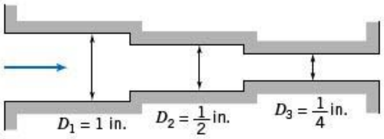
turbulent first? Determine the flow rates at which one, two, then all three sections first become turbulent. At each of these flow rates, determine which sections, if any, attain fully developed flow.



P8.3

8.4 For flow in circular tubes, transition to turbulence usually occurs around $Re \approx 2300$. Investigate the circumstances under which the flows of (a) standard air and (b) water at 15°C become turbulent. On log-log graphs, plot: the average velocity, the volume flow rate, and the mass flow rate, at which turbulence first occurs, as functions of tube diameter.

8.5 For the laminar flow in the section of pipe shown in Fig. 8.1, sketch the expected wall shear stress, pressure, and centerline velocity as functions of distance along the pipe. Explain significant features of the plots, comparing them with fully developed flow. Can the Bernoulli equation be applied anywhere in the flow field? If so, where? Explain briefly.

8.6 An incompressible fluid flows between two infinite stationary parallel plates. The velocity profile is given by $u = u_{\max}(Ay^2 + By + C)$, where A, B, and C are constants and y is measured upward from the lower plate. The total gap width is h units. Use appropriate boundary conditions to express the magnitude and units of the constants in terms of h. Develop an expression for volume flow rate per unit depth and evaluate the ratio $\overline{V}/u_{\max}$.

8.7 The velocity profile for fully developed flow between stationary parallel plates is given by $u = a(h^2/4 - y^2)$, where a is a constant, h is the total gap width between plates, and y is the distance measured from the center of the gap. Determine the ratio $\overline{V}/u_{\max}$.

8.8 A fluid flows steadily between two parallel plates. The flow is fully developed and laminar. The distance between the plates is h.

(a) Derive an equation for the shear stress as a function of y. Sketch this function.

(b) For $\mu = 2.4 \times 10^{-5}$ lbf · s/ft^2, $\partial p/\partial x = -4.0$ lbf/ft^2/ft, and $h = 0.05$ in., calculate the maximum shear stress, in lbf/ft^2.

8.9 Viscous oil flows steadily between parallel plates. The flow is fully developed and laminar. The pressure gradient is 1.25 kPa/m and the channel half-width is $h = 1.5$ mm. Calculate the magnitude and direction of the wall shear stress at the upper plate surface. Find the volume flow rate through the channel ($\mu = 0.50$ N · s/m^2).

8.10 A viscous oil flows steadily between stationary parallel plates. The flow is laminar and fully developed. The total gap width between the plates is $h = 5$ mm. The oil viscosity is 0.5 N · s/m^2 and the pressure gradient is -1000 N/m^2/m. Find the magnitude and direction of the shear stress on the upper plate and the volume flow rate through the channel, per meter of width.

8.11 Oil is confined in a 4-in. diameter cylinder by a piston having a radial clearance of 0.001 in. and a length of 2 in. A steady force of 4500 lbf is applied to the piston. Assume the properties of SAE 30 oil at 120°F. Estimate the rate at which oil leaks past the piston.

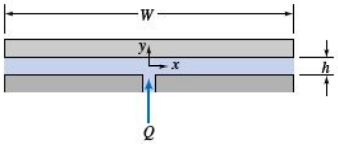
8.12 A hydraulic jack supports a load of 9,000 kg. The following data are given:

Diameter of piston	100 mm
Radial clearance between piston and cylinder	0.05 mm
Length of piston	120 mm

Estimate the rate of leakage of hydraulic fluid past the piston, assuming the fluid is SAE 30 oil at 30°C.

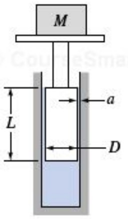
8.13 A high pressure in a system is created by a small piston-cylinder assembly. The piston diameter is 6 mm and it extends 50 mm into the cylinder. The radial clearance between the piston and cylinder is 0.002 mm. Neglect elastic deformations of the piston and cylinder caused by pressure. Assume the fluid properties are those of SAE 10W oil at 35°C. When the pressure in the cylinder is 600 MPa, estimate the leakage rate.

8.14 A hydrostatic bearing is to support a load of 50,000 N per meter of length perpendicular to the diagram. The bearing is supplied with SAE 30 oil at 35°C and 700 kPa (gage) through the central slit. Since the oil is viscous and the gap is small, the flow may be considered fully developed. Calculate (a) the required width of the bearing pad, (b) the resulting pressure gradient, dp/dx, and (c) the gap height, if $Q = 1$ mL/min per meter of length.



P8.14

8.15 The basic component of a pressure gage tester consists of a piston-cylinder apparatus as shown. The piston, 6 mm in diameter, is loaded to develop a pressure of known magnitude. (The piston length is 25 mm.) Calculate the mass, M, required to produce 1.5 MPa (gage) in the cylinder. Determine the leakage flow rate as a function of radial clearance, a, for this load if the liquid is SAE 30 oil at 20°C. Specify the maximum allowable radial clearance so the vertical movement of the piston due to leakage will be less than 1 mm/min.



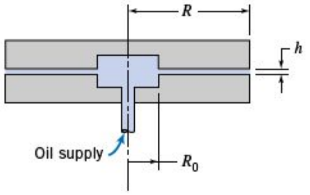
P8.15

8.16 In Section 8-2 we derived the velocity profile between parallel plates (Eq. 8.5) by using a differential control volume. Instead, following the procedure we used in Example 5.9, derive Eq.

8.5 by starting with the Navier-Stokes equations (Eqs. 5.27). Be sure to state all assumptions.

8.17 Viscous liquid, at volume flow rate Q, is pumped through the central opening into the narrow gap between the parallel disks shown. The flow rate is low, so the flow is laminar, and the pressure gradient due to convective acceleration in the gap is negligible compared with the gradient caused by viscous forces (this is termed *creeping flow*). Obtain a general expression for the variation of average velocity in the gap between the disks. For creeping flow, the velocity profile at any cross section in the gap is the same as for fully developed flow between stationary parallel plates. Evaluate the pressure gradient, dp/dr, as a function of radius. Obtain an expression for $p(r)$. Show that the net force required to hold the upper plate in the position shown is

$$F = \frac{3\mu Q R^2}{h^3}\left[1 - \left(\frac{R_0}{R}\right)^2\right]$$



P8.17

8.18 Consider the simple power-law model for a non-Newtonian fluid given by Eq. 2.16. Extend the analysis of Section 8-2 to show that the velocity profile for fully developed laminar flow of a power-law fluid between stationary parallel plates separated by distance $2h$ may be written

$$u = \left(\frac{h}{k}\frac{\Delta p}{L}\right)^{1/n} \frac{nh}{n+1}\left[1 - \left(\frac{y}{h}\right)^{(n+1)/n}\right]$$

where y is the coordinate measured from the channel centerline. Plot the profiles $u/U_{\max}$ versus y/h for $n = 0.7$, 1.0, and 1.3.

8.19 Using the profile of Problem 8.18, show that the flow rate for fully developed laminar flow of a power-law fluid between stationary parallel plates may be written as

$$Q = \left(\frac{h}{k}\frac{\Delta p}{L}\right)^{1/n} \frac{2nwh^2}{2n+1}$$

Here w is the plate width. In such an experimental setup the following data on applied pressure difference Δp and flow rate Q were obtained:

Δp (kPa)	10	20	30	40	50	60	70	80	90	100
Q (L/min)	0.451	0.759	1.01	1.15	1.41	1.57	1.66	1.85	2.05	2.25

Determine if the fluid is pseudoplastic or dilatant, and obtain an experimental value for n.

8.20 A sealed journal bearing is formed from concentric cylinders. The inner and outer radii are 25 and 26 mm, the journal length is 100 mm, and it turns at 2800 rpm. The gap is filled with oil in laminar motion. The velocity profile is linear across the gap. The torque needed to turn the journal is 0.2 N • m. Calculate the viscosity of the oil. Will the torque increase or decrease with time? Why?

8.21 Water at 60°C flows between two large flat plates. The lower plate moves to the left at a speed of 0.3 m/s; the upper plate is stationary. The plate spacing is 3 mm, and the flow is laminar. Determine the pressure gradient required to produce zero net flow at a cross-section.

8.22 Consider fully developed laminar flow between infinite parallel plates separated by gap width $d = 10$ mm. The upper plate moves to the right with speed $U_2 = 0.5$ m/s; the lower plate moves to the left with speed $U_1 = 0.25$ m/s. The pressure gradient in the direction of flow is zero. Develop an expression for the velocity distribution in the gap. Find the volume flow rate per unit depth passing a given cross-section.

8.23 Two immiscible fluids are contained between infinite parallel plates. The plates are separated by distance $2h$, and the two fluid layers are of equal thickness h; the dynamic viscosity of the upper fluid is three times that of the lower fluid. If the lower plate is stationary and the upper plate moves at constant speed $U = 20$ ft/s, what is the velocity at the interface? Assume laminar flows, and that the pressure gradient in the direction of flow is zero.

8.24 Two immiscible fluids are contained between infinite parallel plates. The plates are separated by distance $2h$, and the two fluid layers are of equal thickness $h = 2.5$ mm. The dynamic viscosity of the upper fluid is twice that of the lower fluid, which is $\mu_{\text{lower}} = 0.5$ N • s/m^2. If the plates are stationary and the applied pressure gradient is -1000 N/m^2/m, find the velocity at the interface. What is the maximum velocity of the flow? Plot the velocity distribution.

8.25 The dimensionless velocity profile for fully developed laminar flow between infinite parallel plates with the upper plate moving at constant speed U is shown in Fig. 8.6. Find the pressure gradient $\partial p/\partial x$ at which (a) the upper plate and (b) the lower plate experience zero shear stress, in terms of U, a, and μ. Plot the dimensionless velocity profiles for these cases.

8.26 The record-read head for a computer disk-drive memory storage system rides above the spinning disk on a very thin film of air (the film thickness is 0.25 μm). The head location is 25 mm from the disk centerline; the disk spins at 8500 rpm. The record-read head is 5 mm square. For standard air in the gap between the head and disk, determine (a) the Reynolds number of the flow, (b) the viscous shear stress, and (c) the power required to overcome viscous shear.

8.27 Consider steady, incompressible, and fully developed laminar flow of a viscous liquid down an incline with no pressure gradient. The velocity profile was derived in Example 5.9. Plot the velocity profile. Calculate the kinematic viscosity of the liquid if the film thickness on a 30° slope is 0.8 mm and the maximum velocity is 15.7 mm/s.

8.28 Consider steady, fully developed laminar flow of a viscous liquid down an inclined surface. The liquid layer is of constant thickness, h. Use a suitably chosen differential control volume to obtain the velocity profile. Develop an expression for the volume flow rate.

8.29 The velocity distribution for flow of a thin viscous film down an inclined plane surface was developed in Example 5.9.

Consider a film 7 mm thick, of liquid with SG = 1.2 and dynamic viscosity of 1.60 N • s/m^2. Derive an expression for the shear stress distribution within the film. Calculate the maximum shear stress within the film and indicate its direction. Evaluate the volume flow rate in the film, in mm^3/s per millimeter of surface width. Calculate the film Reynolds number based on average velocity.

8.30 Two immiscible fluids of equal density are flowing down a surface inclined at a 30° angle. The two fluid layers are of equal thickness $h = 2.5$ mm; the kinematic viscosity of the upper fluid is twice that of the lower fluid, which is $\nu_{\text{lower}} = 2 \times 10^{-4}$ m^2/s. Find the velocity at the interface and the velocity at the free surface. Plot the velocity distribution.

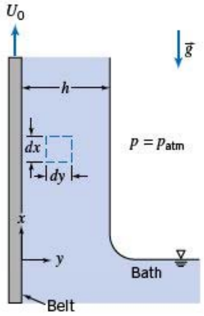
8.31 Consider fully developed flow between parallel plates with the upper plate moving at $U = 5$ ft/s; the spacing between the plates is $a = 0.1$ in. Determine the flow rate per unit depth for the case of zero pressure gradient. If the fluid is air, evaluate the shear stress on the lower plate and plot the shear stress distribution across the channel for the zero pressure gradient case. Will the flow rate increase or decrease if the pressure gradient is adverse? Determine the pressure gradient that will give zero shear stress at $y = 0.25a$. Plot the shear stress distribution across the channel for the latter case.

8.32 Water at 15°C flows between parallel plates with gap width $b = 2.5$ mm. The upper plate moves with speed $U = 0.25$ m/s in the positive x direction. The pressure gradient is $\partial p/\partial x = -175$ Pa/m. Locate the point of maximum velocity and determine its magnitude (let $y = 0$ at the bottom plate). Determine the volume of flow that passes a given cross-section (x = constant) in 10 s. Plot the velocity and shear stress distributions.

8.33 The velocity profile for fully developed flow of air between parallel plates with the upper plate moving is given by Eq. 8.8. Assume $U = 2$ m/s and $a = 2.5$ mm. Find the pressure gradient for which there is no net flow in the x direction. Plot the expected velocity distribution and the expected shear stress distribution across the channel for this flow. For the case where $u = 2U$ at $y/a = 0.5$, plot the expected velocity distribution and shear stress distribution across the channel. Comment on features of the plots.

8.34 The velocity profile for fully developed flow of water between parallel plates with the upper plate moving is given by Eq. 8.8. Assume $U = 2$ m/s and $a = 2.5$ mm. Determine the volume flow rate per unit depth for zero pressure gradient. Evaluate the shear stress on the lower plate and sketch the shear stress distribution across the channel. Would the volume flow rate increase or decrease with a mild adverse pressure gradient? Calculate the pressure gradient that will give zero shear stress at $y/a = 0.25$. Sketch the shear stress distribution for this case.

8.35 A continuous belt, passing upward through a chemical bath at speed U_0, picks up a liquid film of thickness h, density ρ, and viscosity μ. Gravity tends to make the liquid drain down, but the movement of the belt keeps the liquid from running off completely. Assume that the flow is fully developed and laminar with zero pressure gradient, and that the atmosphere produces no shear stress at the outer surface of the film. State clearly the boundary conditions to be satisfied by the velocity at $y = 0$ and $y = h$. Obtain an expression for the velocity profile.



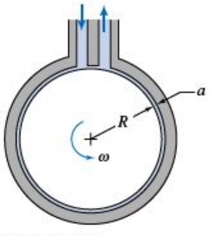
P8.35

8.36 In Example 8.3 we derived the velocity profile for laminar flow on a vertical wall by using a differential control volume. Instead, following the procedure we used in Example 5.9, derive the velocity profile by starting with the Navier-Stokes equations (Eqs. 5.27). Be sure to state all assumptions.

8.37 Microchips are supported on a thin air film on a smooth horizontal surface during one stage of the manufacturing process. The chips are 11.7 mm long and 9.35 mm wide and have a mass of 0.325 g. The air film is 0.125 mm thick. The initial speed of a chip is $V_0 = 1.75$ mm/s; the chip slows as the result of viscous shear in the air film. Analyze the chip motion during deceleration to develop a differential equation for chip speed V versus time t. Calculate the time required for a chip to lose 5 percent of its initial speed. Plot the variation of chip speed versus time during deceleration. Explain why it looks as you have plotted it.

8.38 Free-surface waves begin to form on a laminar liquid film flowing down an inclined surface whenever the Reynolds number, based on mass flow per unit width of film, is larger than about 33. Estimate the maximum thickness of a laminar film of water that remains free from waves while flowing down a vertical surface.

8.39 A viscous-shear pump is made from a stationary housing with a close-fitting rotating drum inside. The clearance is small compared with the diameter of the drum, so flow in the annular space may be treated as flow between parallel plates. Fluid is dragged around the annulus by viscous forces. Evaluate the performance characteristics of the shear pump (pressure differential, input power, and efficiency) as functions of volume flow rate. Assume that the depth normal to the diagram is b.



P8.39, P8.40

case, $k \to 0$, with the corresponding expression for flow in a circular pipe.

8.53 It has been suggested in the design of an agricultural sprinkler that a structural member be held in place by a wire placed along the centerline of a pipe; it is surmised that a relatively small wire would have little effect on the pressure drop for a given flow rate. Using the result of Problem 8.52, derive an expression giving the percentage change in pressure drop as a function of the ratio of wire diameter to pipe diameter for laminar flow. Plot the percentage change in pressure drop as a function of radius ratio k for $0.001 \le k \le 0.10$.

8.54 In a food industry plant two immiscible fluids are pumped through a tube such that fluid 1 ($\mu_1 = 0.02 \text{ lbf} \cdot \text{s/ft}^2$) forms an inner core and fluid 2 ($\mu_2 = 0.03 \text{ lbf} \cdot \text{s/ft}^2$) forms an outer annulus. The tube has $D = 0.2$ in. diameter and length $L = 50$ ft. Derive and plot the velocity distribution if the applied pressure difference, Δp, is 1 psi.

8.55 A horizontal pipe carries fluid in fully developed turbulent flow. The static pressure difference measured between two sections is 35 kPa. The distance between the sections is 10 m and the pipe diameter is 150 mm. Calculate the shear stress, τ_w, that acts on the walls.

8.56 One end of a horizontal pipe is attached using glue to a pressurized tank containing liquid, and the other has a cap attached. The inside diameter of the pipe is 2.5 cm, and the tank pressure is 250 kPa (gage). Find the force the glue must withstand, and the force it must withstand when the cap is off and the liquid is discharging to atmosphere.

8.57 The pressure drop between two taps separated in the streamwise direction by 30 ft in a horizontal, fully developed channel flow of water is 1 psi. The cross-section of the channel is a 1 in. $\times\, 9\frac{1}{2}$ in. rectangle. Calculate the average wall shear stress.

8.58 Kerosine is pumped through a smooth tube with inside diameter $D = 30$ mm at close to the critical Reynolds number. The flow is unstable and fluctuates between laminar and turbulent states, causing the pressure gradient to intermittently change from approximately -4.5 kPa/m to -11 kPa/m. Which pressure gradient corresponds to laminar, and which to turbulent, flow? For each flow, compute the shear stress at the tube wall, and sketch the shear stress distributions.

8.59 A liquid drug, with the viscosity and density of water, is to be administered through a hypodermic needle. The inside diameter of the needle is 0.25 mm and its length is 50 mm. Determine (a) the maximum volume flow rate for which the flow will be laminar, (b) the pressure drop required to deliver the maximum flow rate, and (c) the corresponding wall shear stress.

8.60 Consider the empirical "power-law" profile for turbulent pipe flow, Eq. 8.22. For $n = 7$ determine the value of r/R at which u is equal to the average velocity, $\overline{V}$. Plot the results over the range $6 \le n \le 10$ and compare with the case of fully developed laminar pipe flow, Eq. 8.14.

8.61 Laufer [5] measured the following data for mean velocity in fully developed turbulent pipe flow at $Re_U = 50,000$:

$\overline{u}/U$	0.996	0.981	0.963	0.937	0.907	0.866	0.831
y/r	0.898	0.794	0.691	0.588	0.486	0.383	0.280
$\overline{u}/U$	0.792	0.742	0.700	0.650	0.619	0.551	
y/R	0.216	0.154	0.093	0.062	0.041	0.024	

In addition, Laufer measured the following data for mean velocity in fully developed turbulent pipe flow at $Re_U = 500{,}000$:

$\overline{u}/U$	0.997	0.988	0.975	0.959	0.934	0.908
y/R	0.898	0.794	0.691	0.588	0.486	0.383
$\overline{u}/U$	0.874	0.847	0.818	0.771	0.736	0.690
y/R	0.280	0.216	0.154	0.093	0.062	0.037

Using *Excel*'s trendline analysis, fit each set of data to the "power-law" profile for turbulent flow, Eq. 8.22, and obtain a value of n for each set. Do the data tend to confirm the validity of Eq. 8.22? Plot the data and their corresponding trendlines on the same graph.

8.62 Equation 8.23 gives the power-law velocity profile exponent, n, as a function of centerline Reynolds number, Re_U, for fully developed turbulent flow in smooth pipes. Equation 8.24 relates mean velocity, $\overline{V}$, to centerline velocity, U, for various values of n. Prepare a plot of $\overline{V}/U$ as a function of Reynolds number, $Re_{\overline{V}}$.

8.63 A momentum coefficient, β, is defined by

$$\int_A u\,\rho u\, dA = \beta \int_A \overline{V}\,\rho u\, dA = \beta\, \dot{m}\, \overline{V}$$

Evaluate β for a laminar velocity profile, Eq. 8.14, and for a "power-law" turbulent velocity profile, Eq. 8.22. Plot β as a function of n for turbulent power-law profiles over the range $6 \le n \le 10$ and compare with the case of fully developed laminar pipe flow.

8.64 Consider fully developed laminar flow of water between stationary parallel plates. The maximum flow speed, plate spacing, and width are 20 ft/s, 0.075 in. and 1.25 in. respectively. Find the kinetic energy coefficient, α.

8.65 Consider fully developed laminar flow in a circular tube. Evaluate the kinetic energy coefficient for this flow.

8.66 Show that the kinetic energy coefficient, α, for the "power law" turbulent velocity profile of Eq. 8.22 is given by Eq. 8.27. Plot α as a function of $Re_{\overline{V}}$, for $Re_{\overline{V}} = 1 \times 10^4$ to 1×10^7. When analyzing pipe flow problems it is common practice to assume $\alpha \approx 1$. Plot the error associated with this assumption as a function of $Re_{\overline{V}}$, for $Re_{\overline{V}} = 1 \times 10^4$ to 1×10^7.

8.67 Measurements are made for the flow configuration shown in Fig. 8.12. At the inlet, section ①, the pressure is 70 kPa (gage), the average velocity is 1.75 m/s, and the elevation is 2.25 m. At the outlet, section ②, the pressure, average velocity, and elevation are 45 kPa (gage), 3.5 m/s, and 3 m, respectively. Calculate the head loss in meters. Convert to units of energy per unit mass.

8.68 Water flows in a horizontal constant-area pipe; the pipe diameter is 50 mm and the average flow speed is 1.5 m/s. At the pipe inlet the gage pressure is 588 kPa, and the outlet is at atmospheric pressure. Determine the head loss in the pipe. If the pipe is now aligned so that the outlet is 25 m above the inlet, what will the inlet pressure need to be to maintain the same flow rate? If the pipe is now aligned so that the outlet is 25 m below the inlet, what will the inlet pressure need to be to maintain the same flow rate? Finally, how much lower than the inlet must the outlet be so that the same flow rate is maintained if both ends of the pipe are at atmospheric pressure (i.e., gravity feed)?