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## 296 CHAPTER 7 / DIMENSIONAL ANALYSIS AND SIMILITUDE

7.7 The equation describing motion of fluid in a pipe due to an applied pressure gradient, when the flow starts from rest, is

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right)$$

Use the average velocity  $\bar{V}$ , pressure drop  $\Delta p$ , pipe length  $L$ , and diameter  $D$  to nondimensionalize this equation. Obtain the dimensionless groups that characterize this flow.

7.8 An unsteady, two dimensional, compressible, inviscid flow can be described by the equation

$$\frac{\partial^2 \psi}{\partial t^2} + \frac{\partial}{\partial t} (u^2 + v^2) + (u^2 - c^2) \frac{\partial^2 \psi}{\partial x^2} + (v^2 - c^2) \frac{\partial^2 \psi}{\partial y^2} + 2uv \frac{\partial^2 \psi}{\partial x \partial y} = 0$$

where  $\psi$  is the stream function,  $u$  and  $v$  are the  $x$  and  $y$  components of velocity, respectively,  $c$  is the speed of sound, and  $t$  is the time. Using  $L$  as a characteristic length and  $c_0$  (the speed of sound at the stagnation point) to nondimensionalize this equation, obtain the dimensionless groups that characterize the equation.

7.9 At very low speeds, the drag on an object is independent of fluid density. Thus the drag force,  $F$ , on a small sphere is a function only of speed,  $V$ , fluid viscosity,  $\mu$ , and sphere diameter,  $D$ . Use dimensional analysis to determine how the drag force  $F$  depends on the speed  $V$ .

7.10 At relatively high speeds the drag on an object is independent of fluid viscosity. Thus the aerodynamic drag force,  $F$ , on an automobile, is a function only of speed,  $V$ , air density  $\rho$ , and vehicle size, characterized by its frontal area  $A$ . Use dimensional analysis to determine how the drag force  $F$  depends on the speed  $V$ .

7.11 Experiments show that the pressure drop for flow through an orifice plate of diameter  $d$  mounted in a length of pipe of diameter  $D$  may be expressed as  $\Delta p = p_1 - p_2 = f(\rho, \mu, \bar{V}, d, D)$ . You are asked to organize some experimental data. Obtain the resulting dimensionless parameters.

7.12 The speed,  $V$ , of a free-surface wave in shallow liquid is a function of depth,  $D$ , density,  $\rho$ , gravity,  $g$ , and surface tension,  $\sigma$ . Use dimensional analysis to find the functional dependence of  $V$  on the other variables. Express  $V$  in the simplest form possible.

7.13 The wall shear stress,  $\tau_w$ , in a boundary layer depends on distance from the leading edge of the body,  $x$ , the density,  $\rho$ , and viscosity,  $\mu$ , of the fluid, and the freestream speed of the flow,  $U$ . Obtain the dimensionless groups and express the functional relationship among them.

7.14 The boundary-layer thickness,  $\delta$ , on a smooth flat plate in an incompressible flow without pressure gradients depends on the freestream speed,  $U$ , the fluid density,  $\rho$ , the fluid viscosity,  $\mu$ , and the distance from the leading edge of the plate,  $x$ . Express these variables in dimensionless form.

7.15 If an object is light enough it can be supported on the surface of a fluid by surface tension. Tests are to be done to investigate this phenomenon. The weight,  $W$ , supportable in this way depends on the object's perimeter,  $p$ , and the fluid's density,  $\rho$ , surface tension  $\sigma$ , and gravity,  $g$ . Determine the dimensionless parameters that characterize this problem.

7.16 The mean velocity,  $\bar{u}$ , for turbulent flow in a pipe or a boundary layer may be correlated using the wall shear stress,  $\tau_w$ , distance from the wall,  $y$ , and the fluid properties,  $\rho$  and  $\mu$ . Use

dimensional analysis to find one dimensionless parameter containing  $\bar{u}$  and one containing  $y$  that are suitable for organizing experimental data. Show that the result may be written

$$\frac{\bar{u}}{u_*} = f\left(\frac{yu_*}{\nu}\right)$$

where  $u_* = (\tau_w/\rho)^{1/2}$  is the friction velocity.

7.17 The speed,  $V$ , of a free-surface gravity wave in deep water is a function of wavelength,  $\lambda$ , depth,  $D$ , density,  $\rho$ , and acceleration of gravity,  $g$ . Use dimensional analysis to find the functional dependence of  $V$  on the other variables. Express  $V$  in the simplest form possible.

7.18 The torque,  $T$ , of a handheld automobile buffer is a function of rotational speed,  $\omega$ , applied normal force,  $F$ , automobile surface roughness,  $e$ , buffing paste viscosity,  $\mu$ , and surface tension,  $\sigma$ . Determine the dimensionless parameters that characterize this problem.

7.19 Measurements of the liquid height upstream from an obstruction placed in an open-channel flow can be used to determine volume flow rate. (Such obstructions, designed and calibrated to measure rate of open-channel flow, are called *weirs*.) Assume the volume flow rate,  $Q$ , over a weir is a function of upstream height,  $h$ , gravity,  $g$ , and channel width,  $b$ . Use dimensional analysis to find the functional dependence of  $Q$  on the other variables.

7.20 Capillary waves are formed on a liquid free surface as a result of surface tension. They have short wavelengths. The speed of a capillary wave depends on surface tension,  $\sigma$ , wavelength,  $\lambda$ , and liquid density,  $\rho$ . Use dimensional analysis to express wave speed as a function of these variables.

7.21 The load-carrying capacity,  $W$ , of a journal bearing is known to depend on its diameter,  $D$ , length,  $l$ , and clearance,  $c$ , in addition to its angular speed,  $\omega$ , and lubricant viscosity,  $\mu$ . Determine the dimensionless parameters that characterize this problem.

7.22 The time,  $t$ , for oil to drain out of a viscosity calibration container depends on the fluid viscosity,  $\mu$ , and density,  $\rho$ , the orifice diameter,  $d$ , and gravity,  $g$ . Use dimensional analysis to find the functional dependence of  $t$  on the other variables. Express  $t$  in the simplest possible form.

7.23 The power,  $\mathcal{P}$ , used by a vacuum cleaner is to be correlated with the amount of suction provided (indicated by the pressure drop,  $\Delta p$ , below the ambient room pressure). It also depends on impeller diameter,  $D$ , and width,  $d$ , motor speed,  $\omega$ , air density,  $\rho$ , and cleaner inlet and exit widths,  $d_i$  and  $d_o$ , respectively. Determine the dimensionless parameters that characterize this problem.

7.24 The power per unit cross-sectional area,  $E$ , transmitted by a sound wave is a function of wave speed,  $V$ , medium density,  $\rho$ , wave amplitude,  $r$ , and wave frequency,  $n$ . Determine, by dimensional analysis, the general form of the expression for  $E$  in terms of the other variables.

7.25 You are asked to find a set of dimensionless parameters to organize data from a laboratory experiment, in which a tank is drained through an orifice from initial liquid level  $h_0$ . The time,  $\tau$ , to drain the tank depends on tank diameter,  $D$ , orifice diameter,  $d$ , acceleration of gravity,  $g$ , liquid density,  $\rho$ , and liquid viscosity,  $\mu$ . How many dimensionless parameters will result? How many repeating variables must be selected to determine the

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dimensionless parameters? Obtain the  $\Pi$  parameter that contains the viscosity.

**7.26** The power,  $\mathcal{P}$ , required to drive a fan is believed to depend on fluid density,  $\rho$ , volume flow rate,  $Q$ , impeller diameter,  $D$ , and angular velocity,  $\omega$ . Use dimensional analysis to determine the dependence of  $\mathcal{P}$  on the other variables.

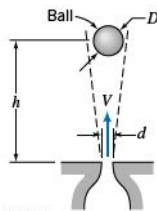
**7.27** A continuous belt moving vertically through a bath of viscous liquid drags a layer of liquid, of thickness  $h$ , along with it. The volume flow rate of liquid,  $Q$ , is assumed to depend on  $\mu$ ,  $\rho$ ,  $g$ ,  $h$ , and  $V$ , where  $V$  is the belt speed. Apply dimensional analysis to predict the form of dependence of  $Q$  on the other variables.

**7.28** In a fluid mechanics laboratory experiment a tank of water, with diameter  $D$ , is drained from initial level  $h_0$ . The smoothly rounded drain hole has diameter  $d$ . Assume the mass flow rate from the tank is a function of  $h$ ,  $D$ ,  $d$ ,  $g$ ,  $\rho$ , and  $\mu$ , where  $g$  is the acceleration of gravity and  $\rho$  and  $\mu$  are fluid properties. Measured data are to be correlated in dimensionless form. Determine the number of dimensionless parameters that will result. Specify the number of repeating parameters that must be selected to determine the dimensionless parameters. Obtain the  $\Pi$  parameter that contains the viscosity.

**7.29** Small droplets of liquid are formed when a liquid jet breaks up in spray and fuel injection processes. The resulting droplet diameter,  $d$ , is thought to depend on liquid density, viscosity, and surface tension, as well as jet speed,  $V$ , and diameter,  $D$ . How many dimensionless ratios are required to characterize this process? Determine these ratios.

**7.30** The diameter,  $d$ , of the dots made by an ink jet printer depends on the ink viscosity,  $\mu$ , density,  $\rho$ , and surface tension,  $\sigma$ , the nozzle diameter,  $D$ , the distance,  $L$ , of the nozzle from the paper surface, and the ink jet velocity,  $V$ . Use dimensional analysis to find the  $\Pi$  parameters that characterize the ink jet's behavior.

**7.31** The sketch shows an air jet discharging vertically. Experiments show that a ball placed in the jet is suspended in a stable position. The equilibrium height of the ball in the jet is found to depend on  $D$ ,  $d$ ,  $V$ ,  $\rho$ ,  $\mu$ , and  $W$ , where  $W$  is the weight of the ball. Dimensional analysis is suggested to correlate experimental data. Find the  $\Pi$  parameters that characterize this phenomenon.



P7.31

**7.32** The terminal speed  $V$  of shipping boxes sliding down an incline on a layer of air (injected through numerous pinholes in the incline surface) depends on the box mass,  $m$ , and base area,  $A$ , gravity,  $g$ , the incline angle,  $\theta$ , the air viscosity,  $\mu$ , and the air layer thickness,  $\delta$ . Use dimensional analysis to find the  $\Pi$  parameters that characterize this phenomenon.

**7.33** The diameter,  $d$ , of bubbles produced by a bubble-making toy depends on the soapy water viscosity,  $\mu$ , density,  $\rho$ , and surface tension,  $\sigma$ , the ring diameter,  $D$ , and the pressure differential,

$\Delta p$ , generating the bubbles. Use dimensional analysis to find the  $\Pi$  parameters that characterize this phenomenon.

**7.34** A washing machine agitator is to be designed. The power,  $\mathcal{P}$ , required for the agitator is to be correlated with the amount of water used (indicated by the depth,  $H$ , of the water). It also depends on the agitator diameter,  $D$ , height,  $h$ , maximum angular velocity,  $\omega_{\max}$ , and frequency of oscillations,  $f$ , and water density,  $\rho$ , and viscosity,  $\mu$ . Determine the dimensionless parameters that characterize this problem.

**7.35** The time,  $t$ , for a flywheel, with moment of inertia,  $I$ , to reach angular velocity,  $\omega$ , from rest, depends on the applied torque,  $T$ , and the following flywheel bearing properties: the oil viscosity,  $\mu$ , gap,  $\delta$ , diameter,  $D$ , and length,  $L$ . Use dimensional analysis to find the  $\Pi$  parameters that characterize this phenomenon.

**7.36** A large tank of liquid under pressure is drained through a smoothly contoured nozzle of area  $A$ . The mass flow rate is thought to depend on nozzle area,  $A$ , liquid density,  $\rho$ , difference in height between the liquid surface and nozzle,  $h$ , tank gage pressure,  $\Delta p$ , and gravitational acceleration,  $g$ . Determine how many independent  $\Pi$  parameters can be formed for this problem. Find the dimensionless parameters. State the functional relationship for the mass flow rate in terms of the dimensionless parameters.

**7.37** The ventilation in the clubhouse on a cruise ship is insufficient to clear cigarette smoke (the ship is not yet completely smoke-free). Tests are to be done to see if a larger extractor fan will work. The concentration of smoke,  $c$  (particles per cubic meter) depends on the number of smokers,  $N$ , the pressure drop produced by the fan,  $\Delta p$ , the fan diameter,  $D$ , motor speed,  $\omega$ , the particle and air densities,  $\rho_p$  and  $\rho$ , respectively, gravity,  $g$ , and air viscosity,  $\mu$ . Determine the dimensionless parameters that characterize this problem.

**7.38** Spin plays an important role in the flight trajectory of golf, Ping-Pong, and tennis balls. Therefore, it is important to know the rate at which spin decreases for a ball in flight. The aerodynamic torque,  $T$ , acting on a ball in flight, is thought to depend on flight speed,  $V$ , air density,  $\rho$ , air viscosity,  $\mu$ , ball diameter,  $D$ , spin rate (angular speed),  $\omega$ , and diameter of the dimples on the ball,  $d$ . Determine the dimensionless parameters that result.

**7.39** The power loss,  $\mathcal{P}$ , in a journal bearing depends on length,  $l$ , diameter,  $D$ , and clearance,  $c$ , of the bearing, in addition to its angular speed,  $\omega$ . The lubricant viscosity and mean pressure are also important. Obtain the dimensionless parameters that characterize this problem. Determine the functional form of the dependence of  $\mathcal{P}$  on these parameters.

**7.40** The thrust of a marine propeller is to be measured during "open-water" tests at a variety of angular speeds and forward speeds ("speeds of advance"). The thrust,  $F_T$ , is thought to depend on water density,  $\rho$ , propeller diameter,  $D$ , speed of advance,  $V$ , acceleration of gravity,  $g$ , angular speed,  $\omega$ , pressure in the liquid,  $p$ , and liquid viscosity,  $\mu$ . Develop a set of dimensionless parameters to characterize the performance of the propeller. (One of the resulting parameters,  $gDV^2$ , is known as the *Froude speed of advance*.)

**7.41** In a fan-assisted convection oven, the heat transfer rate to a roast,  $\dot{Q}$  (energy per unit time), is thought to depend on the specific heat of air,  $c_p$ , temperature difference,  $\Theta$ , a length scale,  $L$ , air density,  $\rho$ , air viscosity,  $\mu$ , and air speed,  $V$ . How many basic dimensions are included in these variables? Determine the

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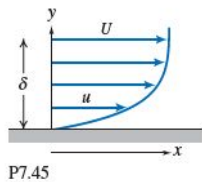
number of  $\Pi$  parameters needed to characterize the oven. Evaluate the  $\Pi$  parameters.

**7.42** The power,  $\mathcal{P}$ , required to drive a propeller is known to depend on the following variables: freestream speed,  $V$ , propeller diameter,  $D$ , angular speed,  $\omega$ , fluid viscosity,  $\mu$ , fluid density,  $\rho$ , and speed of sound in the fluid,  $c$ . How many dimensionless groups are required to characterize this situation? Obtain these dimensionless groups.

**7.43** The rate  $dT/dt$  at which the temperature  $T$  at the center of a rice kernel falls during a food technology process is critical—too high a value leads to cracking of the kernel, and too low a value makes the process slow and costly. The rate depends on the rice specific heat,  $c$ , thermal conductivity,  $k$ , and size,  $L$ , as well as the cooling air specific heat,  $c_p$ , density,  $\rho$ , viscosity,  $\mu$ , and speed,  $V$ . How many basic dimensions are included in these variables? Determine the  $\Pi$  parameters for this problem.

**7.44** When a valve is closed suddenly in a pipe with flowing water, a water hammer pressure wave is set up. The very high pressures generated by such waves can damage the pipe. The maximum pressure,  $p_{\max}$ , generated by water hammer is a function of liquid density,  $\rho$ , initial flow speed,  $U_0$ , and liquid bulk modulus,  $E_v$ . How many dimensionless groups are needed to characterize water hammer? Determine the functional relationship among the variables in terms of the necessary  $\Pi$  groups.

**7.45** The fluid velocity  $u$  at any point in a boundary layer depends on the distance  $y$  of the point above the surface, the free-stream velocity  $U$  and free-stream velocity gradient  $dU/dx$ , the fluid kinematic viscosity  $\nu$ , and the boundary layer thickness  $\delta$ . How many dimensionless groups are required to describe this problem? Find: (a) two  $\Pi$  groups by inspection, (b) one  $\Pi$  that is a standard fluid mechanics group, and (c) any remaining  $\Pi$  groups using the Buckingham Pi theorem.



**7.46** An airship is to operate at 20 m/s in air at standard conditions. A model is constructed to  $\frac{1}{20}$ -scale and tested in a wind tunnel at the same air temperature to determine drag. What criterion should be considered to obtain dynamic similarity? If the model is tested at 75 m/s, what pressure should be used in the wind tunnel? If the model drag force is 250 N, what will be the drag of the prototype?

**7.47** The designers of a large tethered pollution-sampling balloon wish to know what the drag will be on the balloon for the maximum anticipated wind speed of 5 m/s (the air is assumed to be at 20°C). A  $\frac{1}{20}$ -scale model is built for testing in water at 20°C. What water speed is required to model the prototype? At this speed the model drag is measured to be 2 kN. What will be the corresponding drag on the prototype?

**7.48** An ocean-going vessel is to be powered by a rotating circular cylinder. Model tests are planned to estimate the power required to rotate the prototype cylinder. A dimensional analysis is needed to scale the power requirements from model test results

to the prototype. List the parameters that should be included in the dimensional analysis. Perform a dimensional analysis to identify the important dimensionless groups.

**7.49** To match the Reynolds number in an air flow and a water flow using the same size model, which flow will require the higher flow speed? How much higher must it be?

**7.50** Measurements of drag force are made on a model automobile in a towing tank filled with fresh water. The model length scale is  $\frac{1}{5}$  that of the prototype. State the conditions required to ensure dynamic similarity between the model and prototype. Determine the fraction of the prototype speed in air at which the model test should be made in water to ensure dynamically similar conditions. Measurements made at various speeds show that the dimensionless force ratio becomes constant at model test speeds above  $V_m = 4$  m/s. The drag force measured during a test at this speed is  $F_{Dm} = 182$  N. Calculate the drag force expected on the prototype vehicle operating at 90 km/hr in air.

**7.51** On a cruise ship, passengers complain about the noise emanating from the ship's propellers (probably due to turbulent flow effects between propeller and ship). You have been hired to find out the source of this noise. You will study the flow pattern around the propellers and have decided to use a 1:10 scale water tank. If the ship's propellers rotate at 125 rpm, estimate the model propeller rotation speed if a) the Froude number or b) the Reynolds number is the governing dimensionless group. Which is most likely to lead to the best modeling?

**7.52** A  $\frac{1}{5}$ -scale model of a torpedo is tested in a wind tunnel to determine the drag force. The prototype operates in water, has 533 mm diameter, and is 6.7 m long. The desired operating speed of the prototype is 28 m/s. To avoid compressibility effects in the wind tunnel, the maximum speed is limited to 110 m/s. However, the pressure in the wind tunnel can be varied while holding the temperature constant at 20°C. At what minimum pressure should the wind tunnel be operated to achieve a dynamically similar test? At dynamically similar test conditions, the drag force on the model is measured as 618 N. Evaluate the drag force expected on the full-scale torpedo.

**7.53** The drag of an airfoil at zero angle of attack is a function of density, viscosity, and velocity, in addition to a length parameter. A  $\frac{1}{10}$ -scale model of an airfoil was tested in a wind tunnel at a Reynolds number of  $5.5 \times 10^6$ , based on chord length. Test conditions in the wind tunnel air stream were 15°C and 10 atmospheres absolute pressure. The prototype airfoil has a chord length of 2 m, and it is to be flown in air at standard conditions. Determine the speed at which the wind tunnel model was tested, and the corresponding prototype speed.

**7.54** Consider a smooth sphere, of diameter  $D$ , immersed in a fluid moving with speed  $V$ . The drag force on a 10-ft diameter weather balloon in air moving at 5 ft/s is to be calculated from test data. The test is to be performed in water using a 2-in. diameter model. Under dynamically similar conditions, the model drag force is measured as 0.85 lbf. Evaluate the model test speed and the drag force expected on the full-scale balloon.

**7.55** An airplane wing, with chord length of 1.5 m and span of 9 m, is designed to move through standard air at a speed of 7.5 m/s. A  $\frac{1}{10}$ -scale model of this wing is to be tested in a water tunnel. What speed is necessary in the water tunnel to achieve dynamic

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similarity? What will be the ratio of forces measured in the model flow to those on the prototype wing?

**7.56** The fluid dynamic characteristics of a golf ball are to be tested using a model in a wind tunnel. Dependent parameters are the drag force,  $F_D$ , and lift force,  $F_L$ , on the ball. The independent parameters should include angular speed,  $\omega$ , and dimple depth,  $d$ . Determine suitable dimensionless parameters and express the functional dependence among them. A golf pro can hit a ball at  $V = 240$  ft/s and  $\omega = 9000$  rpm. To model these conditions in a wind tunnel with maximum speed of 80 ft/s, what diameter model should be used? How fast must the model rotate? (The diameter of a U.S. golf ball is 1.68 in.)

**7.57** A water pump with impeller diameter 60 cm is to be designed to move  $0.4$  m<sup>3</sup>/s when running at 800 rpm. Testing is performed on a  $\frac{1}{2}$  scale model running at 2000 rpm using air (20°C) as the fluid. For similar conditions (neglecting Reynolds number effects), what will be the model flow rate? If the model draws 75 W, what will be the power requirement of the prototype?

**7.58** A model test is performed to determine the flight characteristics of a Frisbee. Dependent parameters are drag force,  $F_D$ , and lift force,  $F_L$ . The independent parameters should include angular speed,  $\omega$ , and roughness height,  $h$ . Determine suitable dimensionless parameters and express the functional dependence among them. The test (using air) on a  $\frac{1}{4}$ -scale model Frisbee is to be geometrically, kinematically, and dynamically similar to the prototype. The prototype values are  $V_p = 5$  m/s and  $\omega_p = 100$  rpm. What values of  $V_m$  and  $\omega_m$  should be used?

**7.59** A model hydrofoil is to be tested at 1:20 scale. The test speed is chosen to duplicate the Froude number corresponding to the 60 knot prototype speed. To model cavitation correctly, the cavitation number also must be duplicated. At what ambient pressure must the test be run? Water in the model test basin can be heated to 130°F, compared to 45°F for the prototype.

**7.60** SAE 10W oil at 25°C flowing in a 25-mm diameter horizontal pipe, at an average speed of 1 m/s, produces a pressure drop of 450 kPa (gauge) over a 150-m length. Water at 15°C flows through the same pipe under dynamically similar conditions. Using the results of Example 7.2, calculate the average speed of the water flow and the corresponding pressure drop.

**7.61** In some speed ranges, vortices are shed from the rear of bluff cylinders placed across a flow. The vortices alternately leave the top and bottom of the cylinder, as shown, causing an alternating force normal to the freestream velocity. The vortex shedding frequency,  $f$ , is thought to depend on  $\rho$ ,  $d$ ,  $V$ , and  $\mu$ . Use dimensional analysis to develop a functional relationship for  $f$ . Vortex shedding occurs in standard air on two cylinders with a diameter ratio of 2. Determine the velocity ratio for dynamic similarity, and the ratio of vortex shedding frequencies.



P7.61

**7.62** A  $\frac{1}{8}$ -scale model of a tractor-trailer rig is tested in a pressurized wind tunnel. The rig width, height, and length are  $W = 0.305$  m,  $H = 0.476$  m, and  $L = 2.48$  m, respectively. At wind speed  $V = 75.0$  m/s, the model drag force is  $F_D = 128$  N. (Air density in the tunnel is  $\rho = 3.23$  kg/m<sup>3</sup>.) Calculate the aerodynamic drag

coefficient for the model. Compare the Reynolds numbers for the model test and for the prototype vehicle at 55 mph. Calculate the aerodynamic drag force on the prototype vehicle at a road speed of 55 mph into a headwind of 10 mph.

**7.63** On a cruise ship, passengers complain about the amount of smoke that becomes entrained behind the cylindrical smoke stack. You have been hired to study the flow pattern around the stack, and have decided to use a 1:12.5 scale model of the 4.75 m smoke stack. What range of wind tunnel speeds could you use if the ship speed for which the problem occurs is 15 knots to 25 knots?

**7.64** The aerodynamic behavior of a flying insect is to be investigated in a wind tunnel using a ten-times scale model. If the insect flaps its wings 50 times a second when flying at 4 ft/s, determine the wind tunnel air speed and wing oscillation frequency required for dynamic similarity. Do you expect that this would be a successful or practical model for generating an easily measurable wing lift? If not, can you suggest a different fluid (e.g., water, or air at a different pressure and/or temperature) that would produce a better modeling?

**7.65** A model test of a tractor-trailer rig is performed in a wind tunnel. The drag force,  $F_D$ , is found to depend on frontal area,  $A$ , wind speed,  $V$ , air density,  $\rho$ , and air viscosity,  $\mu$ . The model scale is 1:4; frontal area of the model is  $A = 0.625$  m<sup>2</sup>. Obtain a set of dimensionless parameters suitable to characterize the model test results. State the conditions required to obtain dynamic similarity between model and prototype flows. When tested at wind speed  $V = 89.6$  m/s, in standard air, the measured drag force on the model was  $F_D = 2.46$  kN. Assuming dynamic similarity, estimate the aerodynamic drag force on the full-scale vehicle at  $V = 22.4$  m/s. Calculate the power needed to overcome this drag force if there is no wind.

**7.66** Tests are performed on a 1:5 scale boat model. What must be the kinematic viscosity of the model fluid if friction and wave drag phenomena are to be correctly modeled? The full size boat will be used in a freshwater lake where the average water temperature is 10°C.

**7.67** Your favorite professor likes mountain climbing, so there is always a possibility that the professor may fall into a crevasse in some glacier. If that happened today, and the professor was trapped in a slowly moving glacier, you are curious to know whether the professor would reappear at the downstream drop-off of the glacier during this academic year. Assuming ice is a Newtonian fluid with the density of glycerine but a million times as viscous, you decide to build a glycerin model and use dimensional analysis and similarity to estimate when the professor would reappear. Assume the real glacier is 15 m deep and is on a slope that falls 1.5 m in a horizontal distance of 1850 m. Develop the dimensionless parameters and conditions expected to govern dynamic similarity in this problem. If the model professor reappears in the laboratory after 9.6 hours, when should you return to the end of the real glacier to provide help to your favorite professor?

**7.68** An automobile is to travel through standard air at 60 mph. To determine the pressure distribution, a  $\frac{1}{5}$ -scale model is to be tested in water. What factors must be considered to ensure kinematic similarity in the tests? Determine the water speed that should be used. What is the corresponding ratio of drag force between prototype and model flows? The lowest pressure coefficient is  $C_p = -1.4$  at the location of the minimum static pressure on

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the surface. Estimate the minimum tunnel pressure required to avoid cavitation, if the onset of cavitation occurs at a cavitation number of 0.5.

**7.69** A 1:30 scale model of a submarine is to be tested in a towing tank under two conditions: motion at the free surface and motion far below the free surface. The tests are performed in fresh water. On the surface, the submarine cruises at 20 knots. At what speed should the model be towed to ensure dynamic similarity? Far below the surface, the sub cruises at 0.5 knots. At what speed should the model be towed to ensure dynamic similarity? What must the drag of the model be multiplied by under each condition to give the drag of the full-scale submarine?

**7.70** Consider water flow around a circular cylinder, of diameter  $D$  and length  $l$ . In addition to geometry, the drag force is known to depend on liquid speed,  $V$ , density,  $\rho$ , and viscosity,  $\mu$ . Express drag force,  $F_D$ , in dimensionless form as a function of all relevant variables. The static pressure distribution on a circular cylinder, measured in the laboratory, can be expressed in terms of the dimensionless pressure coefficient; the lowest pressure coefficient is  $C_p = -2.4$  at the location of the minimum static pressure on the cylinder surface. Estimate the maximum speed at which a cylinder could be towed in water at atmospheric pressure, without causing cavitation, if the onset of cavitation occurs at a cavitation number of 0.5.

**7.71** A  $\frac{1}{10}$ -scale model of a tractor-trailer rig is tested in a wind tunnel. The model frontal area is  $A_m = 0.1 \text{ m}^2$ . When tested at  $V_m = 75 \text{ m/s}$  in standard air, the measured drag force is  $F_D = 350 \text{ N}$ . Evaluate the drag coefficient for the model conditions given. Assuming that the drag coefficient is the same for model and prototype, calculate the drag force on a prototype rig at a highway speed of 90 km/hr. Determine the air speed at which a model should be tested to ensure dynamically similar results if the prototype speed is 90 km/hr. Is this air speed practical? Why or why not?

**7.72** A circular container, partially filled with water, is rotated about its axis at constant angular speed,  $\omega$ . At any time,  $\tau$ , from the start of rotation, the speed,  $V_\theta$ , at distance  $r$  from the axis of rotation, was found to be a function of  $\tau$ ,  $\omega$ , and the properties of the liquid. Write the dimensionless parameters that characterize this problem. If, in another experiment, honey is rotated in the same cylinder at the same angular speed, determine from your dimensionless parameters whether honey will attain steady motion as quickly as water. Explain why the Reynolds number would not be an important dimensionless parameter in scaling the steady-state motion of liquid in the container.

**7.73** It is recommended in [9] that the frontal area of a model be less than 5 percent of the wind tunnel test section area and  $Re = Vw/\nu > 2 \times 10^6$ , where  $w$  is the model width. Further, the model height must be less than 30 percent of the test section height, and the maximum projected width of the model at maximum yaw ( $20^\circ$ ) must be less than 30 percent of the test section width. The maximum air speed should be less than 300 ft/s to avoid compressibility effects. A model of a tractor-trailer rig is to be tested in a wind tunnel that has a test section 1.5 ft high and 2 ft wide. The height, width, and length of the full-scale rig are 13 ft 6 in., 8 ft, and 65 ft, respectively. Evaluate the scale ratio of the largest model that meets the recommended criteria. Assess whether an adequate Reynolds number can be achieved in this test facility.

**7.74** The power,  $\mathcal{P}$ , required to drive a fan is assumed to depend on fluid density,  $\rho$ , volume flow rate,  $Q$ , impeller diameter,  $D$ , and angular speed,  $\omega$ . If a fan with  $D_1 = 200 \text{ mm}$  delivers  $Q_1 = 0.4 \text{ m}^3/\text{s}$  of air at  $\omega_1 = 2400 \text{ rpm}$ , what volume flow rate could be expected for a geometrically similar fan with  $D_2 = 400 \text{ mm}$  at  $\omega_2 = 1850 \text{ rpm}$ ?

**7.75** Over a certain range of air speeds,  $V$ , the lift,  $F_L$ , produced by a model of a complete aircraft in a wind tunnel depends on the air speed, air density,  $\rho$ , and a characteristic length (the wing base chord length,  $c = 150 \text{ mm}$ ). The following experimental data is obtained for air at standard atmospheric conditions:

$V \text{ (m/s)}$	10	15	20	25	30	35	40	45	50
$F_L \text{ (N)}$	2.2	4.8	8.7	13.3	19.6	26.5	34.5	43.8	54

Plot the lift versus speed curve. By using *Excel* to perform a trendline analysis on this curve, generate and plot data for the lift produced by the prototype, which has a wing base chord length of 5 m, over a speed range of 75 m/s to 250 m/s.

**7.76** The pressure rise,  $\Delta p$ , of a liquid flowing steadily through a centrifugal pump depends on pump diameter,  $D$ , angular speed of the rotor,  $\omega$ , volume flow rate,  $Q$ , and density,  $\rho$ . The table gives data for the prototype and for a geometrically similar model pump. For conditions corresponding to dynamic similarity between the model and prototype pumps, calculate the missing values in the table.

Variable	Prototype	Model
$\Delta p$		29.3 kPa
$Q$	1.25 m <sup>3</sup> /min	
$\rho$	800 kg/m <sup>3</sup>	999 kg/m <sup>3</sup>
$\omega$	183 rad/s	367 rad/s
$D$	150 mm	50 mm

**7.77** Tests are performed on a 1-m long ship model in a water tank. Results obtained (after doing some data analysis) are as follows:

$V \text{ (m/s)}$	3	6	9	12	15	18	20
$D_{\text{Wave}} \text{ (N)}$	0	0.125	0.5	1.5	3	4	5.5
$D_{\text{Friction}} \text{ (N)}$	0.1	0.35	0.75	1.25	2	2.75	3.25

The assumption is that wave drag modeling is done using the Froude number, and friction drag by the Reynolds number. The full size ship will be 50 m long when built. Estimate the total drag when it is cruising at 15 knots, and at 20 knots, in a freshwater lake.

**7.78** A centrifugal water pump running at speed  $\omega = 750 \text{ rpm}$  has the following data for flow rate  $Q$  and pressure head  $\Delta p$ :

$Q \text{ (m}^3/\text{hr)}$	0	100	150	200	250	300	325	350
$\Delta p \text{ (kPa)}$	361	349	328	293	230	145	114	59

The pressure head  $\Delta p$  is a function of flow rate,  $Q$ , and speed,  $\omega$ , and also impeller diameter,  $D$ , and water density,  $\rho$ . Plot the pressure head versus flow rate curve. Find the two  $\Pi$  parameters for this problem, and from the above data plot one against the other. By using *Excel* to perform a trendline analysis on this latter curve, generate and plot data for pressure head versus flow rate for impeller speeds of 500 rpm and 1000 rpm.