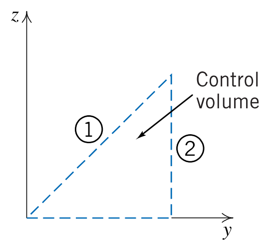
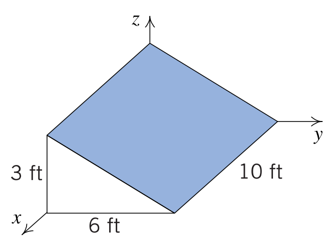
4.10 The velocity field in the region shown is given by $\vec{V} = az\hat{j} + b\hat{k}$, where $a = 10\ \text{s}^{-1}$ and $b = 5$ m/s. For the 1 m × 1 m triangular control volume (depth $w = 1$ m perpendicular to the diagram), an element of area ① may be represented by $w(-dz\hat{j} + dy\hat{k})$ and an element of area ② by $wdz\hat{j}$.

a. Find an expression for $\vec{V} \cdot d\vec{A}_1$.
b. Evaluate $\int_{A_1} \vec{V} \cdot d\vec{A}_1$.
c. Find an expression for $\vec{V} \cdot d\vec{A}_2$.
d. Find an expression for $\vec{V}(\vec{V} \cdot d\vec{A}_2)$.
e. Evaluate $\int_{A_2} \vec{V}(\vec{V} \cdot d\vec{A}_2)$.



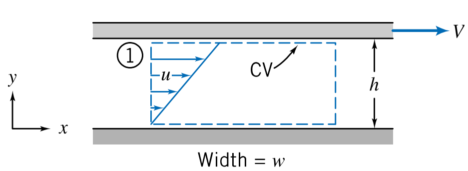
P4.10



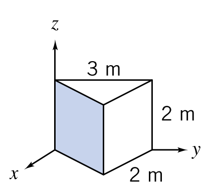
P4.11

4.11 The shaded area shown is in a flow where the velocity field is given by $\vec{V} = ax\hat{i} - by\hat{j}$; $a = b = 1\ \text{s}^{-1}$, and the coordinates are measured in meters. Evaluate the volume flow rate and the momentum flux through the shaded area.

4.12 Obtain expressions for the volume flow rate and the momentum flux through cross section ① of the control volume shown in the diagram.



P4.12



P4.13

4.13 The area shown shaded is in a flow where the velocity field is given by $\vec{V} = -ax\hat{i} + by\hat{j} + c\hat{k}$; $a = b = 2\ \text{s}^{-1}$ and $c = 2.5$ m/s. Write a vector expression for an element of the shaded area. Evaluate the integrals $\int \vec{V} \cdot d\vec{A}$ and $\int \vec{V}(\vec{V} \cdot d\vec{A})$ over the shaded area.

4.14 For the flow of Problem 4.12, obtain an expression for the kinetic energy flux, $\int (V^2/2)\rho\vec{V} \cdot d\vec{A}$, through cross section ① of the control volume shown.

4.15 The velocity distribution for laminar flow in a long circular tube of radius R is given by the one-dimensional expression,

$$\vec{V} = u\hat{i} = u_{\text{max}}\left[1 - \left(\frac{r}{R}\right)^2\right]\hat{i}$$

For this profile obtain expressions for the volume flow rate and the momentum flux through a section normal to the pipe axis.

4.16 For the flow of Problem 4.15, obtain an expression for the kinetic energy flux, $\int (V^2/2)\rho\vec{V} \cdot d\vec{A}$, through a section normal to the pipe axis.

4.17 A farmer is spraying a liquid through 10 nozzles, ⅛th in. ID, at an average exit velocity of 10 ft/s. What is the average velocity in the 1-in. ID head feeder? What is the system flow rate, in gpm?

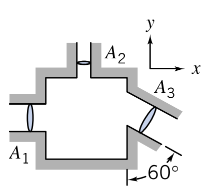
4.18 A cylindrical holding water tank has a 3 m ID, and a height of 3 m. There is one inlet of diameter 10 cm, an exit of diameter 8 cm, and a drain. The tank is initially empty when the inlet pump is turned on, producing an average inlet velocity of 5 m/s. When the level in the tank reaches 0.7 m, the exit pump turns on, causing flow out of the exit; the exit average velocity is 3 m/s. When the water level reaches 2 m the drain is opened such that the level remains at 2 m. Find (a) the time at which the exit pump is switched on, (b) the time at which the drain is opened, and (c) the flow rate into the drain (m^3/min).

4.19 A wet cooling tower cools warm water by spraying it into a forced dry-air flow. Some of the water evaporates in this air and is carried out of the tower into the atmosphere; the evaporation cools the remaining water droplets, which are collected at the exit pipe (6 in. ID) of the tower. Measurements indicate the warm water mass flow rate is 250,000 lb/hr, and the cool water (70°F) flows at an average speed of 5.55 ft/s in the exit pipe. The flow rate of the moist air is to be obtained from measurements of the velocity at four points, each representing 1/4 of the air stream cross-sectional area of 13.2 ft^2. The moist air density is 0.066 lb/ft^3. Find (a) the volume and mass flow rates of the cool water, (b) the mass flow rate of the moist air, and (c) the mass flow rate of the dry air.

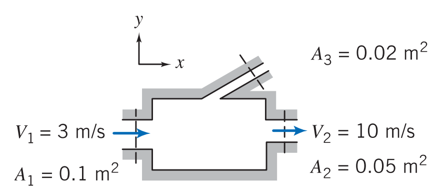
4.20 A university laboratory wishes to build a wind tunnel with variable speeds. Rather than use a variable speed fan, it is proposed to build the tunnel with a sequence of three circular test sections: Section 1 will have a diameter of 5 ft, Section 2 a diameter of 3 ft, and Section 3 a diameter of 2 ft. If the average speed in Section 1 is 20 mph, what will be the speeds in the other two sections? What will be the flow rate (ft^3/min)?

4.21 Fluid with 65 lbm/ft^3 density is flowing steadily through the rectangular box shown. Given $A_1 = 0.5\ \text{ft}^2$, $A_2 = 0.1\ \text{ft}^2$, $A_3 = 0.6\ \text{ft}^2$, $\vec{V}_1 = 10\hat{i}$ ft/s, and $\vec{V}_2 = 20\hat{j}$ ft/s, determine velocity $\vec{V}_3$.

4.22 Consider steady, incompressible flow through the device shown. Determine the magnitude and direction of the volume flow rate through port 3.



P4.21



P4.22

4.23 A rice farmer needs to fill her 5 acre field with water to a depth of 3 in. in 1 hr. How many 6 in. diameter supply pipes are needed if the average velocity in each must be less than 10 ft/s?

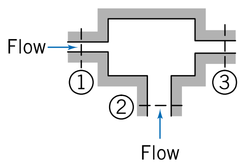
4.24 You are filling your car with gasoline at a rate of 5.3 gals/min. Although you can't see it, the gasoline is rising in the tank at a rate of 4.3 in. per minute. What is the horizontal cross-sectional area of your gas tank? Is this a realistic answer?

4.25 For your sink at home, the flow rate in is 5000 units/hr. Accumulation is 2500 units. What is the accumulation rate if the outflow is 60 units/min? Suddenly, the outflow becomes 13 units/min: What is the accumulation rate? At another time, the flow rate in is 5 units/sec. The accumulation is 50 units. The accumulation rate is −4 units/sec. What is the flow rate out?

4.26 You are trying to pump storm water out of your basement during a storm. The pump can extract 10 gpm. The water level in the basement is now sinking about 1 in./hr. What is the flow rate (gpm) from the storm into the basement? The basement is 25 ft by 20 ft.

4.27 In steady-state flow downstream, the density is 4 lb/ft^3, the velocity is 10 ft/sec, and the area is 1 ft^2. Upstream, the velocity is 15 ft/sec, and the area is 0.25 ft^2. What is the density upstream?

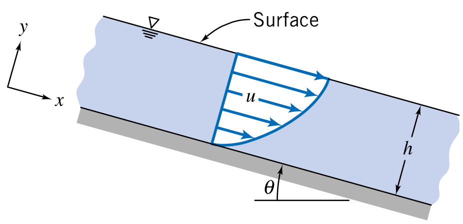
4.28 In the incompressible flow through the device shown, velocities may be considered uniform over the inlet and outlet sections. The following conditions are known: $A_1 = 0.1\ \text{m}^2$, $A_2 = 0.2\ \text{m}^2$, $A_3 = 0.15\ \text{m}^2$, $V_1 = 10e^{-t/2}$ m/s, and $V_2 = 2\cos(2\pi t)$ m/s (t in seconds). Obtain an expression for the velocity at section ③, and plot V_3 as a function of time. At what instant does V_3 first become zero? What is the total mean volumetric flow at section ③?



P4.28

4.29 Oil flows steadily in a thin layer down an inclined plane. The velocity profile is

$$u = \frac{\rho g \sin\theta}{\mu}\left[hy - \frac{y^2}{2}\right]$$



P4.29

Express the mass flow rate per unit width in terms of ρ, μ, g, θ, and h.

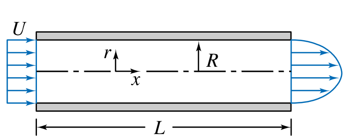
4.30 Water enters a wide, flat channel of height $2h$ with a uniform velocity of 2.5 m/s. At the channel outlet the velocity distribution is given by

$$\frac{u}{u_{\max}} = 1 - \left(\frac{y}{h}\right)^2$$

where y is measured from the centerline of the channel. Determine the exit centerline velocity, $u_{\max}$.

4.31 Water flows steadily through a pipe of length L and radius R = 75 mm. Calculate the uniform inlet velocity, U, if the velocity distribution across the outlet is given by

$$u = u_{\max}\left[1 - \frac{r^2}{R^2}\right]$$



P4.31

and $u_{\max}$ = 3 m/s.

4.32 Incompressible fluid flows steadily through a plane diverging channel. At the inlet, of height H, the flow is uniform with magnitude V_1. At the outlet, of height $2H$, the velocity profile is

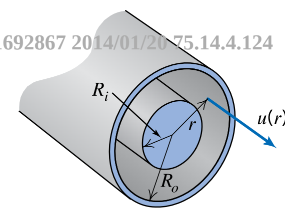
$$V_2 = V_m \cos\left(\frac{\pi y}{2H}\right)$$

where y is measured from the channel centerline. Express V_m in terms of V_1.

4.33 The velocity profile for laminar flow in an annulus is given by

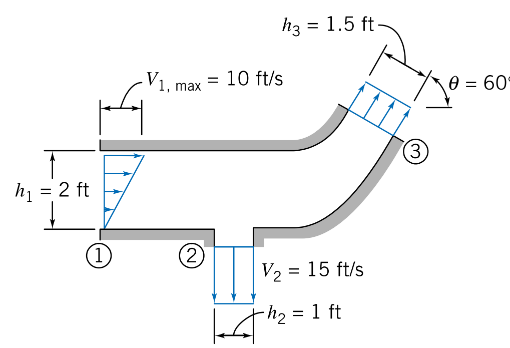
$$u(r) = -\frac{\Delta p}{4\mu L}\left[R_o^2 - r^2 + \frac{R_o^2 - R_i^2}{\ln(R_i/R_o)}\ln\frac{R_o}{r}\right]$$

where $\Delta p/L = -10$ kPa/m is the pressure gradient, μ is the viscosity (SAE 10 oil at 20°C), and R_o = 5 mm and R_i = 1 mm are the outer and inner radii. Find the volume flow rate, the average velocity, and the maximum velocity. Plot the velocity distribution.



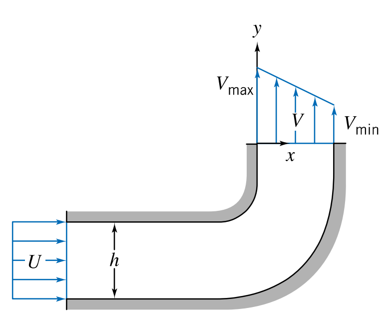
P4.33

4.34 A two-dimensional reducing bend has a linear velocity profile at section ①. The flow is uniform at sections ② and ③. The fluid is incompressible and the flow is steady. Find the magnitude and direction of the uniform velocity at section ③.



P4.34

4.35 Water enters a two-dimensional, square channel of constant width, h = 75.5 mm, with uniform velocity, U. The channel makes a 90° bend that distorts the flow to produce the linear velocity profile shown at the exit, with $v_{\max} = 2\ v_{\min}$. Evaluate $v_{\min}$, if U = 7.5 m/s.



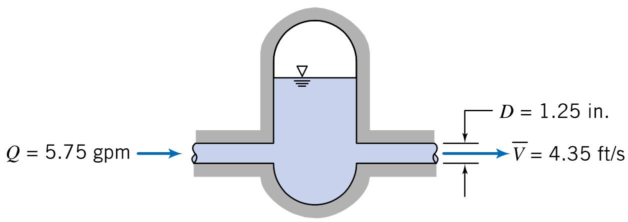
P4.35, 4.75, 4.92

4.36 A porous round tube with $D = 60$ mm carries water. The inlet velocity is uniform with $V_1 = 7.0$ m/s. Water flows radially and axisymmetrically outward through the porous walls with velocity distribution

$$v = V_0\left[1 - \left(\frac{x}{L}\right)^2\right]$$

where $V_0 = 0.03$ m/s and $L = 0.950$ m. Calculate the mass flow rate inside the tube at $x = L$.

4.37 A *hydraulic accumulator* is designed to reduce pressure pulsations in a machine tool hydraulic system. For the instant shown, determine the rate at which the accumulator gains or loses hydraulic oil.



P4.37

4.38 A tank of 0.4 m^3 volume contains compressed air. A valve is opened and air escapes with a velocity of 250 m/s through an opening of 100 mm^2 area. Air temperature passing through the opening is −20°C and the absolute pressure is 300 kPa. Find the rate of change of density of the air in the tank at this moment.

4.39 Viscous liquid from a circular tank, $D = 300$ mm in diameter, drains through a long circular tube of radius $R = 50$ mm. The velocity profile at the tube discharge is

$$u = u_{\text{max}}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Show that the average speed of flow in the drain tube is $\overline{V} = \frac{1}{2}u_{\text{max}}$. Evaluate the rate of change of liquid level in the tank at the instant when $u_{\text{max}} = 0.155$ m/s.

4.40 Air enters a tank through an area of 0.2 ft^2 with a velocity of 15 ft/s and a density of 0.03 slug/ft^3. Air leaves with a velocity of 5 ft/s and a density equal to that in the tank. The initial density of the air in the tank is 0.02 slug/ft^3. The total tank volume is 20 ft^3 and the exit area is 0.4 ft^2. Find the initial rate of change of density in the tank.

4.41 A rectangular tank used to supply water for a Reynolds flow experiment is 230 mm deep. Its width and length are $W = 150$ mm and $L = 230$ mm. Water flows from the outlet tube (inside diameter $D = 6.35$ mm) at Reynolds number $Re = 2000$, when the tank is half full. The supply valve is closed. Find the rate of change of water level in the tank at this instant.

4.42 A cylindrical tank, 0.3 m in diameter, drains through a hole in its bottom. At the instant when the water depth is 0.6 m, the flow rate from the tank is observed to be 4 kg/s. Determine the rate of change of water level at this instant.

4.43 A recent TV news story about lowering Lake Shafer near Monticello, Indiana, by increasing the discharge through the dam that impounds the lake, gave the following information for flow through the dam:

Normal flow rate	290 cfs
Flow rate during draining of lake	2000 cfs

(The flow rate during draining was stated to be equivalent to 16,000 gal/s.) The announcer also said that during draining the lake level was expected to fall at the rate of 1 ft every 8 hr. Calculate the actual flow rate during draining in gal/s. Estimate the surface area of the lake.

4.44 A cylindrical tank, of diameter $D = 50$ mm, drains through an opening, $d = 5$ mm, in the bottom of the tank. The speed of the liquid leaving the tank is approximately $V = \sqrt{2gy}$, where y is the height from the tank bottom to the free surface. If the tank is initially filled with water to $y_0 = 0.4$ m, determine the water depth at $t = 12$ s. Plot y/y_0 versus t with y_0 as a parameter for $0.1 \leq y_0 \leq 1$ m. Plot y/y_0 versus t with D/d as a parameter for $2 \leq D/d \leq 10$ and $y_0 = 0.4$ m.

4.45 For the conditions of Problem 4.44, estimate the time required to drain the tank to depth $y = 20$ mm. Plot time to drain the tank as a function of y/y_0 for $0.1 \leq y_0 \leq 1$ m with d/D as a parameter for $0.1 \leq d/D \leq 0.5$.

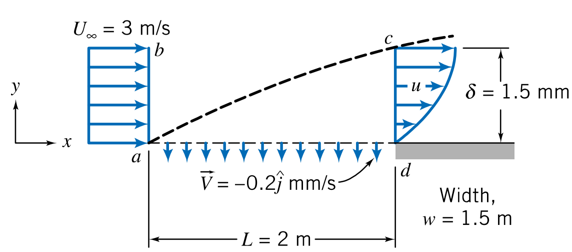
4.46 A conical flask contains water to height $H = 36.8$ mm, where the flask diameter is $D = 29.4$ mm. Water drains out through a smoothly rounded hole of diameter $d = 7.35$ mm at the apex of the cone. The flow speed at the exit is approximately $V = (2gy)^{1/2}$, where y is the height of the liquid free surface above the hole. A stream of water flows into the top of the flask at constant volume flow rate, $Q = 3.75 \times 10^{-7}$ m^3/hr. Find the volume flow rate from the bottom of the flask. Evaluate the direction and rate of change of water surface level in the flask at this instant.

4.47 A conical funnel of half-angle $\theta = 15°$, with maximum diameter $D = 70$ mm and height H, drains through a hole (diameter $d = 3.12$ mm) in its bottom. The speed of the liquid leaving the funnel is approximately $V = (2gy)^{1/2}$, where y is the height of the liquid free surface above the hole. Find the rate of change of surface level in the funnel at the instant when $y = H/2$.

4.48 Water flows steadily past a porous flat plate. Constant suction is applied along the porous section. The velocity profile at section cd is

$$\frac{u}{U_\infty} = 3\left[\frac{y}{\delta}\right] - 2\left[\frac{y}{\delta}\right]^{1.5}$$

Evaluate the mass flow rate across section bc.



P4.48, 4.49

4.49 Consider incompressible steady flow of standard air in a boundary layer on the length of porous surface shown. Assume the boundary layer at the downstream end of the surface has an approximately parabolic velocity profile, $u/U_\infty = 2(y/\delta) - (y/\delta)^2$. Uniform suction is applied along the porous surface, as shown. Calculate the volume flow rate across surface cd, through the porous suction surface, and across surface bc.

4.50 A tank of fixed volume contains brine with initial density, ρ_i, greater than water. Pure water enters the tank steadily and