (a + bx)^{3/2}$$

$$\int \sqrt{a^2 + b^2x^2} \, dx = \frac{x}{2} \sqrt{a^2 + b^2x^2} + \frac{a^2}{2b} \ln \left(\frac{bx}{a} + \sqrt{1 + \frac{b^2x^2}{a^2}} \right)$$

$$\int \frac{dx}{\sqrt{a^2 + b^2x^2}} = \frac{1}{b} \ln \left(\frac{bx}{a} + \sqrt{1 + \frac{b^2x^2}{a^2}} \right)$$

$$\int \sqrt{a^2 - b^2x^2} \, dx = \frac{x}{2} \sqrt{a^2 - b^2x^2} + \frac{a^2}{2b} \sin^{-1} \frac{bx}{a}$$

Definite Integrals

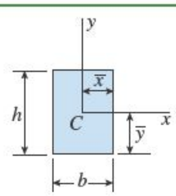
$$\int_a^b f(x) \, dx = -\int_b^a f(x) \, dx \qquad \int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx$$

D

Properties of Plane Areas

Notation: A = area
$\bar{x}, \bar{y}$ = distances to centroid C
I_x, I_y = moments of inertia with respect to the x and y axes, respectively
I_{xy} = product of inertia with respect to the x and y axes
$I_P = I_x + I_y$ = polar moment of inertia with respect to the origin of the x and y axes
I_{BB} = moment of inertia with respect to axis B-B

1

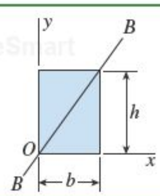


Rectangle (Origin of axes at centroid)

$$A = bh \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{2}$$

$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12} \qquad I_{xy} = 0 \qquad I_P = \frac{bh}{12}(h^2 + b^2)$$

2

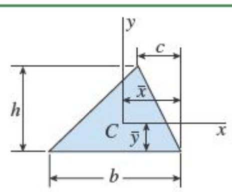


Rectangle (Origin of axes at corner)

$$I_x = \frac{bh^3}{3} \qquad I_y = \frac{hb^3}{3} \qquad I_{xy} = \frac{b^2h^2}{4} \qquad I_P = \frac{bh}{3}(h^2 + b^2)$$

$$I_{BB} = \frac{b^3h^3}{6(b^2 + h^2)}$$

3



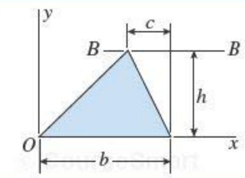
Triangle (Origin of axes at centroid)

$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b + c}{3} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{bh}{36}(b^2 - bc + c^2)$$

$$I_{xy} = \frac{bh^2}{72}(b - 2c) \qquad I_P = \frac{bh}{36}(h^2 + b^2 - bc + c^2)$$

4

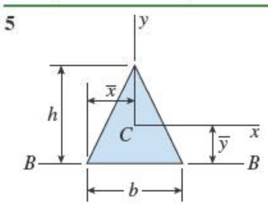


Triangle (Origin of axes at vertex)

$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{bh}{12}(3b^2 - 3bc + c^2)$$

$$I_{xy} = \frac{bh^2}{24}(3b - 2c) \qquad I_{BB} = \frac{bh^3}{4}$$

5



Isosceles triangle (Origin of axes at centroid)

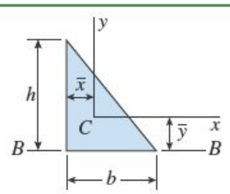
$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{48} \qquad I_{xy} = 0$$

$$I_P = \frac{bh}{144}(4h^2 + 3b^2) \qquad I_{BB} = \frac{bh^3}{12}$$

(*Note:* For an equilateral triangle, $h = \sqrt{3}\, b/2$.)

6



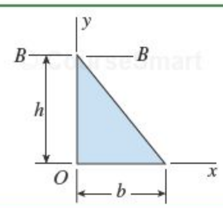
Right triangle (Origin of axes at centroid)

$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b}{3} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{36} \qquad I_{xy} = -\frac{b^2h^2}{72}$$

$$I_P = \frac{bh}{36}(h^2 + b^2) \qquad I_{BB} = \frac{bh^3}{12}$$

7

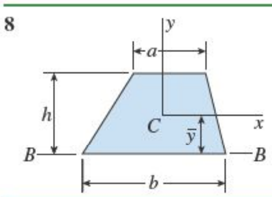


Right triangle (Origin of axes at vertex)

$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12} \qquad I_{xy} = \frac{b^2h^2}{24}$$

$$I_P = \frac{bh}{12}(h^2 + b^2) \qquad I_{BB} = \frac{bh^3}{4}$$

8

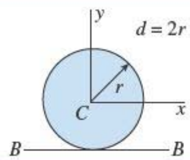


Trapezoid (Origin of axes at centroid)

$$A = \frac{h(a + b)}{2} \qquad \bar{y} = \frac{h(2a + b)}{3(a + b)}$$

$$I_x = \frac{h^3(a^2 + 4ab + b^2)}{36(a + b)} \qquad I_{BB} = \frac{h^3(3a + b)}{12}$$

9

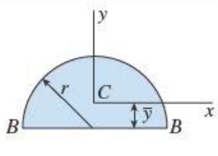


Circle (Origin of axes at center)

$$A = \pi r^2 = \frac{\pi d^2}{4} \qquad I_x = I_y = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$$

$$I_{xy} = 0 \qquad I_P = \frac{\pi r^4}{2} = \frac{\pi d^4}{32} \qquad I_{BB} = \frac{5\pi r^4}{4} = \frac{5\pi d^4}{64}$$

10

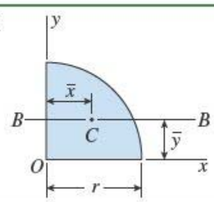


Semicircle (Origin of axes at centroid)

$$A = \frac{\pi r^2}{2} \qquad \bar{y} = \frac{4r}{3\pi}$$

$$I_x = \frac{(9\pi^2 - 64)r^4}{72\pi} \approx 0.1098r^4 \qquad I_y = \frac{\pi r^4}{8} \qquad I_{xy} = 0 \qquad I_{BB} = \frac{\pi r^4}{8}$$

11

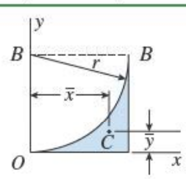


Quarter circle (Origin of axes at center of circle)

$$A = \frac{\pi r^2}{4} \qquad \bar{x} = \bar{y} = \frac{4r}{3\pi}$$

$$I_x = I_y = \frac{\pi r^4}{16} \qquad I_{xy} = \frac{r^4}{8} \qquad I_{BB} = \frac{(9\pi^2 - 64)r^4}{144\pi} \approx 0.05488r^4$$

12

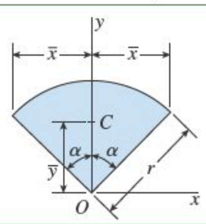


Quarter-circular spandrel (Origin of axes at point of tangency)

$$A = \left(1 - \frac{\pi}{4}\right)r^2 \qquad \bar{x} = \frac{2r}{3(4 - \pi)} \approx 0.7766r \qquad \bar{y} = \frac{(10 - 3\pi)r}{3(4 - \pi)} \approx 0.2234r$$

$$I_x = \left(1 - \frac{5\pi}{16}\right)r^4 \approx 0.01825r^4 \qquad I_y = I_{BB} = \left(\frac{1}{3} - \frac{\pi}{16}\right)r^4 \approx 0.1370r^4$$

13



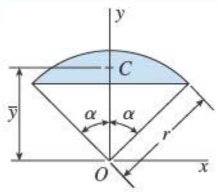
Circular sector (Origin of axes at center of circle)

α = angle in radians $\qquad (\alpha \leq \pi/2)$

$$A = \alpha r^2 \qquad \bar{x} = r \sin\alpha \qquad \bar{y} = \frac{2r \sin\alpha}{3\alpha}$$

$$I_x = \frac{r^4}{4}(\alpha + \sin\alpha\cos\alpha) \qquad I_y = \frac{r^4}{4}(\alpha - \sin\alpha\cos\alpha) \qquad I_{xy} = 0 \qquad I_P = \frac{\alpha r^4}{2}$$

14



Circular segment (Origin of axes at center of circle)

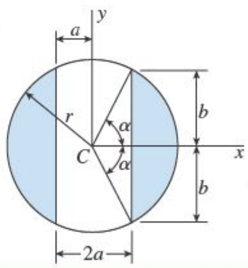
α = angle in radians $\quad (\alpha \le \pi/2)$

$$A = r^2(\alpha - \sin\alpha\cos\alpha) \qquad \bar{y} = \frac{2r}{3}\left(\frac{\sin^3\alpha}{\alpha - \sin\alpha\cos\alpha}\right)$$

$$I_x = \frac{r^4}{4}(\alpha - \sin\alpha\cos\alpha + 2\sin^3\alpha\cos\alpha) \qquad I_{xy} = 0$$

$$I_y = \frac{r^4}{12}(3\alpha - 3\sin\alpha\cos\alpha - 2\sin^3\alpha\cos\alpha)$$

15



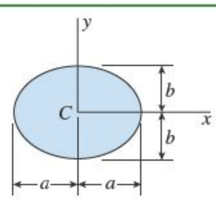
Circle with core removed (Origin of axes at center of circle)

α = angle in radians $\quad (\alpha \le \pi/2)$

$$\alpha = \arccos\frac{a}{r} \qquad b = \sqrt{r^2 - a^2} \qquad A = 2r^2\left(\alpha - \frac{ab}{r^2}\right)$$

$$I_x = \frac{r^4}{6}\left(3\alpha - \frac{3ab}{r^2} - \frac{2ab^3}{r^4}\right) \qquad I_y = \frac{r^4}{2}\left(\alpha - \frac{ab}{r^2} + \frac{2ab^3}{r^4}\right) \qquad I_{xy} = 0$$

16



Ellipse (Origin of axes at centroid)

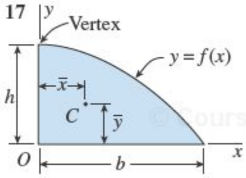
$$A = \pi ab \qquad I_x = \frac{\pi ab^3}{4} \qquad I_y = \frac{\pi ba^3}{4}$$

$$I_{xy} = 0 \qquad I_P = \frac{\pi ab}{4}(b^2 + a^2)$$

Circumference $\approx \pi[1.5(a + b) - \sqrt{ab}\,] \qquad (a/3 \le b \le a)$

$\approx 4.17b^2/a + 4a \qquad (0 \le b \le a/3)$

17



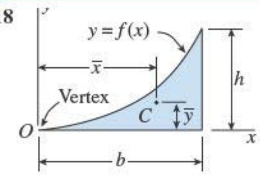
Parabolic semisegment (Origin of axes at corner)

$$y = f(x) = h\left(1 - \frac{x^2}{b^2}\right)$$

$$A = \frac{2bh}{3} \qquad \bar{x} = \frac{3b}{8} \qquad \bar{y} = \frac{2h}{5}$$

$$I_x = \frac{16bh^3}{105} \qquad I_y = \frac{2hb^3}{15} \qquad I_{xy} = \frac{b^2h^2}{12}$$

18



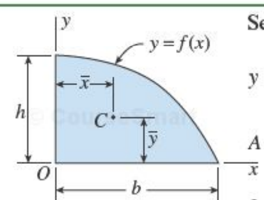
Parabolic spandrel (Origin of axes at vertex)

$$y = f(x) = \frac{hx^2}{b^2}$$

$$A = \frac{bh}{3} \qquad \bar{x} = \frac{3b}{4} \qquad \bar{y} = \frac{3h}{10}$$

$$I_x = \frac{bh^3}{21} \qquad I_y = \frac{hb^3}{5} \qquad I_{xy} = \frac{b^2h^2}{12}$$

19



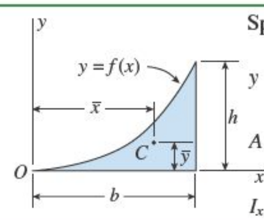
Semisegment of *n*th degree (Origin of axes at corner)

$$y = f(x) = h\left(1 - \frac{x^n}{b^n}\right) \qquad (n > 0)$$

$$A = bh\left(\frac{n}{n+1}\right) \qquad \bar{x} = \frac{b(n+1)}{2(n+2)} \qquad \bar{y} = \frac{hn}{2n+1}$$

$$I_x = \frac{2bh^3n^3}{(n+1)(2n+1)(3n+1)} \qquad I_y = \frac{hb^3n}{3(n+3)} \qquad I_{xy} = \frac{b^2h^2n^2}{4(n+1)(n+2)}$$

20



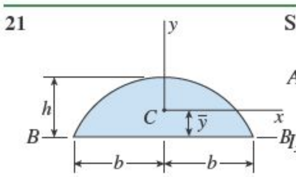
Spandrel of *n*th degree (Origin of axes at point of tangency)

$$y = f(x) = \frac{hx^n}{b^n} \qquad (n > 0)$$

$$A = \frac{bh}{n+1} \qquad \bar{x} = \frac{b(n+1)}{n+2} \qquad \bar{y} = \frac{h(n+1)}{2(2n+1)}$$

$$I_x = \frac{bh^3}{3(3n+1)} \qquad I_y = \frac{hb^3}{n+3} \qquad I_{xy} = \frac{b^2h^2}{4(n+1)}$$

21



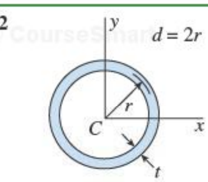
Sine wave (Origin of axes at centroid)

$$A = \frac{4bh}{\pi} \qquad \bar{y} = \frac{\pi h}{8}$$

$$I_x = \left(\frac{8}{9\pi} - \frac{\pi}{16}\right)bh^3 \approx 0.08659bh^3 \qquad I_y = \left(\frac{4}{\pi} - \frac{32}{\pi^3}\right)hb^3 \approx 0.2412hb^3$$

$$I_{xy} = 0 \qquad I_{BB} = \frac{8bh^3}{9\pi}$$

22



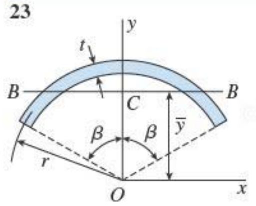
Thin circular ring (Origin of axes at center)

Approximate formulas for case when t is small

$$A = 2\pi rt = \pi dt \qquad I_x = I_y = \pi r^3 t = \frac{\pi d^3 t}{8}$$

$$I_{xy} = 0 \qquad I_P = 2\pi r^3 t = \frac{\pi d^3 t}{4}$$

23



Thin circular arc (Origin of axes at center of circle)
Approximate formulas for case when t is small

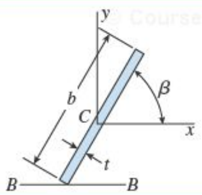
β = angle in radians (*Note:* For a semicircular arc, $\beta = \pi/2$.)

$A = 2\beta rt \qquad \bar{y} = \dfrac{r \sin \beta}{\beta}$

$I_x = r^3 t(\beta + \sin\beta\cos\beta) \qquad I_y = r^3 t(\beta - \sin\beta\cos\beta)$

$I_{xy} = 0 \qquad I_{BB} = r^3 t\left(\dfrac{2\beta + \sin 2\beta}{2} - \dfrac{1 - \cos 2\beta}{\beta}\right)$

24

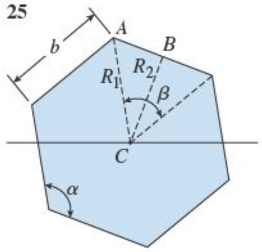


Thin rectangle (Origin of axes at centroid)
Approximate formulas for case when t is small

$A = bt$

$I_x = \dfrac{tb^3}{12}\sin^2\beta \qquad I_y = \dfrac{tb^3}{12}\cos^2\beta \qquad I_{BB} = \dfrac{tb^3}{3}\sin^2\beta$

25



Regular polygon with n sides (Origin of axes at centroid)

C = centroid (at center of polygon)

n = number of sides ($n \geq 3$) $\qquad b$ = length of a side

β = central angle for a side $\qquad \alpha$ = interior angle (or vertex angle)

$\beta = \dfrac{360°}{n} \qquad \alpha = \left(\dfrac{n-2}{n}\right)180° \qquad \alpha + \beta = 180°$

R_1 = radius of circumscribed circle (line CA) $\qquad R_2$ = radius of inscribed circle (line CB)

$R_1 = \dfrac{b}{2}\csc\dfrac{\beta}{2} \qquad R_2 = \dfrac{b}{2}\cot\dfrac{\beta}{2} \qquad A = \dfrac{nb^2}{4}\cot\dfrac{\beta}{2}$

I_c = moment of inertia about any axis through C (the centroid C is a principal point and every axis through C is a principal axis)

$I_c = \dfrac{nb^4}{192}\left(\cot\dfrac{\beta}{2}\right)\left(3\cot^2\dfrac{\beta}{2} + 1\right) \qquad I_P = 2I_c$