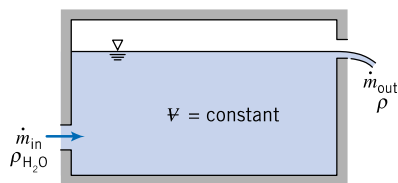


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mixes thoroughly with the brine in the tank. The liquid level in the tank remains constant. Derive expressions for (a) the rate of change of density of the liquid mixture in the tank and (b) the time required for the density to reach the value ρ_f , where $\rho_i > \rho_f > \rho_{H_2O}$.



P4.50

4.51 A conical funnel of half-angle θ drains through a small hole of diameter d at the vertex. The speed of the liquid leaving the funnel is approximately $V = \sqrt{2gy}$, where y is the height of the liquid free surface above the hole. The funnel initially is filled to height y_0 . Obtain an expression for the time, t , required to drain the funnel. Express the result in terms of the initial volume, V_0 , of liquid in the funnel and the initial volume flow rate, $Q_0 = A\sqrt{2gy_0} = AV_0$. If the hole diameter is $d = 5$ mm, plot the time to drain the funnel as a function of y_0 over the range $0.1 \leq y_0 \leq 1$ m with angle θ as a parameter for $15^\circ \leq \theta \leq 45^\circ$.

4.52 Over time, air seeps through pores in the rubber of high-pressure bicycle tires. The saying is that a tire loses pressure at the rate of “a pound [1 psi] a day.” The true rate of pressure loss is not constant; instead, the instantaneous leakage mass flow rate is proportional to the air density in the tire and to the gage pressure in the tire, $\dot{m} \propto \rho p$. Because the leakage rate is slow, air in the tire is nearly isothermal. Consider a tire that initially is inflated to 0.6 MPa (gage). Assume the initial rate of pressure loss is 1 psi per day. Estimate how long it will take for the pressure to drop to 500 kPa. How accurate is “a pound a day” over the entire 30 day period? Plot the pressure as a function of time over the 30 day period. Show the rule-of-thumb results for comparison.

4.53 Evaluate the net rate of flux of momentum out through the control surface of Problem 4.21.

4.54 For the conditions of Problem 4.30, evaluate the ratio of the x -direction momentum flux at the channel outlet to that at the inlet.

4.55 For the conditions of Problem 4.31, evaluate the ratio of the x -direction momentum flux at the pipe outlet to that at the inlet.

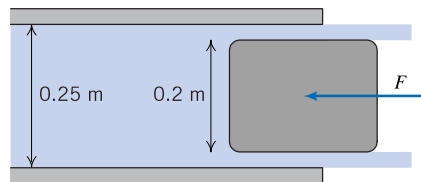
4.56 Evaluate the net momentum flux through the bend of Problem 4.34, if the depth normal to the diagram is $w = 3$ ft.

4.57 Evaluate the net momentum flux through the channel of Problem 4.35. Would you expect the outlet pressure to be higher, lower, or the same as the inlet pressure? Why?

4.58 What force (lbf) will a horizontal 2-in.-diameter stream of water moving at 20 ft/s generate upon hitting a vertical flat plate?

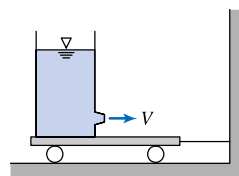
4.59 Considering that in the fully developed region of a pipe, the integral of the axial momentum is the same at all cross sections, explain the reason for the pressure drop along the pipe.

4.60 Find the force required to hold the plug in place at the exit of the water pipe. The flow rate is $1.5 \text{ m}^3/\text{s}$, and the upstream pressure is 3.5 MPa.

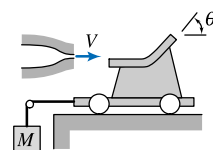


P4.60

4.61 A large tank of height $h = 1$ m and diameter $D = 0.75$ m is affixed to a cart as shown. Water issues from the tank through a nozzle of diameter $d = 15$ mm. The speed of the liquid leaving the tank is approximately $V = \sqrt{2gy}$ where y is the height from the nozzle to the free surface. Determine the tension in the wire when $y = 0.9$ m. Plot the tension in the wire as a function of water depth for $0 \leq y \leq 0.9$ m.



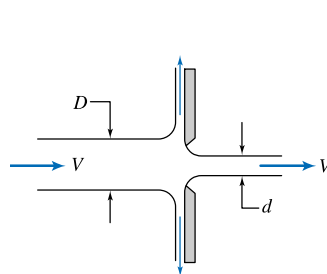
P4.61



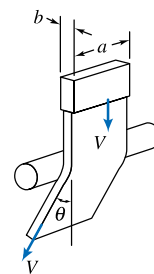
P4.62

4.62 A jet of water issuing from a stationary nozzle at 10 m/s ($A_j = 0.1 \text{ m}^2$) strikes a turning vane mounted on a cart as shown. The vane turns the jet through angle $\theta = 40^\circ$. Determine the value of M required to hold the cart stationary. If the vane angle θ is adjustable, plot the mass, M , needed to hold the cart stationary versus θ for $0 \leq \theta \leq 180^\circ$.

4.63 A vertical plate has a sharp-edged orifice at its center. A water jet of speed V strikes the plate concentrically. Obtain an expression for the external force needed to hold the plate in place, if the jet leaving the orifice also has speed V . Evaluate the force for $V = 15$ ft/s, $D = 4$ in., and $d = 1$ in. Plot the required force as a function of diameter ratio for a suitable range of diameter d .



P4.63

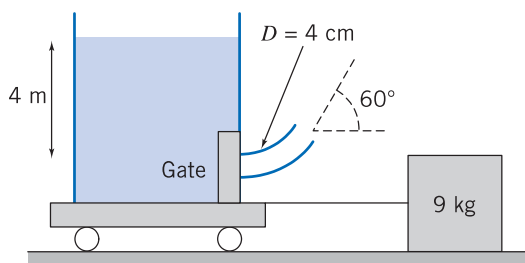


P4.64

4.64 A circular cylinder inserted across a stream of flowing water deflects the stream through angle θ , as shown. (This is termed the “Coanda effect.”) For $a = 12.5$ mm, $b = 2.5$ mm, $V = 3$ m/s, and $\theta = 20^\circ$, determine the horizontal component of the force on the cylinder caused by the flowing water.

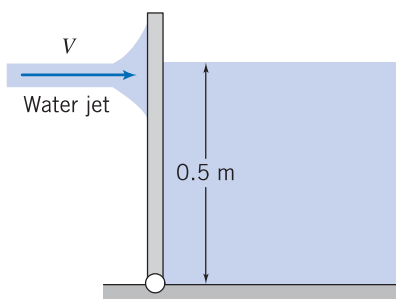
4.65 In a laboratory experiment, the water flow rate is to be measured catching the water as it vertically exits a pipe into an empty open cylindrical (3-ft diameter) tank that is on a zeroed balance. The tank bottom is 5 ft directly below the pipe exit, and the pipe diameter is 2 in. One student obtains a flow rate by noting that after 30 seconds the volume of water (at 50°F) in the tank was 15 ft³. Another student obtains a flow rate by reading the instantaneous weight of 960 lb indicated at the 30-second point. Find the mass flow rate each student computes. Why do they disagree? Which one is more accurate? Show that the magnitude of the discrepancy can be explained by any concept you may have.

4.66 A tank of water sits on a cart with frictionless wheels as shown. The cart is attached using a cable to a 9 kg mass, and the coefficient of static friction of the mass with the ground is 0.5. At time $t = 0$, a second cable is used to remove a gate blocking the tank exit. Will the resulting exit flow be sufficient to start the tank moving? (Assume the water flow is frictionless.)



P4.66

4.67 A gate is 0.5 m wide and 0.6 m tall, and is hinged at the bottom. On one side the gate holds back a 0.5-m deep body of water. On the other side, a 10-cm diameter water jet hits the gate at a height of 0.5 m. What jet speed V is required to hold the gate vertical? What will the speed be if the body of water is lowered to 0.25 m? What will the speed be if the water level is at the top of the gate?



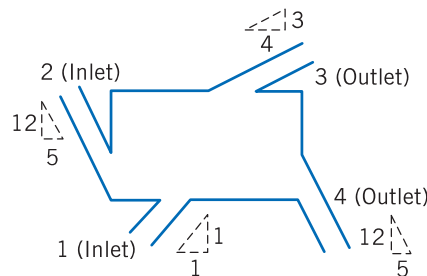
P4.67

4.68 A farmer purchases 675 kg of bulk grain from the local co-op. The grain is loaded into his pickup truck from a hopper with an outlet diameter of 0.3 m. The loading operator determines the payload by observing the indicated gross mass of the truck as a function of time. The grain flow from the hopper ($\dot{m} = 40 \text{ kg/s}$) is terminated when the indicated scale reading reaches the desired gross mass. If the grain density is 600 kg/m^3 , determine the true payload.

4.69 Water flows steadily through a fire hose and nozzle. The hose is 75 mm inside diameter, and the nozzle tip is 25 mm I.D.; water gage pressure in the hose is 510 kPa, and the stream leaving the nozzle is uniform. The exit speed and pressure are 32 m/s and atmospheric, respectively. Find the force transmitted by the

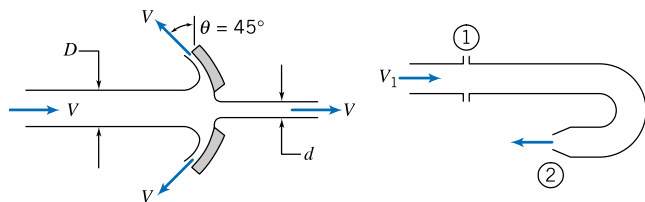
coupling between the nozzle and hose. Indicate whether the coupling is in tension or compression.

4.70 Obtain expressions for the rate of change in mass of the control volume shown, as well as the horizontal and vertical forces required to hold it in place, in terms of $p_1, A_1, V_1, p_2, A_2, V_2, p_3, A_3, V_3, p_4, A_4, V_4$, and the constant density ρ .



P4.70

4.71 A shallow circular dish has a sharp-edged orifice at its center. A water jet, of speed V , strikes the dish concentrically. Obtain an expression for the external force needed to hold the dish in place if the jet issuing from the orifice also has speed V . Evaluate the force for $V = 5 \text{ m/s}$, $D = 100 \text{ mm}$, and $d = 25 \text{ mm}$. Plot the required force as a function of the angle θ ($0 \leq \theta \leq 90^\circ$) with diameter ratio as a parameter for a suitable range of diameter d .



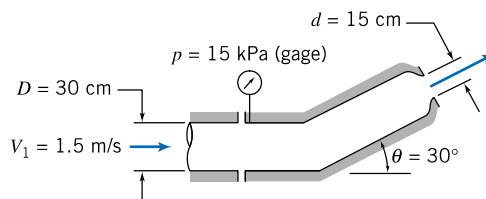
P4.71

P4.72

4.72 Water is flowing steadily through the 180° elbow shown. At the inlet to the elbow the gage pressure is 15 psi. The water discharges to atmospheric pressure. Assume properties are uniform over the inlet and outlet areas: $A_1 = 4 \text{ in.}^2$, $A_2 = 1 \text{ in.}^2$, and $V_1 = 10 \text{ ft/s}$. Find the horizontal component of force required to hold the elbow in place.

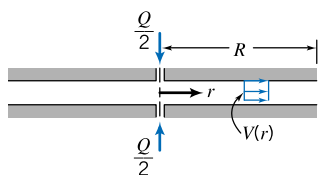
4.73 A 180° elbow takes in water at an average velocity of 0.8 m/s and a pressure of 350 kPa (gage) at the inlet, where the diameter is 0.2 m. The exit pressure is 75 kPa, and the diameter is 0.04 m. What is the force required to hold the elbow in place?

4.74 Water flows steadily through the nozzle shown, discharging to atmosphere. Calculate the horizontal component of force in the flanged joint. Indicate whether the joint is in tension or compression.



P4.74

4.75 Assume the bend of Problem 4.35 is a segment of a larger channel and lies in a horizontal plane. The inlet pressure is

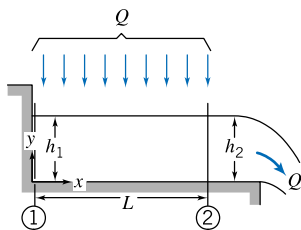


P4.118

***4.118** Incompressible liquid of negligible viscosity is pumped, at total volume flow rate Q , through two small holes into the narrow gap between closely spaced parallel disks as shown. The liquid flowing away from the holes has only radial motion. Assume uniform flow across any vertical section and discharge to atmospheric pressure at $r = R$. Obtain an expression for the pressure variation and plot as a function of radius. *Hint:* Apply conservation of mass and the momentum equation to a differential control volume of thickness dr located at radius r .

***4.119** The narrow gap between two closely spaced circular plates initially is filled with incompressible liquid. At $t = 0$ the upper plate begins to move downward toward the lower plate with constant speed, V_0 , causing the liquid to be squeezed from the narrow gap. Neglecting viscous effects and assuming uniform flow in the radial direction, develop an expression for the velocity field between the parallel plates. *Hint:* Apply conservation of mass to a control volume with outer surface located at radius r . Note that even though the speed of the upper plate is constant, the flow is unsteady.

***4.120** Liquid falls vertically into a short horizontal rectangular open channel of width b . The total volume flow rate, Q , is distributed uniformly over area bL . Neglect viscous effects. Obtain an expression for h_1 in terms of h_2 , Q , and b . *Hint:* Choose a control volume with outer boundary located at $x = L$. Sketch the surface profile, $h(x)$. *Hint:* Use a differential control volume of width dx .

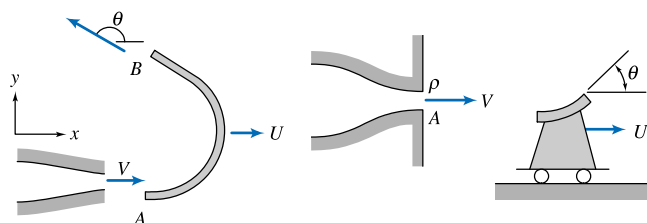


P4.120

***4.121** Design a clepsydra (Egyptian water clock)—a vessel from which water drains by gravity through a hole in the bottom and which indicates time by the level of the remaining water. Specify the dimensions of the vessel and the size of the drain hole; indicate the amount of water needed to fill the vessel and the interval at which it must be filled. Plot the vessel radius as a function of elevation.

4.122 A jet of water is directed against a vane, which could be a blade in a turbine or in any other piece of hydraulic machinery. The water leaves the stationary 40-mm diameter nozzle with a speed of 25 m/s and enters the vane tangent to the surface at A . The inside surface of the vane at B makes angle $\theta = 150^\circ$ with the x direction. Compute the force that must be applied to maintain the vane speed constant at $U = 5$ m/s.

*These problems require material from sections that may be omitted without loss of continuity in the text material.

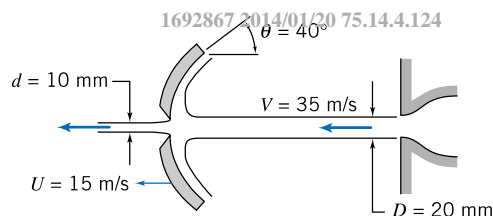


P4.122

P4.123, 4.126, 4.128, 4.140

4.123 Water from a stationary nozzle impinges on a moving vane with turning angle $\theta = 120^\circ$. The vane moves away from the nozzle with constant speed, $U = 10$ m/s, and receives a jet that leaves the nozzle with speed $V = 30$ m/s. The nozzle has an exit area of 0.004 m². Find the force that must be applied to maintain the vane speed constant.

4.124 The circular dish, whose cross section is shown, has an outside diameter of 0.20 m. A water jet with speed of 35 m/s strikes the dish concentrically. The dish moves to the left at 15 m/s. The jet diameter is 20 mm. The dish has a hole at its center that allows a stream of water 10 mm in diameter to pass through without resistance. The remainder of the jet is deflected and flows along the dish. Calculate the force required to maintain the dish motion.



P4.124

4.125 A jet boat takes in water at a constant volumetric rate Q through side vents and ejects it at a high jet speed V_j at the rear. A variable-area exit orifice controls the jet speed. The drag on the boat is given by $F_{\text{drag}} \approx kV^2$, where V is the boat speed. Find an expression for the steady speed V . If a jet speed $V_j = 25$ m/s produces a boat speed of 10 m/s, what jet speed will be required to double the boat speed?

4.126 A jet of oil ($SG = 0.8$) strikes a curved blade that turns the fluid through angle $\theta = 180^\circ$. The jet area is 1200 mm² and its speed relative to the stationary nozzle is 20 m/s. The blade moves toward the nozzle at 10 m/s. Determine the force that must be applied to maintain the blade speed constant.

4.127 The Canadair CL-215T amphibious aircraft is specially designed to fight fires. It is the only production aircraft that can scoop water—1620 gallons in 12 seconds—from any lake, river, or ocean. Determine the added thrust required during water scooping, as a function of aircraft speed, for a reasonable range of speeds.

4.128 Consider a single vane, with turning angle θ , moving horizontally at constant speed, U , under the influence of an impinging jet as in Problem 4.123. The absolute speed of the jet is V . Obtain general expressions for the resultant force and power that the vane could produce. Show that the power is maximized when $U = V/3$.