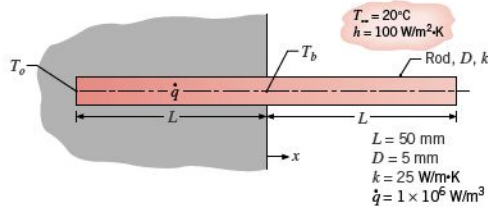


■ Problems

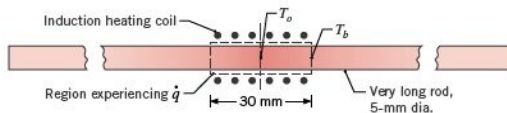
- (a) Derive an expression for the exposed surface temperature  $T_o$  as a function of the prescribed thermal and geometrical parameters. The rod has an exposed length  $L_o$ , and its tip is well insulated.
- (b) Will a rod with  $L_o = 200$  mm meet the specified operating limit? If not, what design parameters would you change? Consider another material, increasing the thickness of the insulation, and increasing the rod length. Also, consider how you might attach the base of the rod to the furnace wall as a means to reduce  $T_o$ .

**3.112** A metal rod of length  $2L$ , diameter  $D$ , and thermal conductivity  $k$  is inserted into a perfectly insulating wall, exposing one-half of its length to an air stream that is of temperature  $T_\infty$  and provides a convection coefficient  $h$  at the surface of the rod. An electromagnetic field induces volumetric energy generation at a uniform rate  $\dot{q}$  within the embedded portion of the rod.



- (a) Derive an expression for the steady-state temperature  $T_b$  at the base of the exposed half of the rod. The exposed region may be approximated as a very long fin.
- (b) Derive an expression for the steady-state temperature  $T_o$  at the end of the embedded half of the rod.
- (c) Using numerical values provided in the schematic, plot the temperature distribution in the rod and describe key features of the distribution. Does the rod behave as a very long fin?

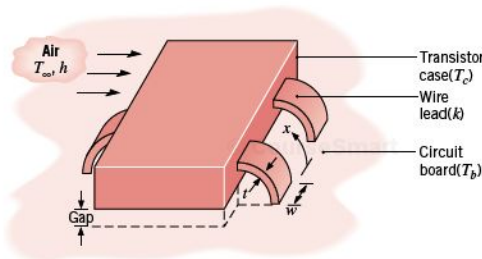
**3.113** A very long rod of 5-mm diameter and uniform thermal conductivity  $k = 25$  W/m · K is subjected to a heat treatment process. The center, 30-mm-long portion of the rod within the induction heating coil experiences uniform volumetric heat generation of  $7.5 \times 10^6$  W/m<sup>3</sup>.



The unheated portions of the rod, which protrude from the heating coil on either side, experience convection with the ambient air at  $T_\infty = 20^\circ\text{C}$  and  $h = 10$  W/m<sup>2</sup> · K. Assume that there is no convection from the surface of the rod within the coil.

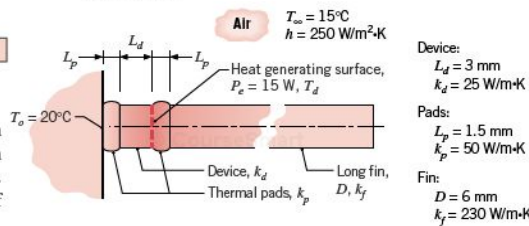
- (a) Calculate the steady-state temperature  $T_o$  of the rod at the midpoint of the heated portion in the coil.
- (b) Calculate the temperature of the rod  $T_b$  at the edge of the heated portion.

**3.114** From Problem 1.71, consider the wire leads connecting the transistor to the circuit board. The leads are of thermal conductivity  $k$ , thickness  $t$ , width  $w$ , and length  $L$ . One end of a lead is maintained at a temperature  $T_c$  corresponding to the transistor case, while the other end assumes the temperature  $T_b$  of the circuit board. During steady-state operation, current flow through the leads provides for uniform volumetric heating in the amount  $\dot{q}$ , while there is convection cooling to air that is at  $T_\infty$  and maintains a convection coefficient  $h$ .



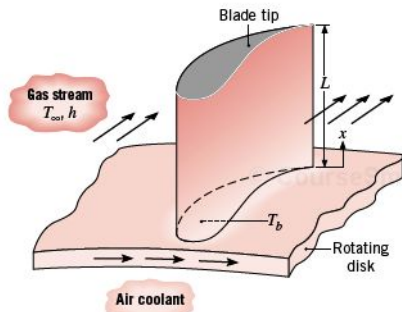
- (a) Derive an equation from which the temperature distribution in a wire lead may be determined. List all pertinent assumptions.
- (b) Determine the temperature distribution in a wire lead, expressing your results in terms of the prescribed variables.

**3.115** A disk-shaped electronic device of thickness  $L_d$ , diameter  $D$ , and thermal conductivity  $k_d$  dissipates electrical power at a steady rate  $P_e$  along one of its surfaces. The device is bonded to a cooled base at  $T_o$  using a thermal pad of thickness  $L_p$  and thermal conductivity  $k_p$ . A long fin of diameter  $D$  and thermal conductivity  $k_f$  is bonded to the heat-generating surface of the device using an identical thermal pad. The fin is cooled by an air stream, which is at a temperature  $T_\infty$  and provides a convection coefficient  $h$ .



- (a) Construct a thermal circuit of the system.
- (b) Derive an expression for the temperature  $T_d$  of the heat-generating surface of the device in terms of the circuit thermal resistances,  $T_o$  and  $T_\infty$ . Express the thermal resistances in terms of appropriate parameters.
- (c) Calculate  $T_d$  for the prescribed conditions.

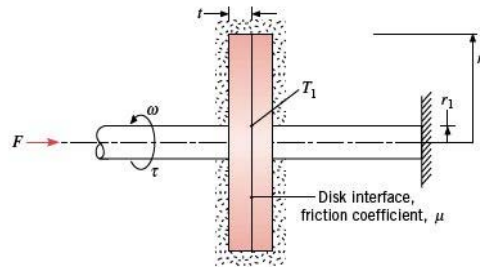
**3.116** Turbine blades mounted to a rotating disc in a gas turbine engine are exposed to a gas stream that is at  $T_\infty = 1200^\circ\text{C}$  and maintains a convection coefficient of  $h = 250 \text{ W/m}^2 \cdot \text{K}$  over the blade.



The blades, which are fabricated from Inconel,  $k \approx 20 \text{ W/m} \cdot \text{K}$ , have a length of  $L = 50 \text{ mm}$ . The blade profile has a uniform cross-sectional area of  $A_c = 6 \times 10^{-4} \text{ m}^2$  and a perimeter of  $P = 110 \text{ mm}$ . A proposed blade-cooling scheme, which involves routing air through the supporting disc, is able to maintain the base of each blade at a temperature of  $T_b = 300^\circ\text{C}$ .

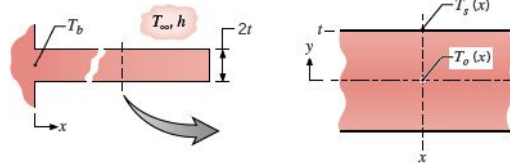
- (a) If the maximum allowable blade temperature is  $1050^\circ\text{C}$  and the blade tip may be assumed to be adiabatic, is the proposed cooling scheme satisfactory?
- (b) For the proposed cooling scheme, what is the rate at which heat is transferred from each blade to the coolant?

**3.117** In a test to determine the friction coefficient,  $\mu$ , associated with a disk brake, one disk and its shaft are rotated at a constant angular velocity  $\omega$ , while an equivalent disk/shaft assembly is stationary. Each disk has an outer radius of  $r_2 = 180 \text{ mm}$ , a shaft radius of  $r_1 = 20 \text{ mm}$ , a thickness of  $t = 12 \text{ mm}$ , and a thermal conductivity of  $k = 15 \text{ W/m} \cdot \text{K}$ . A known force  $F$  is applied to the system, and the corresponding torque  $\tau$  required to maintain rotation is measured. The disk contact pressure may be assumed to be uniform (i.e., independent of location on the interface), and the disks may be assumed to be well insulated from the surroundings.



- (a) Obtain an expression that may be used to evaluate  $\mu$  from known quantities.
- (b) For the region  $r_1 \leq r \leq r_2$ , determine the radial temperature distribution,  $T(r)$ , in the disk, where  $T(r_1) = T_1$  is presumed to be known.
- (c) Consider test conditions for which  $F = 200 \text{ N}$ ,  $\omega = 40 \text{ rad/s}$ ,  $\tau = 8 \text{ N} \cdot \text{m}$ , and  $T_1 = 80^\circ\text{C}$ . Evaluate the friction coefficient and the maximum disk temperature.

**3.118** Consider an extended surface of rectangular cross section with heat flow in the longitudinal direction.



In this problem we seek to determine conditions for which the transverse (y-direction) temperature difference within the extended surface is negligible compared to the temperature difference between the surface and the environment, such that the one-dimensional analysis of Section 3.6.1 is valid.

- (a) Assume that the transverse temperature distribution is parabolic and of the form

$$\frac{T(y) - T_o(x)}{T_s(x) - T_o(x)} = \left(\frac{y}{t}\right)^2$$

where  $T_s(x)$  is the surface temperature and  $T_o(x)$  is the centerline temperature at any  $x$ -location. Using Fourier's law, write an expression for the conduction heat flux at the surface,  $q''_y(t)$ , in terms of  $T_s$  and  $T_o$ .

- (b) Write an expression for the convection heat flux at the surface for the  $x$ -location. Equating the two expressions for the heat flux by conduction and

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- (d) Derive an expression for the steady-state temperature  $T(x, \infty) = T_j$ , leaving your result in terms of plate parameters ( $M, c_p$ ), thermal conditions ( $T_s, T_\infty, h$ ), the surface temperature  $T(L, t)$ , and the heating time  $t_o$ .

### Lumped Capacitance Method

**5.5** Steel balls 12 mm in diameter are annealed by heating to 1150 K and then slowly cooling to 400 K in an air environment for which  $T_\infty = 325$  K and  $h = 20$  W/m<sup>2</sup> · K. Assuming the properties of the steel to be  $k = 40$  W/m · K,  $\rho = 7800$  kg/m<sup>3</sup>, and  $c = 600$  J/kg · K, estimate the time required for the cooling process.

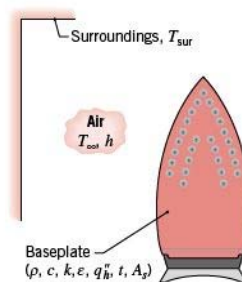
**5.6** Consider the steel balls of Problem 5.5, except now the air temperature increases with time as  $T_\infty(t) = 325$  K +  $at$  where  $a = 0.1875$  K/s.

- (a) Sketch the ball temperature versus time for  $0 \leq t \leq 1$  h. Also show the ambient temperature,  $T_\infty$ , in your graph. Explain special features of the ball temperature behavior.
- (b) Find an expression for the ball temperature as a function of time,  $T(t)$ , and plot the ball temperature for  $0 \leq t \leq 1$  h. Was your sketch correct?

**5.7** The heat transfer coefficient for air flowing over a sphere is to be determined by observing the temperature–time history of a sphere fabricated from pure copper. The sphere, which is 12.7 mm in diameter, is at 66°C before it is inserted into an airstream having a temperature of 27°C. A thermocouple on the outer surface of the sphere indicates 55°C 69 s after the sphere is inserted in the airstream. Assume, and then justify, that the sphere behaves as a spacewise isothermal object and calculate the heat transfer coefficient.

**5.8** A solid steel sphere (AISI 1010), 300 mm in diameter, is coated with a dielectric material layer of thickness 2 mm and thermal conductivity 0.04 W/m · K. The coated sphere is initially at a uniform temperature of 500°C and is suddenly quenched in a large oil bath for which  $T_\infty = 100^\circ\text{C}$  and  $h = 3300$  W/m<sup>2</sup> · K. Estimate the time required for the coated sphere temperature to reach 140°C. *Hint:* Neglect the effect of energy storage in the dielectric material, since its thermal capacitance ( $\rho cV$ ) is small compared to that of the steel sphere.

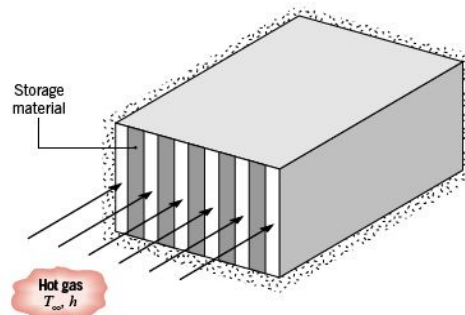
**5.9** The base plate of an iron has a thickness of  $L = 7$  mm and is made from an aluminum alloy ( $\rho = 2800$  kg/m<sup>3</sup>,  $c = 900$  J/kg · K,  $k = 180$  W/m · K,  $\varepsilon = 0.80$ ). An electric resistance heater is attached to the inner surface of the plate, while the outer surface is exposed to ambient air and large surroundings at  $T_\infty = T_{\text{sur}} = 25^\circ\text{C}$ . The areas of both the inner and outer surfaces are  $A_s = 0.040$  m<sup>2</sup>.



If an approximately uniform heat flux of  $q''_h = 1.25 \times 10^4$  W/m<sup>2</sup> is applied to the inner surface of the base plate and the convection coefficient at the outer surface is  $h = 10$  W/m<sup>2</sup> · K, estimate the time required for the plate to reach a temperature of 135°C. *Hint:* Numerical integration is suggested in order to solve the problem.

**5.10** Carbon steel (AISI 1010) shafts of 0.1-m diameter are heat treated in a gas-fired furnace whose gases are at 1200 K and provide a convection coefficient of 100 W/m<sup>2</sup> · K. If the shafts enter the furnace at 300 K, how long must they remain in the furnace to achieve a centerline temperature of 800 K?

**5.11** A thermal energy storage unit consists of a large rectangular channel, which is well insulated on its outer surface and encloses alternating layers of the storage material and the flow passage.



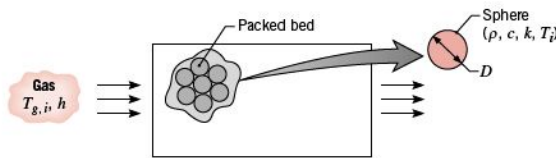
Each layer of the storage material is an aluminum slab of width  $W = 0.05$  m, which is at an initial temperature of 25°C. Consider conditions for which the storage unit is charged by passing a hot gas through the passages, with the gas temperature and the convection coefficient assumed to have constant values of  $T_\infty = 600^\circ\text{C}$  and  $h = 100$  W/m<sup>2</sup> · K throughout the channel. How long will it take to achieve 75% of the maximum possible



■ Problems

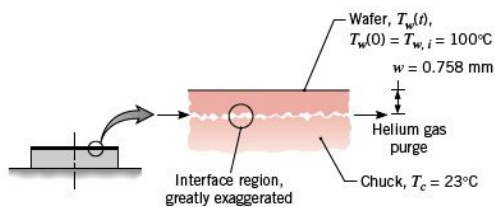
energy storage? What is the temperature of the aluminum at this time?

- 5.12 Thermal energy storage systems commonly involve a packed bed of solid spheres, through which a hot gas flows if the system is being charged, or a cold gas if it is being discharged. In a charging process, heat transfer from the hot gas increases thermal energy stored within the colder spheres; during discharge, the stored energy decreases as heat is transferred from the warmer spheres to the cooler gas.



Consider a packed bed of 75-mm-diameter aluminum spheres ( $\rho = 2700 \text{ kg/m}^3$ ,  $c = 950 \text{ J/kg}\cdot\text{K}$ ,  $k = 240 \text{ W/m}\cdot\text{K}$ ) and a charging process for which gas enters the storage unit at a temperature of  $T_{g,i} = 300^\circ\text{C}$ . If the initial temperature of the spheres is  $T_i = 25^\circ\text{C}$  and the convection coefficient is  $h = 75 \text{ W/m}^2\cdot\text{K}$ , how long does it take a sphere near the inlet of the system to accumulate 90% of the maximum possible thermal energy? What is the corresponding temperature at the center of the sphere? Is there any advantage to using copper instead of aluminum?

- 5.13 A tool used for fabricating semiconductor devices consists of a chuck (thick metallic, cylindrical disk) onto which a very thin silicon wafer ( $\rho = 2700 \text{ kg/m}^3$ ,  $c = 875 \text{ J/kg}\cdot\text{K}$ ,  $k = 177 \text{ W/m}\cdot\text{K}$ ) is placed by a robotic arm. Once in position, an electric field in the chuck is energized, creating an electrostatic force that holds the wafer firmly to the chuck. To ensure a reproducible thermal contact resistance between the chuck and the wafer from cycle-to-cycle, pressurized helium gas is introduced at the center of the chuck and flows (very slowly) radially outward between the asperities of the interface region.

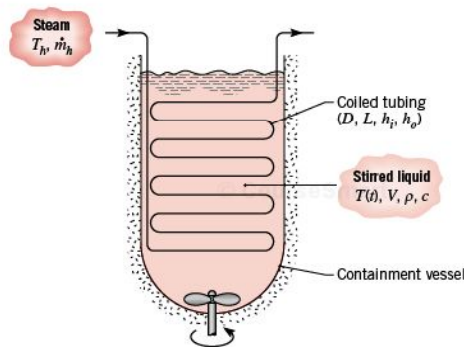


An experiment has been performed under conditions for which the wafer, initially at a uniform temperature  $T_{w,i} = 100^\circ\text{C}$ , is suddenly placed on the chuck, which is at a uniform and constant temperature  $T_c = 23^\circ\text{C}$ . With the wafer in place, the electrostatic force and the helium gas flow are applied. After 15 seconds, the temperature of the wafer is determined to be  $33^\circ\text{C}$ . What is the thermal contact resistance  $R''_{t,c}$  ( $\text{m}^2\cdot\text{K/W}$ ) between the wafer and chuck? Will the value of  $R''_{t,c}$  increase, decrease, or remain the same if air, instead of helium, is used as the purge gas?

- 5.14 A spherical vessel used as a reactor for producing pharmaceuticals has a 5-mm-thick stainless steel wall ( $k = 17 \text{ W/m}\cdot\text{K}$ ) and an inner diameter of  $D_i = 1.0 \text{ m}$ . During production, the vessel is filled with reactants for which  $\rho = 1100 \text{ kg/m}^3$  and  $c = 2400 \text{ J/kg}\cdot\text{K}$ , while exothermic reactions release energy at a volumetric rate of  $\dot{q} = 10^4 \text{ W/m}^3$ . As first approximations, the reactants may be assumed to be well stirred and the thermal capacitance of the vessel may be neglected.

- (a) The exterior surface of the vessel is exposed to ambient air ( $T_\infty = 25^\circ\text{C}$ ) for which a convection coefficient of  $h = 6 \text{ W/m}^2\cdot\text{K}$  may be assumed. If the initial temperature of the reactants is  $25^\circ\text{C}$ , what is the temperature of the reactants after five hours of process time? What is the corresponding temperature at the outer surface of the vessel?
- (b) Explore the effect of varying the convection coefficient on transient thermal conditions within the reactor.

- 5.15 Batch processes are often used in chemical and pharmaceutical operations to achieve a desired chemical composition for the final product and typically involve a transient heating operation to take the product from room temperature to the desired process temperature.



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corresponding temperature histories of the plate for  $0 \leq t \leq 2500$  s.

- 5.20 An electronic device, such as a power transistor mounted on a finned heat sink, can be modeled as a spatially isothermal object with internal heat generation and an external convection resistance.

- (a) Consider such a system of mass  $M$ , specific heat  $c$ , and surface area  $A_s$ , which is initially in equilibrium with the environment at  $T_\infty$ . Suddenly, the electronic device is energized such that a constant heat generation  $\dot{E}_g$  (W) occurs. Show that the temperature response of the device is

$$\frac{\theta}{\theta_i} = \exp\left(-\frac{t}{RC}\right)$$

where  $\theta \equiv T - T(\infty)$  and  $T(\infty)$  is the steady-state temperature corresponding to  $t \rightarrow \infty$ ;  $\theta_i = T_i - T(\infty)$ ;  $T_i$  = initial temperature of device;  $R$  = thermal resistance  $1/\bar{h}A_s$ ; and  $C$  = thermal capacitance  $Mc$ .

- (b) An electronic device, which generates 60 W of heat, is mounted on an aluminum heat sink weighing 0.31 kg and reaches a temperature of 100°C in ambient air at 20°C under steady-state conditions. If the device is initially at 20°C, what temperature will it reach 5 min after the power is switched on?

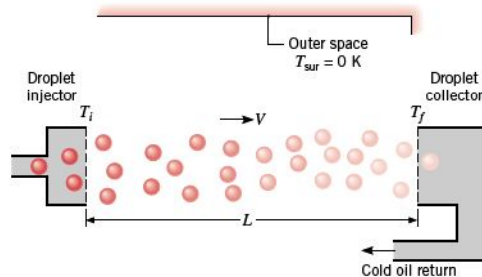
- 5.21 Before being injected into a furnace, pulverized coal is preheated by passing it through a cylindrical tube whose surface is maintained at  $T_{\text{sur}} = 1000^\circ\text{C}$ . The coal pellets are suspended in an airflow and are known to move with a speed of 3 m/s. If the pellets may be approximated as spheres of 1-mm diameter and it may be assumed that they are heated by radiation transfer from the tube surface, how long must the tube be to heat coal entering at 25°C to a temperature of 600°C? Is the use of the lumped capacitance method justified?

- 5.22 A metal sphere of diameter  $D$ , which is at a uniform temperature  $T_i$ , is suddenly removed from a furnace and suspended from a fine wire in a large room with air at a uniform temperature  $T_\infty$  and the surrounding walls at a temperature  $T_{\text{sur}}$ .

- (a) Neglecting heat transfer by radiation, obtain an expression for the time required to cool the sphere to some temperature  $T$ .
- (b) Neglecting heat transfer by convection, obtain an expression for the time required to cool the sphere to the temperature  $T$ .
- (c) How would you go about determining the time required for the sphere to cool to the temperature  $T$  if both convection and radiation are of the same order of magnitude?

- (d) Consider an anodized aluminum sphere ( $\varepsilon = 0.75$ ) 50 mm in diameter, which is at an initial temperature of  $T_i = 800$  K. Both the air and surroundings are at 300 K, and the convection coefficient is  $10 \text{ W/m}^2 \cdot \text{K}$ . For the conditions of parts (a), (b), and (c), determine the time required for the sphere to cool to 400 K. Plot the corresponding temperature histories. Repeat the calculations for a polished aluminum sphere ( $\varepsilon = 0.1$ ).

- 5.23 As permanent space stations increase in size, there is an attendant increase in the amount of electrical power they dissipate. To keep station compartment temperatures from exceeding prescribed limits, it is necessary to transfer the dissipated heat to space. A novel heat rejection scheme that has been proposed for this purpose is termed a Liquid Droplet Radiator (LDR). The heat is first transferred to a high vacuum oil, which is then injected into outer space as a stream of small droplets. The stream is allowed to traverse a distance  $L$ , over which it cools by radiating energy to outer space at absolute zero temperature. The droplets are then collected and routed back to the space station.

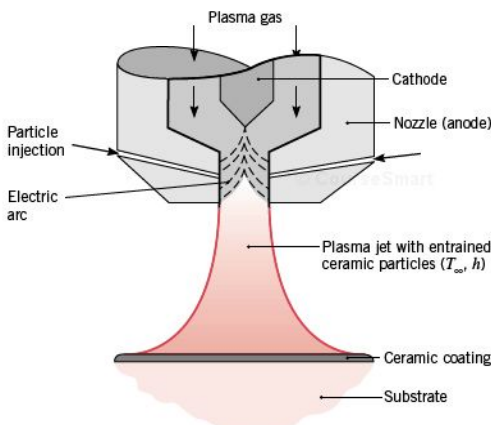


Consider conditions for which droplets of emissivity  $\varepsilon = 0.95$  and diameter  $D = 0.5$  mm are injected at a temperature of  $T_i = 500$  K and a velocity of  $V = 0.1$  m/s. Properties of the oil are  $\rho = 885 \text{ kg/m}^3$ ,  $c = 1900 \text{ J/kg} \cdot \text{K}$ , and  $k = 0.145 \text{ W/m} \cdot \text{K}$ . Assuming each drop to radiate to deep space at  $T_{\text{sur}} = 0$  K, determine the distance  $L$  required for the droplets to impact the collector at a final temperature of  $T_f = 300$  K. What is the amount of thermal energy rejected by each droplet?

- 5.24 In a material processing experiment conducted aboard the space shuttle, a coated niobium sphere of 10-mm diameter is removed from a furnace at 900°C and cooled to a temperature of 300°C. Although properties of the niobium vary over this temperature range, constant values may be assumed to a reasonable approximation, with  $\rho = 8600 \text{ kg/m}^3$ ,  $c = 290 \text{ J/kg} \cdot \text{K}$ , and  $k = 63 \text{ W/m} \cdot \text{K}$ .

- (a) If cooling is implemented in a large evacuated chamber whose walls are at  $25^\circ\text{C}$ , determine the time required to reach the final temperature if the coating is polished and has an emissivity of  $\varepsilon = 0.1$ . How long would it take if the coating is oxidized and  $\varepsilon = 0.6$ ?
- (b) To reduce the time required for cooling, consideration is given to immersion of the sphere in an inert gas stream for which  $T_\infty = 25^\circ\text{C}$  and  $h = 200 \text{ W/m}^2 \cdot \text{K}$ . Neglecting radiation, what is the time required for cooling?
- (c) Considering the effect of both radiation and convection, what is the time required for cooling if  $h = 200 \text{ W/m}^2 \cdot \text{K}$  and  $\varepsilon = 0.6$ ? Explore the effect on the cooling time of independently varying  $h$  and  $\varepsilon$ .

**5.25** Plasma spray-coating processes are often used to provide surface protection for materials exposed to hostile environments, which induce degradation through factors such as wear, corrosion, or outright thermal failure. Ceramic coatings are commonly used for this purpose. By injecting ceramic powder through the nozzle (anode) of a plasma torch, the particles are entrained by the plasma jet, within which they are then accelerated and heated.



During their *time-in-flight*, the ceramic particles must be heated to their melting point and experience complete conversion to the liquid state. The coating is formed as the molten droplets impinge (*splat*) on the substrate material and experience rapid solidification. Consider conditions for which spherical alumina ( $\text{Al}_2\text{O}_3$ ) particles of diameter  $D_p = 50 \mu\text{m}$ , density  $\rho_p = 3970 \text{ kg/m}^3$ , thermal conductivity  $k_p = 10.5 \text{ W/m} \cdot \text{K}$ , and specific heat  $c_p = 1560 \text{ J/kg} \cdot \text{K}$  are injected into an

arc plasma, which is at  $T_\infty = 10,000 \text{ K}$  and provides a coefficient of  $h = 30,000 \text{ W/m}^2 \cdot \text{K}$  for convective heating of the particles. The melting point and latent heat of fusion of alumina are  $T_{\text{mp}} = 2318 \text{ K}$  and  $h_{\text{gf}} = 3577 \text{ kJ/kg}$ , respectively.

- (a) Neglecting radiation, obtain an expression for the time-in-flight,  $t_{i-f}$ , required to heat a particle from its initial temperature  $T_i$  to its melting point  $T_{\text{mp}}$ , and, once at the melting point, for the particle to experience complete melting. Evaluate  $t_{i-f}$  for  $T_i = 300 \text{ K}$  and the prescribed heating conditions.
- (b) Assuming alumina to have an emissivity of  $\varepsilon_p = 0.4$  and the particles to exchange radiation with large surroundings at  $T_{\text{sur}} = 300 \text{ K}$ , assess the validity of neglecting radiation.

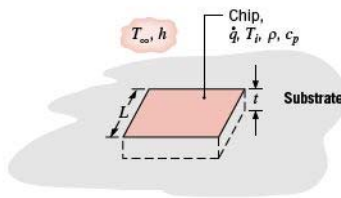
**5.26** Thin film coatings characterized by high resistance to abrasion and fracture may be formed by using micro-scale composite particles in a plasma spraying process. A spherical particle typically consists of a *ceramic core*, such as tungsten carbide (WC), and a *metallic shell*, such as cobalt (Co). The ceramic provides the thin film coating with its desired hardness at elevated temperatures, while the metal serves to coalesce the particles on the coated surface and to inhibit crack formation. In the plasma spraying process, the particles are injected into a plasma gas jet that heats them to a temperature above the melting point of the metallic casing and melts the casing before the particles impact the surface.

Consider spherical particles comprised of a WC core of diameter  $D_i = 16 \mu\text{m}$ , which is encased in a Co shell of outer diameter  $D_o = 20 \mu\text{m}$ . If the particles flow in a plasma gas at  $T_\infty = 10,000 \text{ K}$  and the coefficient associated with convection from the gas to the particles is  $h = 20,000 \text{ W/m}^2 \cdot \text{K}$ , how long does it take to heat the particles from an initial temperature of  $T_i = 300 \text{ K}$  to the melting point of cobalt,  $T_{\text{mp}} = 1770 \text{ K}$ ? The density and specific heat of WC (the core of the particle) are  $\rho_c = 16,000 \text{ kg/m}^3$  and  $c_c = 300 \text{ J/kg} \cdot \text{K}$ , while the corresponding values for Co (the outer shell) are  $\rho_s = 8900 \text{ kg/m}^3$  and  $c_s = 750 \text{ J/kg} \cdot \text{K}$ . Once having reached the melting point, how much additional time is required to completely melt the cobalt if its latent heat of fusion is  $h_{\text{gf}} = 2.59 \times 10^5 \text{ J/kg}$ ? You may use the lumped capacitance method of analysis and neglect radiation exchange between the particle and its surroundings.

**5.27** A chip that is of length  $L = 5 \text{ mm}$  on a side and thickness  $t = 1 \text{ mm}$  is encased in a ceramic substrate, and its exposed surface is convectively cooled by a dielectric liquid for which  $h = 150 \text{ W/m}^2 \cdot \text{K}$  and  $T_\infty = 20^\circ\text{C}$ .



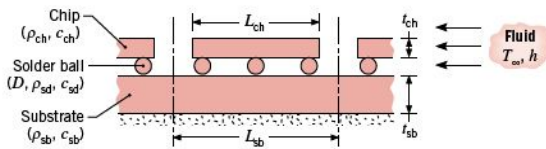
Problems



In the off-mode the chip is in thermal equilibrium with the coolant ( $T_i = T_\infty$ ). When the chip is energized, however, its temperature increases until a new steady-state is established. For purposes of analysis, the energized chip is characterized by uniform volumetric heating with  $\dot{q} = 9 \times 10^6 \text{ W/m}^3$ . Assuming an infinite contact resistance between the chip and substrate and negligible conduction resistance within the chip, determine the steady-state chip temperature  $T_f$ . Following activation of the chip, how long does it take to come within  $1^\circ\text{C}$  of this temperature? The chip density and specific heat are  $\rho = 2000 \text{ kg/m}^3$  and  $c = 700 \text{ J/kg} \cdot \text{K}$ , respectively.

- 5.28 Consider the conditions of Problem 5.27. In addition to treating heat transfer by convection directly from the chip to the coolant, a more realistic analysis would account for indirect transfer from the chip to the substrate and then from the substrate to the coolant. The total thermal resistance associated with this indirect route includes contributions due to the chip–substrate interface (a contact resistance), multidimensional conduction in the substrate, and convection from the surface of the substrate to the coolant. If this total thermal resistance is  $R_t = 200 \text{ K/W}$ , what is the steady-state chip temperature  $T_f$ ? Following activation of the chip, how long does it take to come within  $1^\circ\text{C}$  of this temperature?

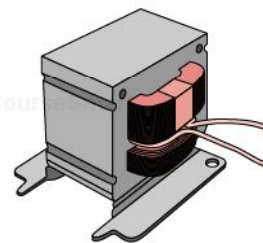
- 5.29 Thermal stress testing is a common procedure used to assess the reliability of an electronic package. Typically, thermal stresses are induced in soldered or wired connections to reveal mechanisms that could cause failure and must therefore be corrected before the product is released. As an example of the procedure, consider an array of silicon chips ( $\rho_{\text{ch}} = 2300 \text{ kg/m}^3$ ,  $c_{\text{ch}} = 710 \text{ J/kg} \cdot \text{K}$ ) joined to an alumina substrate ( $\rho_{\text{sb}} = 4000 \text{ kg/m}^3$ ,  $c_{\text{sb}} = 770 \text{ J/kg} \cdot \text{K}$ ) by solder balls ( $\rho_{\text{sd}} = 11,000 \text{ kg/m}^3$ ,  $c_{\text{sd}} = 130 \text{ J/kg} \cdot \text{K}$ ). Each chip of width  $L_{\text{ch}}$  and thickness  $t_{\text{ch}}$  is joined to a unit substrate section of width  $L_{\text{sb}}$  and thickness  $t_{\text{sb}}$  by solder balls of diameter  $D$ .



A thermal stress test begins by subjecting the multichip module, which is initially at room temperature, to a hot fluid stream and subsequently cooling the module by exposing it to a cold fluid stream. The process is repeated for a prescribed number of cycles to assess the integrity of the soldered connections.

- (a) As a first approximation, assume that there is negligible heat transfer between the components (chip/solder/substrate) of the module and that the thermal response of each component may be determined from a lumped capacitance analysis involving the same convection coefficient  $h$ . Assuming no reduction in surface area due to contact between a solder ball and the chip or substrate, obtain expressions for the thermal time constant of each component. Heat transfer is to all surfaces of a chip, but to only the top surface of the substrate. Evaluate the three time constants for  $L_{\text{ch}} = 15 \text{ mm}$ ,  $t_{\text{ch}} = 2 \text{ mm}$ ,  $L_{\text{sb}} = 25 \text{ mm}$ ,  $t_{\text{sb}} = 10 \text{ mm}$ ,  $D = 2 \text{ mm}$ , and a value of  $h = 50 \text{ W/m}^2 \cdot \text{K}$ , which is characteristic of an air stream. Compute and plot the temperature histories of the three components for the heating portion of a cycle, with  $T_i = 20^\circ\text{C}$  and  $T_\infty = 80^\circ\text{C}$ . At what time does each component experience 99% of its maximum possible temperature rise, that is,  $(T - T_i)/(T_\infty - T_i) = 0.99$ ? If the maximum stress on a solder ball corresponds to the maximum difference between its temperature and that of the chip or substrate, when will this maximum occur?
- (b) To reduce the time required to complete a stress test, a dielectric liquid could be used in lieu of air to provide a larger convection coefficient of  $h = 200 \text{ W/m}^2 \cdot \text{K}$ . What is the corresponding savings in time for each component to achieve 99% of its maximum possible temperature rise?

- 5.30 The objective of this problem is to develop thermal models for estimating the steady-state temperature and the transient temperature history of the electrical transformer shown below.



The external transformer geometry is approximately cubical, with a length of 32 mm to a side. The combined