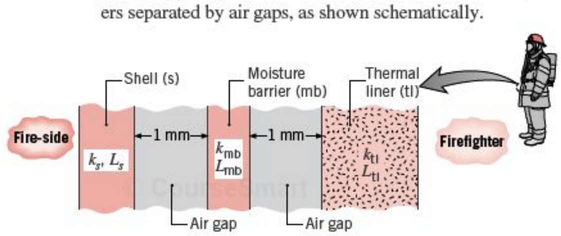
cell is that such a device can internally *reform* readily available liquid fuels into hydrogen that can then be used to produce electrical power in a manner similar to Example 1.4. Consider a portable solid oxide fuel cell, operating at a temperature of $T_{fc} = 800°C$. The fuel cell is housed within a cylindrical canister of diameter $D = 75$ mm and length $L = 120$ mm. The outer surface of the canister is insulated with a low-thermal-conductivity material. For a particular application, it is desired that the *thermal signature* of the canister be small, to avoid its detection by infrared sensors. The degree to which the canister can be detected with an infrared sensor may be estimated by equating the radiation heat flux emitted from the exterior surface of the canister (Equation 1.5; $E_s = \varepsilon_s \sigma T_s^4$) to the heat flux emitted from an *equivalent* black surface, ($E_b = \sigma T_b^4$). If the equivalent black surface temperature, T_b, is near the surroundings temperature, the thermal signature of the canister is too small to be detected—the canister is indistinguishable from the surroundings.

(a) Determine the required thickness of insulation to be applied to the cylindrical wall of the canister to ensure that the canister does not become highly visible to an infrared sensor (i.e., $T_b - T_{sur} < 5$ K). Consider cases where (i) the outer surface is covered with a very thin layer of dirt ($\varepsilon_s = 0.90$) and (ii) the outer surface is comprised of a very thin polished aluminum sheet ($\varepsilon_s = 0.08$). Calculate the required thicknesses for two types of insulating material, calcium silicate ($k = 0.09$ W/m · K) and aerogel ($k = 0.006$ W/m · K). The temperatures of the surroundings and the ambient are $T_{sur} = 300$ K and $T_\infty = 298$ K, respectively. The outer surface is characterized by a convective heat transfer coefficient of $h = 12$ W/m² · K.

(b) Calculate the outer surface temperature of the canister for the four cases (high and low thermal conductivity; high and low surface emissivity).

(c) Calculate the heat loss from the cylindrical walls of the canister for the four cases.

3.19 A firefighter's protective clothing, referred to as a *turnout coat*, is typically constructed as an ensemble of three layers separated by air gaps, as shown schematically.



Representative dimensions and thermal conductivities for the layers are as follows.

Layer	*Thickness* (mm)	k (W/m · K)
Shell (s)	0.8	0.047
Moisture barrier (mb)	0.55	0.012
Thermal liner (tl)	3.5	0.038

The air gaps between the layers are 1 mm thick, and heat is transferred by conduction and radiation exchange through the stagnant air. The linearized radiation coefficient for a gap may be approximated as, $h_{rad} = \sigma(T_1 + T_2)(T_1^2 + T_2^2) \approx 4\sigma T_{avg}^3$, where T_{avg} represents the average temperature of the surfaces comprising the gap, and the radiation flux across the gap may be expressed as $q''_{rad} = h_{rad}(T_1 - T_2)$.

(a) Represent the turnout coat by a thermal circuit, labeling all the thermal resistances. Calculate and tabulate the thermal resistances per unit area (m² · K/W) for each of the layers, as well as for the conduction and radiation processes in the gaps. Assume that a value of $T_{avg} = 470$ K may be used to approximate the radiation resistance of both gaps. Comment on the relative magnitudes of the resistances.

(b) For a *pre-flash-over* fire environment in which firefighters often work, the typical radiant heat flux on the fire-side of the turnout coat is 0.25 W/cm². What is the outer surface temperature of the turnout coat if the inner surface temperature is 66°C, a condition that would result in burn injury?

Contact Resistance

3.20 A composite wall separates combustion gases at 2600°C from a liquid coolant at 100°C, with gas- and liquid-side convection coefficients of 50 and 1000 W/m² · K. The wall is composed of a 10-mm-thick layer of beryllium oxide on the gas side and a 20-mm-thick slab of stainless steel (AISI 304) on the liquid side. The contact resistance between the oxide and the steel is 0.05 m² · K/W. What is the heat loss per unit surface area of the composite? Sketch the temperature distribution from the gas to the liquid.

3.21 Two stainless steel plates 10 mm thick are subjected to a contact pressure of 1 bar under vacuum conditions for which there is an overall temperature drop of 100°C across the plates. What is the heat flux through the plates? What is the temperature drop across the contact plane?

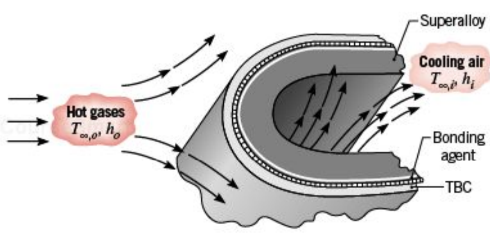
3.22 Consider a plane composite wall that is composed of two materials of thermal conductivities $k_A = 0.1$ W/m · K and

$k_B = 0.04$ W/m · K and thicknesses $L_A = 10$ mm and $L_B = 20$ mm. The contact resistance at the interface between the two materials is known to be 0.30 m^2 · K/W. Material A adjoins a fluid at 200°C for which $h = 10$ W/m^2 · K, and material B adjoins a fluid at 40°C for which $h = 20$ W/m^2 · K.

(a) What is the rate of heat transfer through a wall that is 2 m high by 2.5 m wide?

(b) Sketch the temperature distribution.

3.23 The performance of gas turbine engines may be improved by increasing the tolerance of the turbine blades to hot gases emerging from the combustor. One approach to achieving high operating temperatures involves application of a *thermal barrier coating* (TBC) to the exterior surface of a blade, while passing cooling air through the blade. Typically, the blade is made from a high-temperature superalloy, such as Inconel ($k \approx 25$ W/m · K), while a ceramic, such as zirconia ($k \approx 1.3$ W/m · K), is used as a TBC.



Consider conditions for which hot gases at $T_{\infty,o} = 1700$ K and cooling air at $T_{\infty,i} = 400$ K provide outer and inner surface convection coefficients of $h_o = 1000$ W/m^2 · K and $h_i = 500$ W/m^2 · K, respectively. If a 0.5-mm-thick zirconia TBC is attached to a 5-mm-thick Inconel blade wall by means of a metallic bonding agent, which provides an interfacial thermal resistance of $R''_{t,c} = 10^{-4}$ m^2 · K/W, can the Inconel be maintained at a temperature that is below its maximum allowable value of 1250 K? Radiation effects may be neglected, and the turbine blade may be approximated as a plane wall. Plot the temperature distribution with and without the TBC. Are there any limits to the thickness of the TBC?

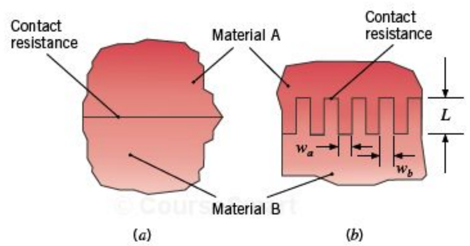
3.24 A commercial grade cubical freezer, 3 m on a side, has a composite wall consisting of an exterior sheet of 6.35-mm-thick plain carbon steel, an intermediate layer of 100-mm-thick cork insulation, and an inner sheet of 6.35-mm-thick aluminum alloy (2024). Adhesive interfaces between the insulation and the metallic strips are each characterized by a thermal contact resistance of $R''_{t,c} = 2.5 \times 10^{-4}$ m^2 · K/W. What is the steady-state cooling load that must be maintained by the refrigerator under conditions for which the outer and inner surface temperatures are 22°C and −6°C respectively?

3.25 Physicists have determined the theoretical value of the thermal conductivity of a carbon nanotube to be $k_{cn,T} = 5000$ W/m · K.

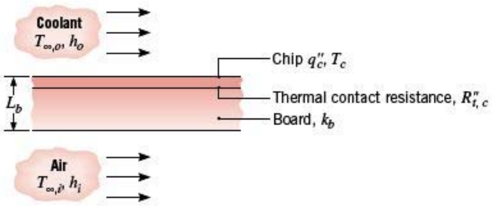
(a) Assuming the actual thermal conductivity of the carbon nanotube is the same as its theoretical value, find the thermal contact resistance, $R_{t,c}$, that exists between the carbon nanotube and the top surfaces of the heated and sensing islands in Example 3.3.

(b) Using the value of the thermal contact resistance calculated in part (a), plot the fraction of the total resistance between the heated and sensing islands that is due to the thermal contact resistances for island separation distances of 5 μm $\leq s \leq$ 20 μm.

3.26 In a particular application, it is desirable to minimize the effects of the thermal contact resistance between two plane mating surfaces as shown in part (*a*) of the schematic. An engineer suggests that the overall resistance to heat transfer can be reduced by cutting relatively deep linear grooves in each surface, resulting in the interlocking fin-like structure shown in part (*b*) of the schematic. If the grooves in material A are of the same width as the grooves in material B, evaluate the merit of the proposed scheme using an appropriate analysis.



3.27 Approximately 10^6 discrete electrical components can be placed on a single integrated circuit (chip), with electrical heat dissipation as high as 30,000 W/m^2. The chip, which is very thin, is exposed to a dielectric liquid at its outer surface, with $h_o = 1000$ W/m^2 · K and $T_{\infty,o} = 20$°C, and is joined to a circuit board at its inner surface. The thermal contact resistance between the chip and the board is 10^{-4} m^2 · K/W, and the board thickness and thermal conductivity are $L_b = 5$ mm and $k_b = 1$ W/m · K, respectively. The other surface of the board is exposed to ambient air for which $h_i = 40$ W/m^2 · K and $T_{\infty,i} = 20$°C.

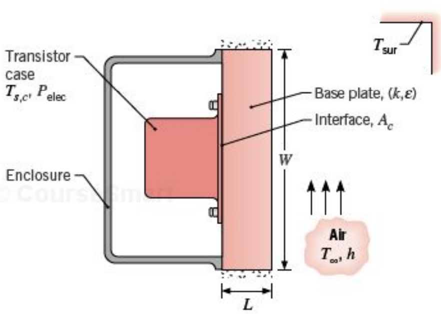


(a) Sketch the equivalent thermal circuit corresponding to steady-state conditions. In variable form, label appropriate resistances, temperatures, and heat fluxes.

(b) Under steady-state conditions for which the chip heat dissipation is $q_c'' = 30{,}000$ W/m^2, what is the chip temperature?

(c) The maximum allowable heat flux, $q''_{c,m}$, is determined by the constraint that the chip temperature must not exceed 85°C. Determine $q''_{c,m}$ for the foregoing conditions. If air is used in lieu of the dielectric liquid, the convection coefficient is reduced by approximately an order of magnitude. What is the value of $q''_{c,m}$ for $h_o = 100$ W/m$^2\cdot$K? With air cooling, can significant improvements be realized by using an aluminum oxide circuit board and/or by using a conductive paste at the chip/board interface for which $R''_{t,c} = 10^{-5}$ m$^2\cdot$K/W?

3.28 Consider a power transistor encapsulated in an aluminum case that is attached at its base to a square aluminum plate of thermal conductivity $k = 240$ W/m$\cdot$K, thickness $L = 6$ mm, and width $W = 20$ mm. The case is joined to the plate by screws that maintain a contact pressure of 1 bar, and the back surface of the plate transfers heat by natural convection and radiation to ambient air and large surroundings at $T_\infty = T_{sur} = 25$°C. The surface has an emissivity of $\varepsilon = 0.9$, and the convection coefficient is $h = 4$ W/m$^2\cdot$K. The case is completely enclosed such that heat transfer may be assumed to occur exclusively through the base plate.

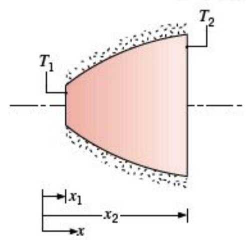


(a) If the air-filled aluminum-to-aluminum interface is characterized by an area of $A_c = 2 \times 10^{-4}$ m^2 and a roughness of 10 μm, what is the maximum allowable power dissipation if the surface temperature of the case, $T_{s,c}$, is not to exceed 85°C?

(b) The convection coefficient may be increased by subjecting the plate surface to a forced flow of air. Explore the effect of increasing the coefficient over the range $4 \leq h \leq 200$ W/m$^2\cdot$K.

Alternative Conduction Analysis

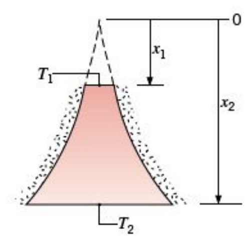
3.29 The diagram shows a conical section fabricated from pure aluminum. It is of circular cross section having diameter $D = ax^{1/2}$, where $a = 0.5$ m$^{1/2}$. The small end is located at $x_1 = 25$ mm and the large end at $x_2 = 125$ mm. The end temperatures are $T_1 = 600$ K and $T_2 = 400$ K, while the lateral surface is well insulated.



(a) Derive an expression for the temperature distribution $T(x)$ in symbolic form, assuming one-dimensional conditions. Sketch the temperature distribution.

(b) Calculate the heat rate q_x.

3.30 A truncated solid cone is of circular cross section, and its diameter is related to the axial coordinate by an expression of the form $D = ax^{3/2}$, where $a = 1.0$ m$^{-1/2}$.



The sides are well insulated, while the top surface of the cone at x_1 is maintained at T_1 and the bottom surface at x_2 is maintained at T_2.

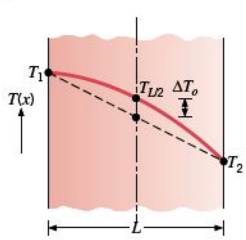
(a) Obtain an expression for the temperature distribution $T(x)$.

(b) What is the rate of heat transfer across the cone if it is constructed of pure aluminum with $x_1 = 0.075$ m, $T_1 = 100°C$, $x_2 = 0.225$ m, and $T_2 = 20°C$?

3.31 From Figure 2.5 it is evident that, over a wide temperature range, the temperature dependence of the thermal conductivity of many solids may be approximated by a linear expression of the form $k = k_o + aT$, where k_o is a positive constant and a is a coefficient that may be positive or negative. Obtain an expression for the heat flux across a plane wall whose inner and outer surfaces are maintained at T_0 and T_1, respectively. Sketch the forms of the temperature distribution corresponding to $a > 0$, $a = 0$, and $a < 0$.

3.32 Consider a tube wall of inner and outer radii r_i and r_o, whose temperatures are maintained at T_i and T_o, respectively. The thermal conductivity of the cylinder is temperature dependent and may be represented by an expression of the form $k = k_o(1 + aT)$, where k_o and a are constants. Obtain an expression for the heat transfer per unit length of the tube. What is the thermal resistance of the tube wall?

3.33 Measurements show that steady-state conduction through a plane wall without heat generation produced a convex temperature distribution such that the midpoint temperature was ΔT_o higher than expected for a linear temperature distribution.



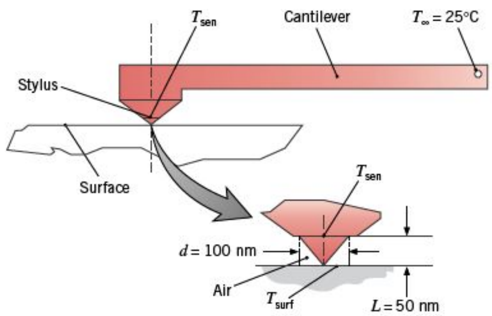
Assuming that the thermal conductivity has a linear dependence on temperature, $k = k_o(1 + \alpha T)$, where α is a constant, develop a relationship to evaluate α in terms of ΔT_o, T_1, and T_2.

3.34 A device used to measure the surface temperature of an object to within a spatial resolution of approximately 50 nm is shown in the schematic. It consists of an extremely sharp-tipped stylus and an extremely small cantilever that is scanned across the surface. The probe tip is of circular cross section and is fabricated of polycrystalline silicon dioxide. The ambient temperature is measured at the pivoted end of the cantilever as $T_\infty = 25°C$, and the device is equipped with a sensor to measure the temperature at the upper end of the sharp tip, T_{sen}. The thermal resistance between the sensing probe and the pivoted end is $R_t = 5 \times 10^6$ K/W.

(a) Determine the thermal resistance between the surface temperature and the sensing temperature.

(b) If the sensing temperature is $T_{sen} = 28.5°C$, determine the surface temperature.

Hint: Although nanoscale heat transfer effects may be important, assume that the conduction occurring in the air adjacent to the probe tip can be described by Fourier's law and the thermal conductivity found in Table A.4.



Cylindrical Wall

3.35 A steam pipe of 0.12-m outside diameter is insulated with a layer of calcium silicate.

(a) If the insulation is 20 mm thick and its inner and outer surfaces are maintained at $T_{s,1} = 800$ K and $T_{s,2} = 490$ K, respectively, what is the heat loss per unit length (q') of the pipe?

(b) We wish to explore the effect of insulation thickness on the heat loss q' and outer surface temperature $T_{s,2}$, with the inner surface temperature fixed at $T_{s,1} = 800$ K. The outer surface is exposed to an airflow ($T_\infty = 25°C$) that maintains a convection coefficient of $h = 25$ W/m$^2 \cdot$K and to large surroundings for which $T_{sur} = T_\infty = 25°C$. The surface emissivity of calcium silicate is approximately 0.8. Compute and plot the temperature distribution in the insulation as a function of the dimensionless radial coordinate, $(r - r_1)/(r_2 - r_1)$, where $r_1 = 0.06$ m and r_2 is a variable ($0.06 < r_2 \leq 0.20$ m). Compute and plot the heat loss as a function of the insulation thickness for $0 \leq (r_2 - r_1) \leq 0.14$ m.

3.36 Consider the water heater described in Problem 1.37. We now wish to determine the energy needed to compensate for heat losses incurred while the water is stored at the prescribed temperature of 55°C. The cylindrical storage tank (with flat ends) has a capacity of 100 gallons, and foamed urethane is used to insulate the side and end walls from ambient air at an annual average temperature of 20°C. The resistance to heat transfer is dominated by conduction in the insulation and by free convection in the air, for which $h \approx 2\ \text{W/m}^2 \cdot \text{K}$. If electric resistance heating is used to compensate for the losses and the cost of electric power is \$0.08/kW · h, specify tank and insulation dimensions for which the annual cost associated with the heat losses is less than \$50.

3.37 A thin electrical heater is wrapped around the outer surface of a long cylindrical tube whose inner surface is maintained at a temperature of 5°C. The tube wall has inner and outer radii of 25 and 75 mm, respectively, and a thermal conductivity of 10 W/m · K. The thermal contact resistance between the heater and the outer surface of the tube (per unit length of the tube) is $R'_{t,c} = 0.01\ \text{m} \cdot \text{K/W}$. The outer surface of the heater is exposed to a fluid with $T_\infty = -10°\text{C}$ and a convection coefficient of $h = 100\ \text{W/m}^2 \cdot \text{K}$. Determine the heater power per unit length of tube required to maintain the heater at $T_o = 25°\text{C}$.

3.38 In the previous problem, the electrical power required to maintain the heater at $T_o = 25°\text{C}$ depends on the thermal conductivity of the wall material k, the thermal contact resistance $R'_{t,c}$, and the convection coefficient h. Compute and plot the separate effect of changes in k ($1 \le k \le 200\ \text{W/m} \cdot \text{K}$), $R'_{t,c}$ ($0 \le R'_{t,c} \le 0.1\ \text{m} \cdot \text{K/W}$), and h ($10 \le h \le 1000\ \text{W/m}^2 \cdot \text{K}$) on the total heater power requirement, as well as the rate of heat transfer to the inner surface of the tube and to the fluid.

3.39 A stainless steel (AISI 304) tube used to transport a chilled pharmaceutical has an inner diameter of 36 mm and a wall thickness of 2 mm. The pharmaceutical and ambient air are at temperatures of 6°C and 23°C, respectively, while the corresponding inner and outer convection coefficients are 400 W/m² · K and 6 W/m² · K, respectively.

(a) What is the heat gain per unit tube length?

(b) What is the heat gain per unit length if a 10-mm-thick layer of calcium silicate insulation ($k_{\text{ins}} = 0.050\ \text{W/m} \cdot \text{K}$) is applied to the tube?

3.40 Superheated steam at 575°C is routed from a boiler to the turbine of an electric power plant through steel tubes ($k = 35\ \text{W/m} \cdot \text{K}$) of 300 mm inner diameter and 30 mm wall thickness. To reduce heat loss to the surroundings and to maintain a *safe-to-touch* outer surface temperature, a layer of calcium silicate insulation ($k = 0.10\ \text{W/m} \cdot \text{K}$) is applied to the tubes, while degradation of the insulation is reduced by wrapping it in a thin sheet of aluminum having an emissivity of $\varepsilon = 0.20$. The air and wall temperatures of the power plant are 27°C.

(a) Assuming that the inner surface temperature of a steel tube corresponds to that of the steam and the convection coefficient outside the aluminum sheet is 6 W/m² · K, what is the minimum insulation thickness needed to insure that the temperature of the aluminum does not exceed 50°C? What is the corresponding heat loss per meter of tube length?

(b) Explore the effect of the insulation thickness on the temperature of the aluminum and the heat loss per unit tube length.

3.41 A thin electrical heater is inserted between a long circular rod and a concentric tube with inner and outer radii of 20 and 40 mm. The rod (A) has a thermal conductivity of $k_A = 0.15\ \text{W/m} \cdot \text{K}$, while the tube (B) has a thermal conductivity of $k_B = 1.5\ \text{W/m} \cdot \text{K}$ and its outer surface is subjected to convection with a fluid of temperature $T_\infty = -15°\text{C}$ and heat transfer coefficient 50 W/m² · K. The thermal contact resistance between the cylinder surfaces and the heater is negligible.

(a) Determine the electrical power per unit length of the cylinders (W/m) that is required to maintain the outer surface of cylinder B at 5°C.

(b) What is the temperature at the center of cylinder A?

3.42 A wire of diameter $D = 2$ mm and uniform temperature T has an electrical resistance of 0.01 Ω/m and a current flow of 20 A.

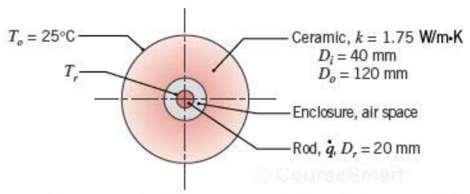
(a) What is the rate at which heat is dissipated per unit length of wire? What is the heat dissipation per unit volume within the wire?

(b) If the wire is not insulated and is in ambient air and large surrounding for which $T_\infty = T_{\text{sur}} = 20°\text{C}$, what is the temperature T of the wire? The wire has an emissivity of 0.3, and the coefficient associated with heat transfer by natural convection may be approximated by an expression of the form, $h = C[(T - T_\infty)/D]^{1/4}$, where $C = 1.25\ \text{W/m}^{7/4} \cdot \text{K}^{5/4}$.

(c) If the wire is coated with plastic insulation of 2-mm thickness and a thermal conductivity of 0.25 W/m · K, what are the inner and outer surface temperatures of the insulation? The insulation has an emissivity of 0.9, and the convection coefficient is given by the expression of part (b). Explore the effect of the insulation thickness on the surface temperatures.

3.43 A 2-mm-diameter electrical wire is insulated by a 2-mm-thick rubberized sheath ($k = 0.13\ \text{W/m} \cdot \text{K}$), and the wire/sheath interface is characterized by a thermal contact resistance of $R''_{t,c} = 3 \times 10^{-4}\,\text{m}^2 \cdot \text{K/W}$. The convection heat transfer coefficient at the outer surface of the sheath is $10\,\text{W/m}^2 \cdot \text{K}$, and the temperature of the ambient air is 20°C. If the temperature of the insulation may not exceed 50°C, what is the maximum allowable electrical power that may be dissipated per unit length of the conductor? What is the critical radius of the insulation?

3.44 Electric current flows through a long rod generating thermal energy at a uniform volumetric rate of $\dot{q} = 2 \times 10^6\,\text{W/m}^3$. The rod is concentric with a hollow ceramic cylinder, creating an enclosure that is filled with air.



The thermal resistance per unit length due to radiation between the enclosure surfaces is $R'_{\text{rad}} = 0.30\,\text{m} \cdot \text{K/W}$, and the coefficient associated with free convection in the enclosure is $h = 20\,\text{W/m}^2 \cdot \text{K}$.

(a) Construct a thermal circuit that can be used to calculate the surface temperature of the rod, T_r. Label all temperatures, heat rates, and thermal resistances, and evaluate each thermal resistance.

(b) Calculate the surface temperature of the rod for the prescribed conditions.

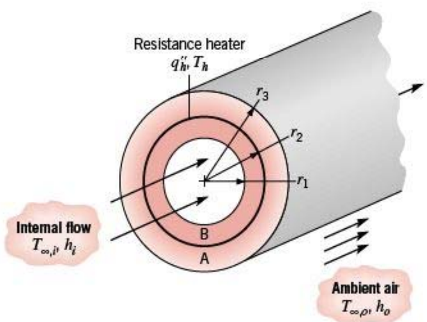
3.45 The evaporator section of a refrigeration unit consists of thin-walled, 10-mm-diameter tubes through which refrigerant passes at a temperature of −18°C. Air is cooled as it flows over the tubes, maintaining a surface convection coefficient of $100\,\text{W/m}^2 \cdot \text{K}$, and is subsequently routed to the refrigerator compartment.

(a) For the foregoing conditions and an air temperature of −3°C, what is the rate at which heat is extracted from the air per unit tube length?

(b) If the refrigerator's defrost unit malfunctions, frost will slowly accumulate on the outer tube surface. Assess the effect of frost formation on the cooling capacity of a tube for frost layer thicknesses in the range $0 \le \delta \le 4$ mm. Frost may be assumed to have a thermal conductivity of 0.4 W/m · K.

(c) The refrigerator is disconnected after the defrost unit malfunctions and a 2-mm-thick layer of frost has formed. If the tubes are in ambient air for which $T_\infty = 20°\text{C}$ and natural convection maintains a convection coefficient of $2\,\text{W/m}^2 \cdot \text{K}$, how long will it take for the frost to melt? The frost may be assumed to have a mass density of $700\,\text{kg/m}^3$ and a latent heat of fusion of 334 kJ/kg.

3.46 A composite cylindrical wall is composed of two materials of thermal conductivity k_A and k_B, which are separated by a very thin, electric resistance heater for which interfacial contact resistances are negligible.



Liquid pumped through the tube is at a temperature $T_{\infty,i}$ and provides a convection coefficient h_i at the inner surface of the composite. The outer surface is exposed to ambient air, which is at $T_{\infty,o}$ and provides a convection coefficient of h_o. Under steady-state conditions, a uniform heat flux of q''_h is dissipated by the heater.

(a) Sketch the equivalent thermal circuit of the system and express all resistances in terms of relevant variables.

(b) Obtain an expression that may be used to determine the heater temperature, T_h.

(c) Obtain an expression for the ratio of heat flows to the outer and inner fluids, q'_o/q'_i. How might the variables of the problem be adjusted to minimize this ratio?

3.47 An electrical current of 700 A flows through a stainless steel cable having a diameter of 5 mm and an electrical resistance of $6 \times 10^{-4}\,\Omega/\text{m}$ (i.e., per meter of cable length). The cable is in an environment having a temperature of 30°C, and the total coefficient associated with convection and radiation between the cable and the environment is approximately $25\,\text{W/m}^2 \cdot \text{K}$.

(a) If the cable is bare, what is its surface temperature?

(b) If a very thin coating of electrical insulation is applied to the cable, with a contact resistance of $0.02\,\text{m}^2 \cdot \text{K/W}$, what are the insulation and cable surface temperatures?

(c) There is some concern about the ability of the insulation to withstand elevated temperatures. What thickness of this insulation ($k = 0.5$ W/m · K) will yield the lowest value of the maximum insulation temperature? What is the value of the maximum temperature when the thickness is used?

3.48 A 0.20-m-diameter, thin-walled steel pipe is used to transport saturated steam at a pressure of 20 bars in a room for which the air temperature is 25°C and the convection heat transfer coefficient at the outer surface of the pipe is 20 W/m^2 · K.

(a) What is the heat loss per unit length from the bare pipe (no insulation)? Estimate the heat loss per unit length if a 50-mm-thick layer of insulation (magnesia, 85%) is added. The steel and magnesia may each be assumed to have an emissivity of 0.8, and the steam-side convection resistance may be neglected.

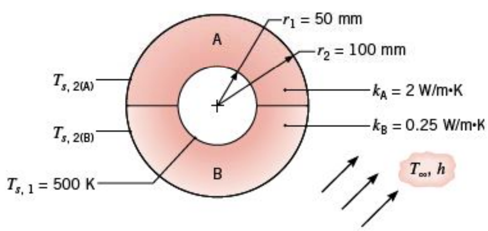
(b) The costs associated with generating the steam and installing the insulation are known to be \$4/10^9 J and \$100/m of pipe length, respectively. If the steam line is to operate 7500 h/yr, how many years are needed to pay back the initial investment in insulation?

3.49 Steam at a temperature of 250°C flows through a steel pipe (AISI 1010) of 60-mm inside diameter and 75-mm outside diameter. The convection coefficient between the steam and the inner surface of the pipe is 500 W/m^2 · K, while that between the outer surface of the pipe and the surroundings is 25 W/m^2 · K. The pipe emissivity is 0.8, and the temperature of the air and the surroundings is 20°C. What is the heat loss per unit length of pipe?

3.50 We wish to determine the effect of adding a layer of magnesia insulation to the steam pipe of the foregoing problem. The convection coefficient at the outer surface of the insulation may be assumed to remain at 25 W/m^2 · K, and the emissivity is $\varepsilon = 0.8$. Determine and plot the heat loss per unit length of pipe and the outer surface temperature as a function of insulation thickness. If the cost of generating the steam is \$4/10^9 J and the steam line operates 7000 h/yr, recommend an insulation thickness and determine the corresponding annual savings in energy costs. Plot the temperature distribution in the insulation for the recommended thickness.

3.51 An uninsulated, thin-walled pipe of 100-mm diameter is used to transport water to equipment that operates outdoors and uses the water as a coolant. During particularly harsh winter conditions, the pipe wall achieves a temperature of −15°C and a cylindrical layer of ice forms on the inner surface of the wall. If the mean water temperature is 3°C and a convection coefficient of 2000 W/m^2 · K is maintained at the inner surface of the ice, which is at 0°C, what is the thickness of the ice layer?

3.52 Steam flowing through a long, thin-walled pipe maintains the pipe wall at a uniform temperature of 500 K. The pipe is covered with an insulation blanket comprised of two different materials, A and B.



The interface between the two materials may be assumed to have an infinite contact resistance, and the entire outer surface is exposed to air for which T_∞ = 300 K and h = 25 W/m^2 · K.

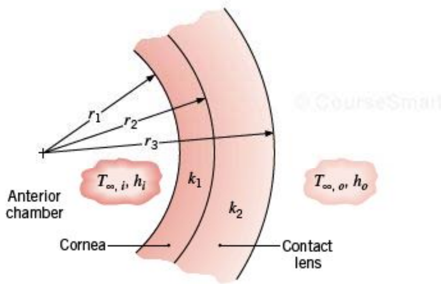
(a) Sketch the thermal circuit of the system. Label (using the above symbols) all pertinent nodes and resistances.

(b) For the prescribed conditions, what is the total heat loss from the pipe? What are the outer surface temperatures $T_{s,2(A)}$ and $T_{s,2(B)}$?

3.53 A bakelite coating is to be used with a 10-mm-diameter conducting rod, whose surface is maintained at 200°C by passage of an electrical current. The rod is in a fluid at 25°C, and the convection coefficient is 140 W/m^2 · K. What is the critical radius associated with the coating? What is the heat transfer rate per unit length for the bare rod and for the rod with a coating of bakelite that corresponds to the critical radius? How much bakelite should be added to reduce the heat transfer associated with the bare rod by 25%?

Spherical Wall

3.54 A storage tank consists of a cylindrical section that has a length and inner diameter of $L = 2$ m and $D_i = 1$ m, respectively, and two hemispherical end sections. The tank is constructed from 20-mm-thick glass (Pyrex) and is exposed to ambient air for which the temperature is 300 K and the convection coefficient is 10 W/m^2 · K. The tank is used to store heated oil, which maintains the inner surface at a temperature of 400 K. Determine the electrical power that must be supplied to a heater submerged in the oil if the prescribed conditions are to be maintained. Radiation spherical system and assume the system to be at steady state. The convection coefficient h_o is unchanged with and without the contact lens in place. The cornea and the lens cover one-third of the spherical surface area.



Values of the parameters representing this situation are as follows:

$r_1 = 10.2$ mm	$r_2 = 12.7$ mm
$r_3 = 16.5$ mm	$T_{\infty,o} = 21°C$
$T_{\infty,i} = 37°C$	$k_2 = 0.80$ W/m · K
$k_1 = 0.35$ W/m · K	$h_o = 6$ W/m^2 · K
$h_i = 12$ W/m^2 · K	

(a) Construct the thermal circuits, labeling all potentials and flows for the systems excluding the contact lens and including the contact lens. Write resistance elements in terms of appropriate parameters.

(b) Determine the heat loss from the anterior chamber with and without the contact lens in place.

(c) Discuss the implication of your results.

3.65 The outer surface of a hollow sphere of radius r_2 is subjected to a uniform heat flux q_2''. The inner surface at r_1 is held at a constant temperature $T_{s,1}$.

(a) Develop an expression for the temperature distribution $T(r)$ in the sphere wall in terms of q_2'', $T_{s,1}$, r_1, r_2, and the thermal conductivity of the wall material k.

(b) If the inner and outer tube radii are $r_1 = 50$ mm and $r_2 = 100$ mm, what heat flux q_2'' is required to maintain the outer surface at $T_{s,2} = 50°C$, while the inner surface is at $T_{s,1} = 20°C$? The thermal conductivity of the wall material is $k = 10$ W/m · K.

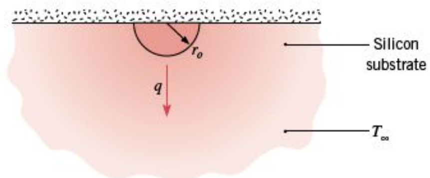
3.66 A spherical shell of inner and outer radii r_i and r_o, respectively, is filled with a heat-generating material that provides for a uniform volumetric generation rate (W/m^3) of $\dot{q}$. The outer surface of the shell is exposed to a fluid having a temperature T_∞ and a convection coefficient h. Obtain an expression for the steady-state temperature distribution $T(r)$ in the shell, expressing your result in terms of r_i, r_o, $\dot{q}$, h, T_∞, and the thermal conductivity k of the shell material.

3.67 A spherical tank of 3-m diameter contains a liquified-petroleum gas at −60°C. Insulation with a thermal conductivity of 0.06 W/m · K and thickness 250 mm is applied to the tank to reduce the heat gain.

(a) Determine the radial position in the insulation layer at which the temperature is 0°C when the ambient air temperature is 20°C and the convection coefficient on the outer surface is 6 W/m^2 · K.

(b) If the insulation is pervious to moisture from the atmospheric air, what conclusions can you reach about the formation of ice in the insulation? What effect will ice formation have on heat gain to the LP gas? How could this situation be avoided?

3.68 A transistor, which may be approximated as a hemispherical heat source of radius $r_o = 0.1$ mm, is embedded in a large silicon substrate ($k = 125$ W/m · K) and dissipates heat at a rate q. All boundaries of the silicon are maintained at an ambient temperature of $T_\infty = 27°C$, except for the top surface, which is well insulated.



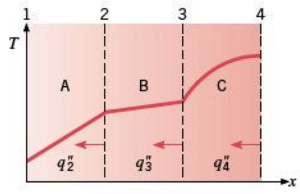
Obtain a general expression for the substrate temperature distribution and evaluate the surface temperature of the heat source for $q = 4$ W.

3.69 One modality for destroying malignant tissue involves imbedding a small spherical heat source of radius r_o within the tissue and maintaining local temperatures above a critical value T_c for an extended period. Tissue that is well removed from the source may be assumed to remain at normal body temperature ($T_b = 37°C$). Obtain a general expression for the radial temperature distribution in the tissue under steady-state conditions for which heat is dissipated at a rate q. If $r_o = 0.5$ mm, what heat rate must be supplied to maintain a tissue temperature of $T \geq T_c = 42°C$ in the domain $0.5 \leq r \leq 5$ mm? The tissue thermal conductivity is approximately 0.5 W/m · K. Assume negligible perfusion.

Conduction with Thermal Energy Generation

3.70 Consider cylindrical and spherical shells with inner and outer surfaces at r_1 and r_2 maintained at uniform temperatures $T_{s,1}$ and $T_{s,2}$, respectively. If there is uniform heat generation within the shells, obtain expressions for the steady-state, one-dimensional radial distributions of the temperature, heat flux, and heat rate. Contrast your results with those summarized in Appendix C.

3.71 The steady-state temperature distribution in a composite plane wall of three different materials, each of constant thermal conductivity, is shown as follows.



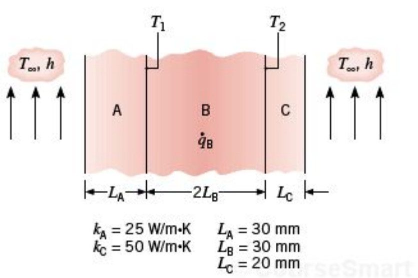
(a) Comment on the relative magnitudes of q''_2 and q''_3 and of q''_3 and q''_4.

(b) Comment on the relative magnitudes of k_A and k_B and of k_B and k_C.

(c) Sketch the heat flux as a function of x.

3.72 A plane wall of thickness 0.1 m and thermal conductivity 25 W/m · K having uniform volumetric heat generation of 0.3 MW/m^3 is insulated on one side, while the other side is exposed to a fluid at 92°C. The convection heat transfer coefficient between the wall and the fluid is 500 W/m^2 · K. Determine the maximum temperature in the wall.

3.73 Consider one-dimensional conduction in a plane composite wall. The outer surfaces are exposed to a fluid at 25°C and a convection heat transfer coefficient of 1000 W/m^2 · K. The middle wall B experiences uniform heat generation $\dot{q}_B$, while there is no generation in walls A and C. The temperatures at the interfaces are $T_1 =$ 261°C and $T_2 =$ 211°C.



(a) Assuming negligible contact resistance at the interfaces, determine the volumetric heat generation $\dot{q}_B$ and the thermal conductivity k_B.

(b) Plot the temperature distribution, showing its important features.

(c) Consider conditions corresponding to a *loss of coolant* at the exposed surface of material A ($h =$ 0). Determine T_1 and T_2 and plot the temperature distribution throughout the system.

3.74 Consider a plane composite wall that is composed of three materials (materials A, B, and C are arranged left to right) of thermal conductivities $k_A = 0.24$ W/m · K, $k_B = 0.13$ W/m · K, and $k_C = 0.50$ W/m · K. The thicknesses of the three sections of the wall are $L_A = 20$ mm, $L_B = 13$ mm, and $L_C = 20$ mm. A contact resistance of $R''_{t,c} = 10^{-2}$ m^2 · K/W exists at the interface between materials A and B, as well as at the interface between materials B and C. The left face of the composite wall is insulated, while the right face is exposed to convective conditions characterized by $h = 10$ W/m^2 · K, $T_\infty =$ 20°C. For Case 1, thermal energy is generated within material A at the rate $\dot{q}_A = 5000$ W/m^3. For Case 2, thermal energy is generated within material C at the rate $\dot{q}_C = 5000$ W/m^3.

(a) Determine the maximum temperature within the composite wall under steady-state conditions for Case 1.

(b) Sketch the steady-state temperature distribution on $T - x$ coordinates for Case 1.

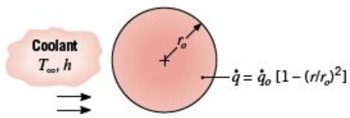
(c) Sketch the steady-state temperature distribution for Case 2 on the same $T - x$ coordinates used for Case 1.

3.75 Consider the composite wall of Example 3.7. In the Comments section, temperature distributions in the wall were determined assuming negligible contact resistance between materials A and B. Compute and plot the temperature distributions if the thermal contact resistance is $R''_{t,c} = 10^{-4}$ m^2 · K/W.

3.76 A plane wall of thickness $2L$ and thermal conductivity k experiences a uniform volumetric generation rate $\dot{q}$. As shown in the sketch for Case 1, the surface at $x = -L$ is perfectly insulated, while the other surface is maintained at a uniform, constant temperature T_o. For Case 2, a very thin dielectric strip is inserted at the midpoint of the wall ($x = 0$) in order to electrically isolate the two sections, A and B. The thermal resistance of the strip is $R''_t = 0.0005$ m^2 · K/W. The parameters associated with the wall are $k = 50$ W/m · K, $L = 20$ mm, $\dot{q} = 5 \times 10^6$ W/m^3, and $T_o = 50$°C.

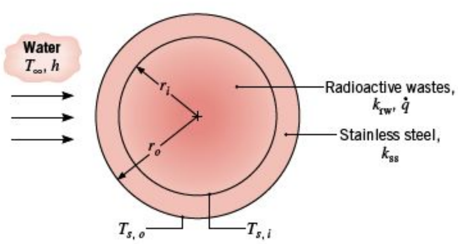
3.93 A radioactive material of thermal conductivity k is cast as a solid sphere of radius r_o and placed in a liquid bath for which the temperature T_∞ and convection coefficient h are known. Heat is uniformly generated within the solid at a volumetric rate of $\dot{q}$. Obtain the steady-state radial temperature distribution in the solid, expressing your result in terms of r_o, $\dot{q}$, k, h, and T_∞.

3.94 Radioactive wastes are packed in a thin-walled spherical container. The wastes generate thermal energy nonuniformly according to the relation $\dot{q} = \dot{q}_o[1 - (r/r_o)^2]$, where $\dot{q}$ is the local rate of energy generation per unit volume, $\dot{q}_o$ is a constant, and r_o is the radius of the container. Steady-state conditions are maintained by submerging the container in a liquid that is at T_∞ and provides a uniform convection coefficient h.



Determine the temperature distribution, $T(r)$, in the container. Express your result in terms of $\dot{q}_o$, r_o, T_∞, h, and the thermal conductivity k of the radioactive wastes.

3.95 Radioactive wastes ($k_{rw} = 20$ W/m · K) are stored in a spherical, stainless steel ($k_{ss} = 15$ W/m · K) container of inner and outer radii equal to $r_i = 0.5$ m and $r_o = 0.6$ m. Heat is generated volumetrically within the wastes at a uniform rate of $\dot{q} = 10^5$ W/m^3, and the outer surface of the container is exposed to a water flow for which $h = 1000$ W/m^2 · K and $T_\infty = 25°$C.



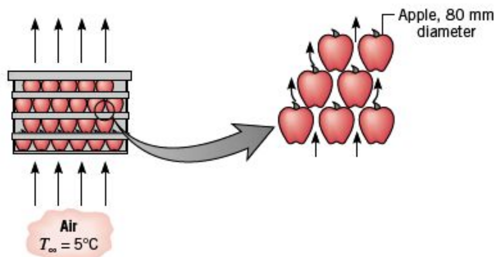
(a) Evaluate the steady-state outer surface temperature, $T_{s,o}$.

(b) Evaluate the steady-state inner surface temperature, $T_{s,i}$.

(c) Obtain an expression for the temperature distribution, $T(r)$, in the radioactive wastes. Express your result in terms of r_i, $T_{s,i}$, k_{rw}, and $\dot{q}$. Evaluate the temperature at $r = 0$.

(d) A proposed extension of the foregoing design involves storing waste materials having the same thermal conductivity but twice the heat generation ($\dot{q} = 2 \times 10^5$ W/m^3) in a stainless steel container of equivalent inner radius ($r_i = 0.5$ m). Safety considerations dictate that the maximum system temperature not exceed 475°C and that the container wall thickness be no less than $t = 0.04$ m and preferably at or close to the original design ($t = 0.1$ m). Assess the effect of varying the outside convection coefficient to a maximum achievable value of $h = 5000$ W/m^2 · K (by increasing the water velocity) and the container wall thickness. Is the proposed extension feasible? If so, recommend suitable operating and design conditions for h and t, respectively.

3.96 Unique characteristics of biologically active materials such as fruits, vegetables, and other products require special care in handling. Following harvest and separation from producing plants, glucose is catabolized to produce carbon dioxide, water vapor, and heat, with attendant internal energy generation. Consider a carton of apples, each of 80-mm diameter, which is ventilated with air at 5°C and a velocity of 0.5 m/s. The corresponding value of the heat transfer coefficient is 7.5 W/m^2 · K. Within each apple thermal energy is uniformly generated at a total rate of 4000 J/kg · day. The density and thermal conductivity of the apple are 840 kg/m^3 and 0.5 W/m · K, respectively.



(a) Determine the apple center and surface temperatures.

(b) For the stacked arrangement of apples within the crate, the convection coefficient depends on the velocity as $h = C_1 V^{0.425}$, where $C_1 = 10.1$ W/m^2 · K · (m/s)$^{0.425}$. Compute and plot the center and surface temperatures as a function of the air velocity for $0.1 \le V \le 1$ m/s.

3.97 Consider the plane wall, long cylinder, and sphere shown schematically, each with the same characteristic length a, thermal conductivity k, and uniform volumetric energy generation rate $\dot{q}$.