Particle acceleration components in cylindrical coordinates:	$a_{r_p} = V_r \frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} - \frac{V_\theta^2}{r} + V_z \frac{\partial V_r}{\partial z} + \frac{\partial V_r}{\partial t}$ (5.12a) $a_{\theta_p} = V_r \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{V_r V_\theta}{r} + V_z \frac{\partial V_\theta}{\partial z} + \frac{\partial V_\theta}{\partial t}$ (5.12b) $a_{z_p} = V_r \frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z \frac{\partial V_z}{\partial z} + \frac{\partial V_z}{\partial t}$ (5.12c)	Page 177
Navier–Stokes equations (incompressible, constant viscosity):	$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$ (5.27a) $\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$ (5.27b) $\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$ (5.27c)	Page 189

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PROBLEMS

5.1 Which of the following sets of equations represent possible two-dimensional incompressible flow cases?

a. $u = 2x^2 + y^2 - x^2y;\ v = x^3 + x(y^2 - 2y)$
b. $u = 2xy - x^2 + y;\ v = 2xy - y^2 + x^2$
c. $u = xt + 2y;\ v = xt^2 - yt$
d. $u = (x + 2y)xt;\ v = -(2x + y)yt$

5.2 Which of the following sets of equations represent possible three-dimensional incompressible flow cases?

a. $u = y^2 + 2xz;\ v = -2yz + x^2yz;\ w = \frac{1}{2}x^2z^2 + x^3y^4$
b. $u = xyzt;\ v = -xyzt^2;\ w = (z^2/2)(xt^2 - yt)$
c. $u = x^2 + y + z^2;\ v = x - y + z;\ w = -2xz + y^2 + z$

5.3 The three components of velocity in a velocity field are given by $u = Ax + By + Cz$, $v = Dx + Ey + Fz$, and $w = Gx + Hy + Jz$. Determine the relationship among the coefficients A through J that is necessary if this is to be a possible incompressible flow field.

5.4 For a flow in the xy plane, the x component of velocity is given by $u = Ax(y - B)$, where $A = 1\ \text{ft}^{-1} \cdot \text{s}^{-1}$, $B = 6$ ft, and x and y are measured in feet. Find a possible y component for steady, incompressible flow. Is it also valid for unsteady, incompressible flow? Why? How many y components are possible?

5.5 For a flow in the xy plane, the x component of velocity is given by $u = x^3 - 3xy^2$. Determine a possible y component for steady, incompressible flow. Is it also valid for unsteady, incompressible flow? Why? How many possible y components are there?

5.6 The x component of velocity in a steady, incompressible flow field in the xy plane is $u = A/x$, where $A = 2\ \text{m}^2/\text{s}$, and x is measured in meters. Find the simplest y component of velocity for this flow field.

5.7 The y component of velocity in a steady, incompressible flow field in the xy plane is $v = Axy(y^2 - x^2)$, where $A = 2$

$m^{-3} \cdot s^{-1}$ and x and y are measured in meters. Find the simplest x component of velocity for this flow field.

5.8 The x component of velocity in a steady incompressible flow field in the xy plane is $u = Ae^{x/b}\cos(y/b)$, where $A = 10$ m/s, $b = 5$ m, and x and y are measured in meters. Find the simplest y component of velocity for this flow field.

5.9 The y component of velocity in a steady incompressible flow field in the xy plane is

$$v = \frac{2xy}{(x^2+y^2)^2}$$

Show that the simplest expression for the x component of velocity is

$$u = \frac{1}{(x^2+y^2)} - \frac{2y^2}{(x^2+y^2)^2}$$

5.10 A crude approximation for the x component of velocity in an incompressible laminar boundary layer is a linear variation from $u = 0$ at the surface ($y = 0$) to the freestream velocity, U, at the boundary-layer edge ($y = \delta$). The equation for the profile is $u = Uy/\delta$, where $\delta = cx^{1/2}$ and c is a constant. Show that the simplest expression for the y component of velocity is $v = uy/4x$. Evaluate the maximum value of the ratio v/U, at a location where $x = 0.5$ m and $\delta = 5$ mm.

5.11 A useful approximation for the x component of velocity in an incompressible laminar boundary layer is a parabolic variation from $u = 0$ at the surface ($y = 0$) to the freestream velocity, U, at the edge of the boundary layer ($y = \delta$). The equation for the profile is $u/U = 2(y/\delta) - (y/\delta)^2$, where $\delta = cx^{1/2}$ and c is a constant. Show that the simplest expression for the y component of velocity is

$$\frac{v}{U} = \frac{\delta}{x}\left[\frac{1}{2}\left(\frac{y}{\delta}\right)^2 - \frac{1}{3}\left(\frac{y}{\delta}\right)^3\right]$$

Plot v/U versus y/δ to find the location of the maximum value of the ratio v/U. Evaluate the ratio where $\delta = 5$ mm and $x = 0.5$ m.

5.12 A useful approximation for the x component of velocity in an incompressible laminar boundary layer is a sinusoidal variation from $u = 0$ at the surface ($y = 0$) to the freestream velocity, U, at the edge of the boundary layer ($y = \delta$). The equation for the profile is $u = U\sin(\pi y/2\delta)$, where $\delta = cx^{1/2}$ and c is a constant. Show that the simplest expression for the y component of velocity is

$$\frac{v}{U} = \frac{1}{\pi}\frac{\delta}{\chi}\left[\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right) + \left(\frac{\pi}{2}\frac{y}{\delta}\right)\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right) - 1\right]$$

Plot u/U and v/U versus y/δ, and find the location of the maximum value of the ratio v/U. Evaluate the ratio where $x = 0.5$ m and $\delta = 5$ mm.

5.13 A useful approximation for the x component of velocity in an incompressible laminar boundary layer is a cubic variation from $u = 0$ at the surface ($y = 0$) to the freestream velocity, U, at the edge of the boundary layer ($y = \delta$). The equation for the profile is $u/U = \frac{3}{2}(y/\delta) - \frac{1}{2}(y/\delta)^3$, where $\delta = cx^{1/2}$ and c is a constant. Derive the simplest expression for v/U, the y component of velocity ratio. Plot u/U and v/U versus y/δ, and find the location of the maximum value of the ratio v/U. Evaluate the ratio where $\delta = 5$ mm and $x = 0.5$ m.

5.14 The y component of velocity in a steady, incompressible flow field in the xy plane is $v = -Bxy^3$, where $B = 0.2\ m^{-3} \cdot s^{-1}$, and x and y are measured in meters. Find the simplest x component of velocity for this flow field. Find the equation of the streamlines for this flow. Plot the streamlines through points (1, 4) and (2, 4).

5.15 For a flow in the xy plane, the x component of velocity is given by $u = Ax^2y^2$, where $A = 0.3\ m^{-3} \cdot s^{-1}$, and x and y are measured in meters. Find a possible y component for steady, incompressible flow. Is it also valid for unsteady, incompressible flow? Why? How many possible y components are there? Determine the equation of the streamline for the simplest y component of velocity. Plot the streamlines through points (1, 4) and (2, 4).

5.16 Derive the differential form of conservation of mass in rectangular coordinates by expanding the *products* of density and the velocity components, ρu, ρv, and ρw, in a Taylor series about a point O. Show that the result is identical to Eq. 5.1a.

5.17 Consider a water stream from a jet of an oscillating lawn sprinkler. Describe the corresponding pathline and streakline.

5.18 Which of the following sets of equations represent possible incompressible flow cases?

a. $V_r = U\cos\theta$; $V_\theta = -U\sin\theta$
b. $V_r = -q/2\pi r$; $V_\theta = K/2\pi r$
c. $V_r = U\cos\theta\,[1-(a/r)^2]$; $V_\theta = -U\sin\theta[1+(a/r)^2]$

5.19 For an incompressible flow in the $r\theta$ plane, the r component of velocity is given as $V_r = -\Lambda\cos\theta/r^2$. Determine a possible θ component of velocity. How many possible θ components are there?

5.20 A viscous liquid is sheared between two parallel disks of radius R, one of which rotates while the other is fixed. The velocity field is purely tangential, and the velocity varies linearly with z from $V_\theta = 0$ at $z = 0$ (the fixed disk) to the velocity of the rotating disk at its surface ($z = h$). Derive an expression for the velocity field between the disks.

5.21 Evaluate $\nabla \cdot \rho\vec{V}$ in cylindrical coordinates. Use the definition of ∇ in cylindrical coordinates. Substitute the velocity vector and perform the indicated operations, using the hint in footnote 1 on page 169. Collect terms and simplify; show that the result is identical to Eq. 5.2c.

5.22 A velocity field in cylindrical coordinates is given as $\vec{V} = \hat{e}_r A/r + \hat{e}_\theta B/r$, where A and B are constants with dimensions of m^2/s. Does this represent a possible incompressible flow? Sketch the streamline that passes through the point $r_0 = 1$ m, $\theta = 90°$ if $A = B = 1\ m^2/s$, if $A = 1\ m^2/s$ and $B = 0$, and if $B = 1\ m^2/s$ and $A = 0$.

***5.23** The velocity field for the viscometric flow of Example 5.7 is $\vec{V} = U(y/h)\hat{i}$. Find the stream function for this flow. Locate the streamline that divides the total flow rate into two equal parts.

***5.24** Determine the family of stream functions ψ that will yield the velocity field $\vec{V} = y(2x+1)\hat{i} + [x(x+1) - y^2]\hat{j}$.

***5.25** The stream function for a certain incompressible flow field is given by the expression $\psi = -Ur\sin\theta + q\theta/2\pi$. Obtain an expression for the velocity field. Find the stagnation point(s) where $|\vec{V}| = 0$, and show that $\psi = 0$ there.

*These problems require material from sections that may be omitted without loss of continuity in the text material.

*5.26 Does the velocity field of Problem 5.22 represent a possible incompressible flow case? If so, evaluate and sketch the stream function for the flow. If not, evaluate the rate of change of density in the flow field.

*5.27 Consider a flow with velocity components $u = 0$, $v = y(y^2 - 3z^2)$, and $w = z(z^2 - 3y^2)$.

a. Is this a one-, two-, or three-dimensional flow?
b. Demonstrate whether this is an incompressible or compressible flow.
c. If possible, derive a stream function for this flow.

*5.28 An incompressible frictionless flow field is specified by the stream function $\psi = -2Ax - 5Ay$, where $A = 1$ m/s, and x and y are coordinates in meters. Sketch the streamlines $\psi = 0$ and $\psi = 5$. Indicate the direction of the velocity vector at the point (0, 0) on the sketch. Determine the magnitude of the flow rate between the streamlines passing through the points (2, 2) and (4, 1).

*5.29 In a parallel one-dimensional flow in the positive x direction, the velocity varies linearly from zero at $y = 0$ to 30 m/s at $y = 1.5$ m. Determine an expression for the stream function, ψ. Also determine the y coordinate above which the volume flow rate is half the total between $y = 0$ and $y = 1.5$ m.

*5.30 A linear velocity profile was used to model flow in a laminar incompressible boundary layer in Problem 5.10. Derive the stream function for this flow field. Locate streamlines at one-quarter and one-half the total volume flow rate in the boundary layer.

*5.31 A parabolic velocity profile was used to model flow in a laminar incompressible boundary layer in Problem 5.11. Derive the stream function for this flow field. Locate streamlines at one-quarter and one-half the total volume flow rate in the boundary layer.

*5.32 Derive the stream function that represents the sinusoidal approximation used to model the x component of velocity for the boundary layer of Problem 5.12. Locate streamlines at one-quarter and one-half the total volume flow rate in the boundary layer.

*5.33 A cubic velocity profile was used to model flow in a laminar incompressible boundary layer in Problem 5.13. Derive the stream function for this flow field. Locate streamlines at one-quarter and one-half the total volume flow rate in the boundary layer.

*5.34 A rigid-body motion was modeled in Example 5.6 by the velocity field $\vec{V} = r\omega\hat{e}_\theta$. Find the stream function for this flow. Evaluate the volume flow rate per unit depth between $r_1 = 0.10$ m and $r_2 = 0.12$ m, if $\omega = 0.5$ rad/s. Sketch the velocity profile along a line of constant θ. Check the flow rate calculated from the stream function by integrating the velocity profile along this line.

*5.35 Example 5.6 showed that the velocity field for a free vortex in the $r\theta$ plane is $\vec{V} = \hat{e}_\theta C/r$. Find the stream function for this flow. Evaluate the volume flow rate per unit depth between $r_1 = 0.10$ m and $r_2 = 0.12$ m, if $C = 0.5$ m²/s. Sketch the velocity profile along a line of constant θ. Check the flow rate calculated from the stream function by integrating the velocity profile along this line.

5.36 Consider the velocity field $\vec{V} = A(x^4 - 6x^2y^2 + y^4)\hat{i} + A(4xy^3 - 4x^3y)\hat{j}$ in the xy plane, where $A = 0.25\ \text{m}^{-3}\cdot\text{s}^{-1}$, and the coordinates are measured in meters. Is this a possible incompressible flow field? Calculate the acceleration of a fluid particle at point $(x,y) = (2, 1)$.

5.37 Consider the flow field given by $\vec{V} = xy^2\hat{i} - \frac{1}{3}y^3\hat{j} + xy\hat{k}$. Determine (a) the number of dimensions of the flow, (b) if it is a possible incompressible flow, and (c) the acceleration of a fluid particle at point $(x, y, z) = (1, 2, 3)$.

5.38 Consider the flow field given by $\vec{V} = ax^2y\hat{i} - by\hat{j} + cz^2\hat{k}$, where $a = 1\ \text{m}^{-2}\cdot\text{s}^{-1}$, $b = 3\ \text{s}^{-1}$, and $c = 2\ \text{m}^{-1}\cdot\text{s}^{-1}$. Determine (a) the number of dimensions of the flow, (b) if it is a possible incompressible flow, and (c) the acceleration of a fluid particle at point $(x, y, z) = (3, 1, 2)$.

5.39 The velocity field within a laminar boundary layer is approximated by the expression

$$\vec{V} = \frac{AUy}{x^{1/2}}\hat{i} + \frac{AUy^2}{4x^{3/2}}\hat{j}$$

In this expression, $A = 141\ \text{m}^{-1/2}$, and $U = 0.240$ m/s is the freestream velocity. Show that this velocity field represents a possible incompressible flow. Calculate the acceleration of a fluid particle at point $(x, y) = (0.5\ \text{m}, 5\ \text{mm})$. Determine the slope of the streamline through the point.

5.40 The x component of velocity in a steady, incompressible flow field in the xy plane is $u = A(x^5 - 10x^3y^2 + 5xy^4)$, where $A = 2\ \text{m}^{-4}\cdot\text{s}^{-1}$ and x is measured in meters. Find the simplest y component of velocity for this flow field. Evaluate the acceleration of a fluid particle at point $(x, y) = (1, 3)$.

5.41 Consider the velocity field $\vec{V} = Ax/(x^2 + y^2)\hat{i} + Ay/(x^2 + y^2)\hat{j}$ in the xy plane, where $A = 10$ m²/s, and x and y are measured in meters. Is this an incompressible flow field? Derive an expression for the fluid acceleration. Evaluate the velocity and acceleration along the x axis, the y axis, and along a line defined by $y = x$. What can you conclude about this flow field?

5.42 The y component of velocity in a two-dimensional, incompressible flow field is given by $v = -Axy$, where v is in m/s, x and y are in meters, and A is a dimensional constant. There is no velocity component or variation in the z direction. Determine the dimensions of the constant, A. Find the simplest x component of velocity in this flow field. Calculate the acceleration of a fluid particle at point $(x, y) = (1, 2)$.

5.43 An incompressible liquid with negligible viscosity flows steadily through a horizontal pipe of constant diameter. In a porous section of length $L = 0.3$ m, liquid is removed at a constant rate per unit length, so the uniform axial velocity in the pipe is $u(x) = U(1 - x/2L)$, where $U = 5$ m/s. Develop an expression for the acceleration of a fluid particle along the centerline of the porous section.

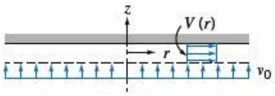
5.44 An incompressible liquid with negligible viscosity flows steadily through a horizontal pipe. The pipe diameter linearly varies from a diameter of 10 cm to a diameter of 2.5 cm over a length of 2 m. Develop an expression for the acceleration of a fluid particle along the pipe centerline. Plot the centerline velocity and acceleration versus position along the pipe, if the inlet centerline velocity is 1 m/s.

5.45 Solve Problem 4.118 to show that the radial velocity in the narrow gap is $V_r = Q/2\pi rh$. Derive an expression for the acceleration of a fluid particle in the gap.

5.46 Consider the low-speed flow of air between parallel disks as shown. Assume that the flow is incompressible and inviscid,

*These problems require material from sections that may be omitted without loss of continuity in the text material.

5.60 Air flows into the narrow gap, of height h, between closely spaced parallel disks through a porous surface as shown. Use a control volume, with outer surface located at position r, to show that the uniform velocity in the r direction is $V = v_0 r/2h$. Find an expression for the velocity component in the z direction ($v_0 \ll V$). Evaluate the components of acceleration for a fluid particle in the gap.



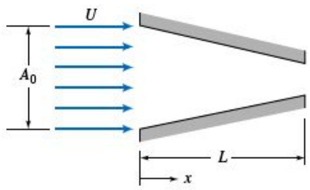
P5.60

5.61 The velocity field for steady inviscid flow from left to right over a circular cylinder, of radius R, is given by

$$\vec{V} = U\cos\theta\left[1-\left(\frac{R}{r}\right)^2\right]\hat{e}_r - U\sin\theta\left[1+\left(\frac{R}{r}\right)^2\right]\hat{e}_\theta$$

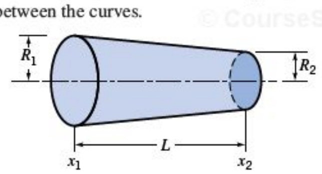
Obtain expressions for the acceleration of a fluid particle moving along the stagnation streamline ($\theta = \pi$) and for the acceleration along the cylinder surface ($r = R$). Plot a_r as a function of r/R for $\theta = \pi$, and as a function of θ for $r = R$; plot a_θ as a function of θ for $r = R$. Comment on the plots. Determine the locations at which these accelerations reach maximum and minimum values.

5.62 Consider the incompressible flow of a fluid through a nozzle as shown. The area of the nozzle is given by $A = A_0(1 - bx)$ and the inlet velocity varies according to $U = U_0(1 - e^{-\lambda t})$, where $A_0 = 0.5\ \text{m}^2$, $L = 5$ m, $b = 0.1\ \text{m}^{-1}$, $\lambda = 0.2\ \text{s}^{-1}$, and $U_0 = 5$ m/s. Find and plot the acceleration on the centerline, with time as a parameter.



P5.62

5.63 Consider the one-dimensional, incompressible flow through the circular channel shown. The velocity at section ① is given by $U = U_0 + U_1 \sin \omega t$, where $U_0 = 20$ m/s, $U_1 = 2$ m/s, and $\omega = 0.3$ rad/s. The channel dimensions are $L = 1$ m, $R_1 = 0.2$ m, and $R_2 = 0.1$ m. Determine the particle acceleration at the channel exit. Plot the results as a function of time over a complete cycle. On the same plot, show the acceleration at the channel exit if the channel is constant area, rather than convergent, and explain the difference between the curves.



P5.63

5.64 Consider again the steady, two-dimensional velocity field of Problem 5.53. Obtain expressions for the particle coordinates, $x_p = f_1(t)$ and $y_p = f_2(t)$, as functions of time and the initial particle position, (x_0, y_0) at $t = 0$. Determine the time required for a particle to travel from initial position, $(x_0, y_0) = (\frac{1}{2}, 2)$ to positions $(x, y) = (1, 1)$ and $(2, \frac{1}{2})$. Compare the particle accelerations determined by differentiating $f_1(t)$ and $f_2(t)$ with those obtained in Problem 5.53.

5.65 Expand $(\vec{V} \cdot \nabla)\vec{V}$ in cylindrical coordinates by direct substitution of the velocity vector to obtain the convective acceleration of a fluid particle. (Recall the hint in footnote 1 on page 169.) Verify the results given in Eqs. 5.12.

5.66 Which, if any, of the flow fields of Problem 5.1 are irrotational?

5.67 A flow is represented by the velocity field $\vec{V} = (x^7 - 21x^5y^2 + 35x^3y^4 - 7xy^6)\hat{i} + (7x^6y - 35x^4y^3 + 21x^2y^5 - y^7)\hat{j}$. Determine if the field is (a) a possible incompressible flow and (b) irrotational.

5.68 Consider again the sinusoidal velocity profile used to model the x component of velocity for a boundary layer in Problem 5.12. Neglect the vertical component of velocity. Evaluate the circulation around the contour bounded by $x = 0.4$ m, $x = 0.6$ m, $y = 0$, and $y = 8$ mm. What would be the results of this evaluation if it were performed 0.2 m further downstream? Assume $U = 0.5$ m/s.

5.69 Consider the velocity field for flow in a rectangular "corner," $\vec{V} = Ax\hat{i} - Ay\hat{j}$, with $A = 0.3\ \text{s}^{-1}$, as in Example 5.8. Evaluate the circulation about the unit square of Example 5.8.

5.70 Consider the two-dimensional flow field in which $u = Axy$ and $v = By^2$, where $A = 1\ \text{m}^{-1} \cdot \text{s}^{-1}$, $B = -\frac{1}{2}\text{m}^{-1} \cdot \text{s}^{-1}$, and the coordinates are measured in meters. Show that the velocity field represents a possible incompressible flow. Determine the rotation at point $(x, y) = (1, 1)$. Evaluate the circulation about the "curve" bounded by $y = 0$, $x = 1$, $y = 1$, and $x = 0$.

***5.71** Consider the flow field represented by the stream function $\psi = x^6 - 15x^4y^2 + 15x^2y^4 - y^6$. Is this a possible two-dimensional, incompressible flow? Is the flow irrotational?

***5.72** Consider a flow field represented by the stream function $\psi = 3x^5y - 10x^3y^3 + 3xy^5$. Is this a possible two-dimensional incompressible flow? Is the flow irrotational?

***5.73** Consider a flow field represented by the stream function $\psi = -A/2(x^2 + y^2)$, where A = constant. Is this a possible two-dimensional incompressible flow? Is the flow irrotational?

***5.74** Consider a velocity field for motion parallel to the x axis with constant shear. The shear rate is $du/dy = A$, where $A = 0.1\ \text{s}^{-1}$. Obtain an expression for the velocity field, $\vec{V}$. Calculate the rate of rotation. Evaluate the stream function for this flow field.

***5.75** A flow field is represented by the stream function $\psi = x^2 - y^2$. Find the corresponding velocity field. Show that this flow field is irrotational. Plot several streamlines and illustrate the velocity field.

***5.76** Consider the velocity field given by $\vec{V} = Axy\hat{i} + By^2\hat{j}$, where $A = 4\ \text{m}^{-1}\cdot\text{s}^{-1}$, $B = -2\ \text{m}^{-1}\cdot\text{s}^{-1}$, and the coordinates are measured in meters. Determine the fluid rotation. Evaluate the circulation about the "curve" bounded by $y = 0$, $x = 1$, $y = 1$, and x

*These problems require material from sections that may be omitted without loss of continuity in the text material.