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Problems

Conduction

1.1 The thermal conductivity of a sheet of rigid, extruded insulation is reported to be $k = 0.029$ W/m · K. The measured temperature difference across a 20-mm-thick sheet of the material is $T_1 - T_2 = 10°C$.

(a) What is the heat flux through a 2 m × 2 m sheet of the insulation?

(b) What is the rate of heat transfer through the sheet of insulation?

1.2 A concrete wall, which has a surface area of 20 m^2 and is 0.30 m thick, separates conditioned room air from ambient air. The temperature of the inner surface of the wall is maintained at 25°C, and the thermal conductivity of the concrete is 1 W/m · K.

(a) Determine the heat loss through the wall for outer surface temperatures ranging from −15°C to 38°C, which correspond to winter and summer extremes, respectively. Display your results graphically.

(b) On your graph, also plot the heat loss as a function of the outer surface temperature for wall materials having thermal conductivities of 0.75 and 1.25 W/m · K. Explain the family of curves you have obtained.

1.3 The concrete slab of a basement is 11 m long, 8 m wide, and 0.20 m thick. During the winter, temperatures are nominally 17°C and 10°C at the top and bottom surfaces, respectively. If the concrete has a thermal conductivity of 1.4 W/m · K, what is the rate of heat loss through the slab? If the basement is heated by a gas furnace operating at an efficiency of $\eta_f = 0.90$ and natural gas is priced at $C_g = \$0.01/\text{MJ}$, what is the daily cost of the heat loss?

1.4 The heat flux through a wood slab 50 mm thick, whose inner and outer surface temperatures are 40 and 20°C, respectively, has been determined to be 40 W/m^2. What is the thermal conductivity of the wood?

1.5 The inner and outer surface temperatures of a glass window 5 mm thick are 15 and 5°C. What is the heat loss through a window that is 1 m by 3 m on a side? The thermal conductivity of glass is 1.4 W/m · K.

1.6 A glass window of width $W = 1$ m and height $H = 2$ m is 5 mm thick and has a thermal conductivity of $k_g = 1.4$ W/m · K. If the inner and outer surface temperatures of the glass are 15°C and −20°C, respectively, on a cold winter day, what is the rate of heat loss through the glass? To reduce heat loss through windows, it is customary to use a double pane construction in which adjoining panes are separated by an air space. If the spacing is 10 mm and the glass surfaces in contact with the air have temperatures of 10°C and −15°C, what is the rate of heat loss from a 1 m × 2 m window? The thermal conductivity of air is $k_a = 0.024$ W/m · K.

1.7 A freezer compartment consists of a cubical cavity that is 2 m on a side. Assume the bottom to be perfectly insulated. What is the minimum thickness of styrofoam insulation ($k = 0.030$ W/m · K) that must be applied to the top and side walls to ensure a heat load of less than 500 W, when the inner and outer surfaces are −10 and 35°C?

1.8 An inexpensive food and beverage container is fabricated from 25-mm-thick polystyrene ($k = 0.023$ W/m · K) and has interior dimensions of 0.8 m × 0.6 m × 0.6 m. Under conditions for which an inner surface temperature of approximately 2°C is maintained by an ice-water mixture and an outer surface temperature of 20°C is maintained by the ambient, what is the heat flux through the container wall? Assuming negligible heat gain through the 0.8 m × 0.6 m base of the cooler, what is the total heat load for the prescribed conditions?

1.9 What is the thickness required of a masonry wall having thermal conductivity 0.75 W/m · K if the heat rate is to be 80% of the heat rate through a composite structural wall having a thermal conductivity of 0.25 W/m · K and a thickness of 100 mm? Both walls are subjected to the same surface temperature difference.

1.10 The 5-mm-thick bottom of a 200-mm-diameter pan may be made from aluminum ($k = 240$ W/m · K) or

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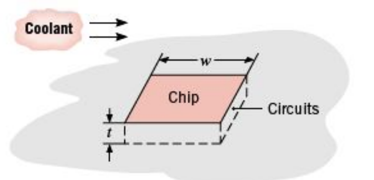
copper ($k = 390$ W/m · K). When used to boil water, the surface of the bottom exposed to the water is nominally at 110°C. If heat is transferred from the stove to the pan at a rate of 600 W, what is the temperature of the surface in contact with the stove for each of the two materials?

1.11 A square silicon chip ($k = 150$ W/m · K) is of width $w = 5$ mm on a side and of thickness $t = 1$ mm. The chip is mounted in a substrate such that its side and back surfaces are insulated, while the front surface is exposed to a coolant.

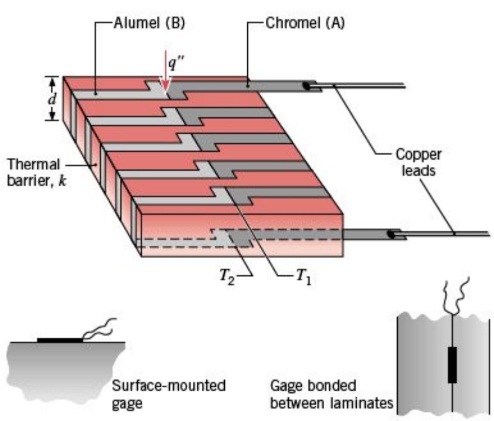


If 4 W are being dissipated in circuits mounted to the back surface of the chip, what is the steady-state temperature difference between back and front surfaces?

1.12 A gage for measuring heat flux to a surface or through a laminated material employs five thin-film, chromel/alumel (type K) thermocouples deposited on the upper and lower surfaces of a wafer with a thermal conductivity of 1.4 W/m · K and a thickness of 0.25 mm.

(a) Determine the heat flux q'' through the gage when the voltage output at the copper leads is 350 μV. The Seebeck coefficient of the type-K thermocouple materials is approximately 40 μV/°C.

(b) What precaution should you take in using a gage of this nature to measure heat flow through the laminated structure shown?



Convection

1.13 You've experienced convection cooling if you've ever extended your hand out the window of a moving vehicle or into a flowing water stream. With the surface of your hand at a temperature of 30°C, determine the convection heat flux for (a) a vehicle speed of 35 km/h in air at −5°C with a convection coefficient of 40 W/m^2 · K and (b) a velocity of 0.2 m/s in a water stream at 10°C with a convection coefficient of 900 W/m^2 · K. Which condition would *feel* colder? Contrast these results with a heat loss of approximately 30 W/m^2 under normal room conditions.

1.14 Air at 40°C flows over a long, 25-mm-diameter cylinder with an embedded electrical heater. In a series of tests, measurements were made of the power per unit length, P', required to maintain the cylinder surface temperature at 300°C for different freestream velocities V of the air. The results are as follows:

Air velocity, V (m/s)	1	2	4	8	12
Power, P' (W/m)	450	658	983	1507	1963

(a) Determine the convection coefficient for each velocity, and display your results graphically.

(b) Assuming the dependence of the convection coefficient on the velocity to be of the form $h = CV^n$, determine the parameters C and n from the results of part (a).

1.15 An electric resistance heater is embedded in a long cylinder of diameter 30 mm. When water with a temperature of 25°C and velocity of 1 m/s flows crosswise over the cylinder, the power per unit length required to maintain the surface at a uniform temperature of 90°C is 28 kW/m. When air, also at 25°C, but with a velocity of 10 m/s is flowing, the power per unit length required to maintain the same surface temperature is 400 W/m. Calculate and compare the convection coefficients for the flows of water and air.

1.16 A cartridge electrical heater is shaped as a cylinder of length $L = 200$ mm and outer diameter $D = 20$ mm. Under normal operating conditions the heater dissipates 2 kW while submerged in a water flow that is at 20°C and provides a convection heat transfer coefficient of $h = 5000$ W/m^2 · K. Neglecting heat transfer from the ends of the heater, determine its surface temperature T_s. If the water flow is inadvertently terminated while the heater continues to operate, the heater surface is exposed to air that is also at 20°C but for which $h = 50$ W/m^2 · K. What is the corresponding surface temperature? What are the consequences of such an event?

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If the transmission efficiency is $\eta = 0.93$ and air flow over the case corresponds to $T_\infty = 30°C$ and $h = 200\ W/m^2 \cdot K$, what is the surface temperature of the transmission?

Radiation

1.24 Under conditions for which the same room temperature is maintained by a heating or cooling system, it is not uncommon for a person to feel chilled in the winter but comfortable in the summer. Provide a plausible explanation for this situation (with supporting calculations) by considering a room whose air temperature is maintained at 20°C throughout the year, while the walls of the room are nominally at 27°C and 14°C in the summer and winter, respectively. The exposed surface of a person in the room may be assumed to be at a temperature of 32°C throughout the year and to have an emissivity of 0.90. The coefficient associated with heat transfer by natural convection between the person and the room air is approximately $2\ W/m^2 \cdot K$.

1.25 A spherical interplanetary probe of 0.5-m diameter contains electronics that dissipate 150 W. If the probe surface has an emissivity of 0.8 and the probe does not receive radiation from other surfaces, as, for example, from the sun, what is its surface temperature?

1.26 An instrumentation package has a spherical outer surface of diameter $D = 100$ mm and emissivity $\varepsilon = 0.25$. The package is placed in a large space simulation chamber whose walls are maintained at 77 K. If operation of the electronic components is restricted to the temperature range $40 \le T \le 85°C$, what is the range of acceptable power dissipation for the package? Display your results graphically, showing also the effect of variations in the emissivity by considering values of 0.20 and 0.30.

1.27 Consider the conditions of Problem 1.22. However, now the plate is in a vacuum with a surrounding temperature of 25°C. What is the emissivity of the plate? What is the rate at which radiation is emitted by the surface?

1.28 An overhead 25-m-long, uninsulated industrial steam pipe of 100 mm diameter is routed through a building whose walls and air are at 25°C. Pressurized steam maintains a pipe surface temperature of 150°C, and the coefficient associated with natural convection is $h = 10\ W/m^2 \cdot K$. The surface emissivity is $\varepsilon = 0.8$.

(a) What is the rate of heat loss from the steam line?
(b) If the steam is generated in a gas-fired boiler operating at an efficiency of $\eta_f = 0.90$ and natural gas is priced at C_g = \$0.01 per MJ, what is the annual cost of heat loss from the line?

1.29 If $T_s \approx T_{sur}$ in Equation 1.9, the radiation heat transfer coefficient may be approximated as

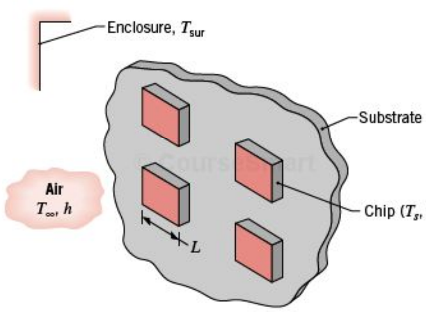
$$h_{r,a} = 4\varepsilon\sigma\overline{T}^3$$

where $\overline{T} \equiv (T_s + T_{sur})/2$. We wish to assess the validity of this approximation by comparing values of h_r and $h_{r,a}$ for the following conditions. In each case represent your results graphically and comment on the validity of the approximation.

(a) Consider a surface of either polished aluminum ($\varepsilon = 0.05$) or black paint ($\varepsilon = 0.9$), whose temperature may exceed that of the surroundings ($T_{sur} = 25°C$) by 10 to 100°C. Also compare your results with values of the coefficient associated with free convection in air ($T_\infty = T_{sur}$), where $h\ (W/m^2 \cdot K) = 0.98\ \Delta T^{1/3}$.
(b) Consider initial conditions associated with placing a workpiece at $T_s = 25°C$ in a large furnace whose wall temperature may be varied over the range $100 \le T_{sur} \le 1000°C$. According to the surface finish or coating, its emissivity may assume values of 0.05, 0.2, and 0.9. For each emissivity, plot the relative error, $(h_r - h_{r,a})/h_r$, as a function of the furnace temperature.

1.30 Consider the conditions of Problem 1.18. With heat transfer by convection to air, the maximum allowable chip power is found to be 0.35 W. If consideration is also given to net heat transfer by radiation from the chip surface to large surroundings at 15°C, what is the percentage increase in the maximum allowable chip power afforded by this consideration? The chip surface has an emissivity of 0.9.

1.31 Chips of width $L = 15$ mm on a side are mounted to a substrate that is installed in an enclosure whose walls and air are maintained at a temperature of $T_{sur} = T_\infty = 25°C$. The chips have an emissivity of $\varepsilon = 0.60$ and a maximum allowable temperature of $T_s = 85°C$.



(a) If heat is rejected from the chips by radiation and natural convection, what is the maximum operating

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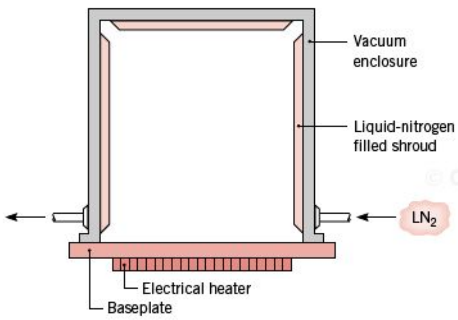
power of each chip? The convection coefficient depends on the chip-to-air temperature difference and may be approximated as $h = C(T_s - T_\infty)^{1/4}$, where $C = 4.2\ \text{W/m}^2 \cdot \text{K}^{5/4}$.

(b) If a fan is used to maintain air flow through the enclosure and heat transfer is by forced convection, with $h = 250\ \text{W/m}^2 \cdot \text{K}$, what is the maximum operating power?

1.32 A vacuum system, as used in sputtering electrically conducting thin films on microcircuits, is comprised of a baseplate maintained by an electrical heater at 300 K and a shroud within the enclosure maintained at 77 K by a liquid-nitrogen coolant loop. The circular baseplate, insulated on the lower side, is 0.3 m in diameter and has an emissivity of 0.25.



(a) How much electrical power must be provided to the baseplate heater?

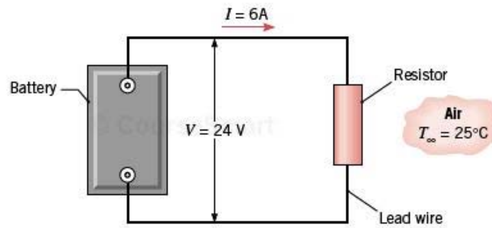
(b) At what rate must liquid nitrogen be supplied to the shroud if its heat of vaporization is 125 kJ/kg?

(c) To reduce the liquid-nitrogen consumption, it is proposed to bond a thin sheet of aluminum foil ($\varepsilon = 0.09$) to the baseplate. Will this have the desired effect?

1.33 Consider the transmission case of Problem 1.23, but now allow for radiation exchange with the ground/chassis, which may be approximated as large surroundings at $T_{sur} = 30°\text{C}$. If the emissivity of the case is $\varepsilon = 0.80$, what is the surface temperature?

Energy Balance and Multimode Effects

1.34 An electrical resistor is connected to a battery, as shown schematically. After a brief transient, the resistor assumes a nearly uniform, steady-state temperature of 95°C, while the battery and lead wires remain at the ambient temperature of 25°C. Neglect the electrical resistance of the lead wires.



(a) Consider the resistor as a system about which a control surface is placed and Equation 1.11c is applied. Determine the corresponding values of $\dot{E}_{in}$(W), $\dot{E}_g$(W), $\dot{E}_{out}$(W), and $\dot{E}_{st}$(W). If a control surface is placed about the entire system, what are the values of $\dot{E}_{in}$, $\dot{E}_g$, $\dot{E}_{out}$, and $\dot{E}_{st}$?

(b) If electrical energy is dissipated uniformly within the resistor, which is a cylinder of diameter $D = 60$ mm and length $L = 250$ mm, what is the volumetric heat generation rate, $\dot{q}$ (W/m³)?

(c) Neglecting radiation from the resistor, what is the convection coefficient?

1.35 An aluminum plate 4 mm thick is mounted in a horizontal position, and its bottom surface is well insulated. A special, thin coating is applied to the top surface such that it absorbs 80% of any incident solar radiation, while having an emissivity of 0.25. The density ρ and specific heat c of aluminum are known to be 2700 kg/m³ and 900 J/kg · K, respectively.

(a) Consider conditions for which the plate is at a temperature of 25°C and its top surface is suddenly exposed to ambient air at $T_\infty = 20°\text{C}$ and to solar radiation that provides an incident flux of 900 W/m². The convection heat transfer coefficient between the surface and the air is $h = 20\ \text{W/m}^2 \cdot \text{K}$. What is the initial rate of change of the plate temperature?

(b) What will be the equilibrium temperature of the plate when steady-state conditions are reached?

(c) The surface radiative properties depend on the specific nature of the applied coating. Compute and plot the steady-state temperature as a function of the emissivity for $0.05 \leq \varepsilon \leq 1$, with all other conditions remaining as prescribed. Repeat your calculations for values of $\alpha_S = 0.5$ and 1.0, and plot the results with those obtained for $\alpha_S = 0.8$. If the intent is to maximize the plate temperature, what is the most desirable combination of the plate emissivity and its absorptivity to solar radiation?

1.36 A blood warmer is to be used during the transfusion of blood to a patient. This device is to heat blood taken from the blood bank at 10°C to 37°C at a flow rate of 200 ml/min. The blood passes through tubing of length

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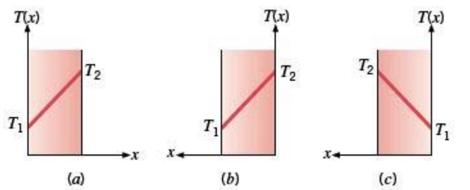
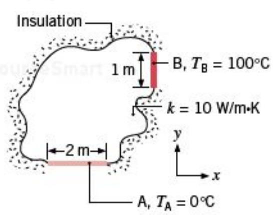


2.10 A cylinder of radius r_o, length L, and thermal conductivity k is immersed in a fluid of convection coefficient h and unknown temperature T_∞. At a certain instant the temperature distribution in the cylinder is $T(r) = a + br^2$, where a and b are constants. Obtain expressions for the heat transfer rate at r_o and the fluid temperature.

2.11 In the two-dimensional body illustrated, the gradient at surface A is found to be $\partial T/\partial y = 30$ K/m. What are $\partial T/\partial y$ and $\partial T/\partial x$ at surface B?

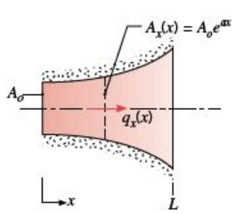


2.12 Sections of the trans-Alaska pipeline run above the ground and are supported by vertical steel shafts ($k = 25$ W/m · K) that are 1 m long and have a cross-sectional area of 0.005 m^2. Under normal operating conditions, the temperature variation along the length of a shaft is known to be governed by an expression of the form

$$T = 100 - 150x + 10x^2$$

where T and x have units of °C and meters, respectively. Temperature variations are small over the shaft cross section. Evaluate the temperature and conduction heat rate at the shaft–pipeline joint ($x = 0$) and at the shaft–ground interface ($x = 1$ m). Explain the difference in the heat rates.

2.13 Steady-state, one-dimensional conduction occurs in a rod of constant thermal conductivity k and variable cross-sectional area $A_x(x) = A_o e^{ax}$, where A_o and a are constants. The lateral surface of the rod is well insulated.



(a) Write an expression for the conduction heat rate, $q_x(x)$. Use this expression to determine the temperature distribution $T(x)$ and qualitatively sketch the distribution for $T(0) > T(L)$.

(b) Now consider conditions for which thermal energy is generated in the rod at a volumetric rate $\dot{q} = \dot{q}_o \exp(-ax)$, where $\dot{q}_o$ is a constant. Obtain an expression for $q_x(x)$ when the left face ($x = 0$) is well insulated.

Thermophysical Properties

2.14 Consider a 300 mm × 300 mm window in an aircraft. For a temperature difference of 80°C from the inner to the outer surface of the window, calculate the heat loss through $L = 10$-mm-thick polycarbonate, soda lime glass, and aerogel windows, respectively. The thermal conductivities of the aerogel and polycarbonate are $k_{ag} = 0.014$ W/m · K and $k_{pc} = 0.21$ W/m · K, respectively. Evaluate the thermal conductivity of the soda lime glass at 300 K. If the aircraft has 130 windows and the cost to heat the cabin air is \$1/kW · h, compare the costs associated with the heat loss through the windows for an 8-hour intercontinental flight.

2.15 Gold is commonly used in semiconductor packaging to form interconnections (also known as interconnects) that carry electrical signals between different devices in the package. In addition to being a good electrical conductor, gold interconnects are also effective at protecting the heat-generating devices to which they are attached by conducting thermal energy away from the devices to surrounding, cooler regions. Consider a thin film of gold that has a cross section of 60 nm × 250 nm.

(a) For an applied temperature difference of 20°C, determine the energy conducted along a 1-μm-long, thin-film interconnect. Evaluate properties at 300 K.

(b) Plot the lengthwise (in the 1-μm direction) and spanwise (in the thinnest direction) thermal conductivities of the gold film as a function of the film thickness, L, for $30 \leq L \leq 140$ nm.

2.16 A TV advertisement by a well-known insulation manufacturer states: it isn't the thickness of the insulating material that counts, it's the R-value. The ad shows that to obtain an R-value of 19, you need 18 ft of rock, 15 in. of wood, or just 6 in. of the manufacturer's insulation. Is this advertisement technically reasonable? If you are like most TV viewers, you don't know the R-value is defined as L/k, where L (in.) is the thickness of the insulation and k (Btu · in./hr · ft^2 · °F) is the thermal conductivity of the material.

2.17 An apparatus for measuring thermal conductivity employs an electrical heater sandwiched between two

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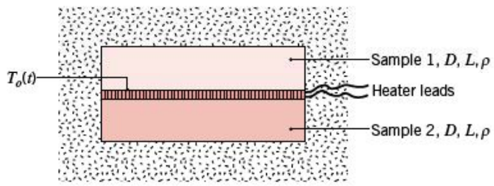
For a particular test run, the electrical heater dissipates 15.0 W for a period of $\Delta t_o = 120$ s and the temperature at the interface is $T_o(30 \text{ s}) = 24.57°\text{C}$ after 30 s of heating. A long time after the heater is deenergized, $t \gg \Delta t_o$, the samples reach the uniform temperature of $T_o(\infty) = 33.50°\text{C}$. The density of the sample materials, determined by measurement of volume and mass, is $\rho = 3965 \text{ kg/m}^3$.



Determine the specific heat and thermal conductivity of the test material. By looking at values of the thermophysical properties in Table A.1 or A.2, identify the test sample material.

The Heat Equation

2.20 At a given instant of time the temperature distribution within an infinite homogeneous body is given by the function

$$T(x, y, z) = x^2 - 2y^2 + z^2 - xy + 2yz$$

Assuming constant properties and no internal heat generation, determine the regions where the temperature changes with time.

2.21 A pan is used to boil water by placing it on a stove, from which heat is transferred at a fixed rate q_o. There are two stages to the process. In Stage 1, the water is taken from its initial (room) temperature T_i to the boiling point, as heat is transferred from the pan by natural convection. During this stage, a constant value of the convection coefficient h may be assumed, while the bulk temperature of the water increases with time, $T_\infty = T_\infty(t)$. In Stage 2, the water has come to a boil, and its temperature remains at a fixed value, $T_\infty = T_b$, as heating continues. Consider a pan bottom of thickness L and diameter D, with a coordinate system corresponding to $x = 0$ and $x = L$ for the surfaces in contact with the stove and water, respectively.

(a) Write the form of the heat equation and the boundary/initial conditions that determine the variation of temperature with position and time, $T(x,t)$, in the pan bottom during Stage 1. Express your result in terms of the parameters q_o, D, L, h, and T_∞, as well as appropriate properties of the pan material.

(b) During Stage 2, the surface of the pan in contact with the water is at a fixed temperature, $T(L, t) = T_L > T_b$. Write the form of the heat equation and boundary conditions that determine the temperature distribution, $T(x)$, in the pan bottom. Express your result in terms of the parameters q_o, D, L, and T_L, as well as appropriate properties of the pan material.

2.22 Uniform internal heat generation at $\dot{q} = 5 \times 10^7 \text{ W/m}^3$ is occurring in a cylindrical nuclear reactor fuel rod of 50-mm diameter, and under steady-state conditions the temperature distribution is of the form $T(r) = a + br^2$, where T is in degrees Celsius and r is in meters, while $a = 800°\text{C}$ and $b = -4.167 \times 10^5 °\text{C/m}^2$. The fuel rod properties are $k = 30 \text{ W/m} \cdot \text{K}$, $\rho = 1100 \text{ kg/m}^3$, and $c_p = 800 \text{ J/kg} \cdot \text{K}$.

(a) What is the rate of heat transfer per unit length of the rod at $r = 0$ (the centerline) and at $r = 25$ mm (the surface)?

(b) If the reactor power level is suddenly increased to $\dot{q}_2 = 10^8 \text{ W/m}^3$, what is the initial time rate of temperature change at $r = 0$ and $r = 25$ mm?

2.23 The steady-state temperature distribution in a one-dimensional wall of thermal conductivity 50 W/m · K and thickness 50 mm is observed to be $T(°\text{C}) = a + bx^2$, where $a = 200°\text{C}$, $b = -2000°\text{C/m}^2$, and x is in meters.

(a) What is the heat generation rate $\dot{q}$ in the wall?

(b) Determine the heat fluxes at the two wall faces. In what manner are these heat fluxes related to the heat generation rate?

2.24 The temperature distribution across a wall 0.3 m thick at a certain instant of time is $T(x) = a + bx + cx^2$, where T is in degrees Celsius and x is in meters, $a = 200°\text{C}$, $b = -200°\text{C/m}$, and $c = 30°\text{C/m}^2$. The wall has a thermal conductivity of 1 W/m · K.

(a) On a unit surface area basis, determine the rate of heat transfer into and out of the wall and the rate of change of energy stored by the wall.

(b) If the cold surface is exposed to a fluid at 100°C, what is the convection coefficient?

2.25 A plane wall of thickness $2L = 40$ mm and thermal conductivity $k = 5 \text{ W/m} \cdot \text{K}$ experiences uniform volumetric heat generation at a rate $\dot{q}$, while convection heat transfer occurs at both of its surfaces ($x = -L, +L$), each of which is exposed to a fluid of temperature $T_\infty = 20°\text{C}$. Under steady-state conditions, the temperature distribution in the wall is of the form $T(x) = a + bx + cx^2$, where $a = 82.0°\text{C}$, $b = -210°\text{C/m}$, $c = -2 \times 10^4 °\text{C/m}^2$, and x is in meters. The origin of the x-coordinate is at the midplane of the wall.

(a) Sketch the temperature distribution and identify significant physical features.

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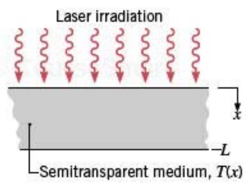
where A, a, B, and C are known constants. For this situation, radiation absorption in the material is manifested by a distributed heat generation term, $\dot{q}(x)$.



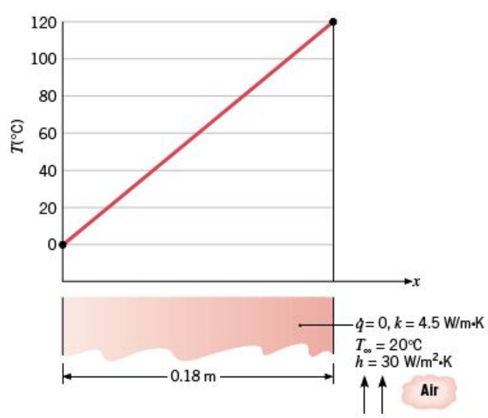
(a) Obtain expressions for the conduction heat fluxes at the front and rear surfaces.

(b) Derive an expression for $\dot{q}(x)$.

(c) Derive an expression for the rate at which radiation is absorbed in the entire material, per unit surface area. Express your result in terms of the known constants for the temperature distribution, the thermal conductivity of the material, and its thickness.

2.29 The steady-state temperature distribution in a one-dimensional wall of thermal conductivity k and thickness L is of the form $T = ax^3 + bx^2 + cx + d$. Derive expressions for the heat generation rate per unit volume in the wall and the heat fluxes at the two wall faces ($x = 0, L$).

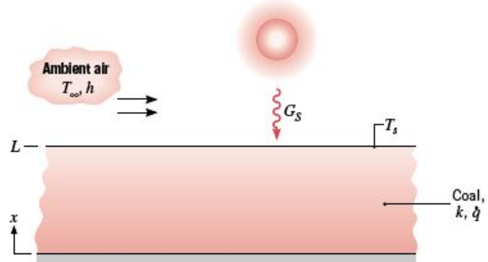
2.30 One-dimensional, steady-state conduction with no internal energy generation is occurring in a plane wall of constant thermal conductivity.



(a) Is the prescribed temperature distribution possible? Briefly explain your reasoning.

(b) With the temperature at $x = 0$ and the fluid temperature fixed at $T(0) = 0°C$ and $T_\infty = 20°C$, respectively, compute and plot the temperature at $x = L$, $T(L)$, as a function of h for $10 \leq h \leq 100$ $W/m^2 \cdot K$. Briefly explain your results.

2.31 A plane layer of coal of thickness $L = 1$ m experiences uniform volumetric generation at a rate of $\dot{q} = 20$ W/m^3 due to slow oxidation of the coal particles. Averaged over a daily period, the top surface of the layer transfers heat by convection to ambient air for which $h = 5$ $W/m^2 \cdot K$ and $T_\infty = 25°C$, while receiving solar irradiation in the amount $G_S = 400$ W/m^2. Irradiation from the atmosphere may be neglected. The solar absorptivity and emissivity of the surface are each $\alpha_S = \varepsilon = 0.95$.



(a) Write the steady-state form of the heat diffusion equation for the layer of coal. Verify that this equation is satisfied by a temperature distribution of the form

$$T(x) = T_s + \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right)$$

From this distribution, what can you say about conditions at the bottom surface ($x = 0$)? Sketch the temperature distribution and label key features.

(b) Obtain an expression for the rate of heat transfer by conduction per unit area at $x = L$. Applying an energy balance to a control surface about the top surface of the layer, obtain an expression for T_s. Evaluate T_s and $T(0)$ for the prescribed conditions.

(c) Daily average values of G_S and h depend on a number of factors such as time of year, cloud cover, and wind conditions. For $h = 5$ $W/m^2 \cdot K$, compute and plot T_s and $T(0)$ as a function of G_S for $50 \leq G_S \leq 500$ W/m^2. For $G_S = 400$ W/m^2, compute and plot T_s and $T(0)$ as a function of h for $5 \leq h \leq 50$ $W/m^2 \cdot K$.

2.32 The cylindrical system illustrated has negligible variation of temperature in the r and z directions. Assume that $\Delta r = r_o - r_i$ is small compared to r_i and denote the length in the z direction, normal to the page, as L.

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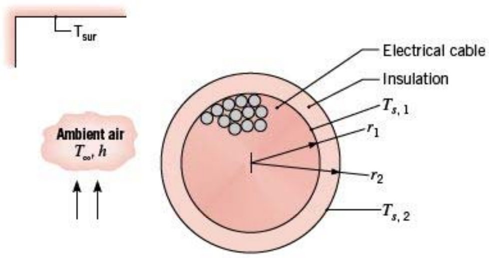
through the cable, thermal energy is generated within the cable at a volumetric rate $\dot{q}$.



(a) Write the steady-state forms of the heat diffusion equation for the insulation and the cable. Verify that these equations are satisfied by the following temperature distributions:

$$\textit{Insulation}: \ T(r) = T_{s,2} + (T_{s,1} - T_{s,2})\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$$

$$\textit{Cable}: \quad T(r) = T_{s,1} + \frac{\dot{q}r_1^2}{4k_c}\left(1 - \frac{r^2}{r_1^2}\right)$$

Sketch the temperature distribution, $T(r)$, in the cable and the sleeve, labeling key features.

(b) Applying Fourier's law, show that the rate of conduction heat transfer per unit length through the sleeve may be expressed as

$$q_r' = \frac{2\pi k_s(T_{s,1} - T_{s,2})}{\ln(r_2/r_1)}$$

Applying an energy balance to a control surface placed around the cable, obtain an alternative expression for q_r', expressing your result in terms of $\dot{q}$ and r_1.

(c) Applying an energy balance to a control surface placed around the outer surface of the sleeve, obtain an expression from which $T_{s,2}$ may be determined as a function of $\dot{q}$, r_1, h, T_∞, ε, and T_{sur}.

(d) Consider conditions for which 250 A are passing through a cable having an electric resistance per unit length of $R_e' = 0.005\ \Omega/\text{m}$, a radius of $r_1 = 15$ mm, and a thermal conductivity of $k_c = 200\ \text{W/m}\cdot\text{K}$. For $k_s = 0.15\ \text{W/m}\cdot\text{K}$, $r_2 = 15.5$ mm, $h = 25\ \text{W/m}^2\cdot\text{K}$, $\varepsilon = 0.9$, $T_\infty = 25°\text{C}$, and $T_{sur} = 35°\text{C}$, evaluate the surface temperatures, $T_{s,1}$ and $T_{s,2}$, as well as the temperature T_o at the centerline of the cable.

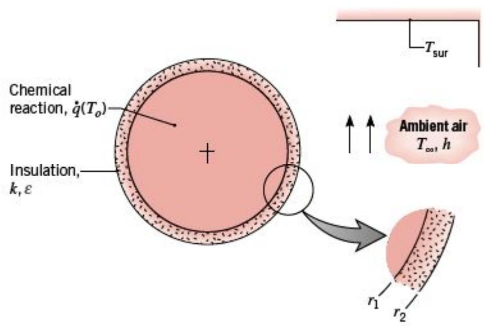
(e) With all other conditions remaining the same, compute and plot T_o, $T_{s,1}$, and $T_{s,2}$ as a function of r_2 for $15.5 \le r_2 \le 20$ mm.

2.42 A spherical shell of inner and outer radii r_i and r_o, respectively, contains heat-dissipating components, and at a particular instant the temperature distribution in the shell is known to be of the form

$$T(r) = \frac{C_1}{r} + C_2$$

Are conditions steady-state or transient? How do the heat flux and heat rate vary with radius?

2.43 A chemically reacting mixture is stored in a thin-walled spherical container of radius $r_1 = 200$ mm, and the exothermic reaction generates heat at a uniform, but temperature-dependent volumetric rate of $\dot{q} = \dot{q}_o \exp(-A/T_o)$, where $\dot{q}_o = 5000\ \text{W/m}^3$, $A = 75$ K, and T_o is the mixture temperature in kelvins. The vessel is enclosed by an insulating material of outer radius r_2, thermal conductivity k, and emissivity ε. The outer surface of the insulation experiences convection heat transfer and net radiation exchange with the adjoining air and large surroundings, respectively.



(a) Write the steady-state form of the heat diffusion equation for the insulation. Verify that this equation is satisfied by the temperature distribution

$$T(r) = T_{s,1} - (T_{s,1} - T_{s,2})\left[\frac{1 - (r_1/r)}{1 - (r_1/r_2)}\right]$$

Sketch the temperature distribution, $T(r)$, labeling key features.

(b) Applying Fourier's law, show that the rate of heat transfer by conduction through the insulation may be expressed as

$$q_r = \frac{4\pi k(T_{s,1} - T_{s,2})}{(1/r_1) - (1/r_2)}$$

Applying an energy balance to a control surface about the container, obtain an alternative expression for q_r, expressing your result in terms of $\dot{q}$ and r_1.