






meters, obtain an equation for the flow streamlines. Plot several streamlines for positive y .

 **2.4** The velocity field $\vec{V} = ax\hat{i} - by\hat{j}$, where $a = b = 1 \text{ s}^{-1}$, can be interpreted to represent flow in a corner. Find an equation for the flow streamlines. Plot several streamlines in the first quadrant, including the one that passes through the point $(x, y) = (0, 0)$.


 **2.5** A velocity field is given by $\vec{V} = ax\hat{i} - by\hat{j}$, where $a = 1 \text{ s}^{-1}$ and $b = 1 \text{ s}^{-2}$. Find the equation of the streamlines at any time t . Plot several streamlines in the first quadrant at $t = 0 \text{ s}$, $t = 1 \text{ s}$, and $t = 20 \text{ s}$.

 **2.6** A velocity field is specified as $\vec{V} = axy\hat{i} + by^2\hat{j}$, where $a = 2 \text{ m}^{-1}\text{s}^{-1}$, $b = -6 \text{ m}^{-1}\text{s}^{-1}$, and the coordinates are measured in meters. Is the flow field one-, two-, or three-dimensional? Why? Calculate the velocity components at the point $(2, \frac{1}{2})$. Develop an equation for the streamline passing through this point. Plot several streamlines in the first quadrant including the one that passes through the point $(2, \frac{1}{2})$.

 **2.7** A velocity field is given by $\vec{V} = ax^3\hat{i} + bxy^3\hat{j}$, where $a = 1 \text{ m}^{-2}\text{s}^{-1}$ and $b = 1 \text{ m}^{-3}\text{s}^{-1}$. Find the equation of the streamlines. Plot several streamlines in the first quadrant.


 **2.8** A flow is described by the velocity field $\vec{V} = (Ax + B)\hat{i} + (-Ay)\hat{j}$, where $A = 10 \text{ ft/s/ft}$ and $B = 20 \text{ ft/s}$. Plot a few streamlines in the xy plane, including the one that passes through the point $(x, y) = (1, 2)$.

2.9 The velocity for a steady, incompressible flow in the xy plane is given by $\vec{V} = \hat{i}Ax + \hat{j}Ay/x^2$, where $A = 2 \text{ m}^2/\text{s}$, and the coordinates are measured in meters. Obtain an equation for the streamline that passes through the point $(x, y) = (1, 3)$. Calculate the time required for a fluid particle to move from $x = 1 \text{ m}$ to $x = 2 \text{ m}$ in this flow field.

 **2.10** The flow field for an atmospheric flow is given by


$$\vec{V} = -\frac{Ky}{2\pi(x^2 + y^2)}\hat{i} + \frac{Kx}{2\pi(x^2 + y^2)}\hat{j}$$

where $K = 5 \times 10^4 \text{ m}^2/\text{s}$ and the x and y coordinates are parallel to the local latitude and longitude. Plot the velocity magnitude along the x axis, along the y axis, and along the line $y = x$. For each plot use the range $-10 \text{ km} \leq x$ or $y \leq 10 \text{ km}$, excluding $|x|$ or $|y| \leq 100 \text{ m}$. Find the equation for the streamlines and sketch several of them. What does this flow field model?

 **2.11** The flow field for an atmospheric flow is given by

$$\vec{V} = -\frac{My}{2\pi}\hat{i} + \frac{Mx}{2\pi}\hat{j}$$

where $M = 0.5 \text{ s}^{-1}$ and the x and y coordinates are parallel to the local latitude and longitude. Plot the velocity magnitude along the x axis, along the y axis, and along the line $y = x$. For each plot use the range $-10 \text{ km} \leq x$ or $y \leq 10 \text{ km}$, excluding $|x|$ or $|y| \leq 100 \text{ m}$. Find the equation for the streamlines and sketch several of them. What does this flow field model?


 **2.12** A flow field flow is given by


$$\vec{V} = -\frac{qx}{2\pi(x^2 + y^2)}\hat{i} - \frac{qy}{2\pi(x^2 + y^2)}\hat{j}$$


where $q = 2 \times 10^4 \text{ m}^2/\text{s}$. Plot the velocity magnitude along the x axis, along the y axis, and along the line $y = x$. For each plot use the range $-10 \text{ km} \leq x$ or $y \leq 10 \text{ km}$, excluding $|x|$ or $|y| \leq 100 \text{ m}$.


Find the equation for the streamlines and sketch several of them. What does this flow field model?


2.13 Beginning with the velocity field of Problem 2.4, verify that the parametric equations for particle motion are given by $x_p = c_1 e^{at}$ and $y_p = c_2 e^{-bt}$. Obtain the equation for the pathline of the particle located at the point $(x, y) = (1, 2)$ at the instant $t = 0$. Compare this pathline with the streamline through the same point.


 **2.14** A velocity field is given by $\vec{V} = ay\hat{i} - bx\hat{j}$, where $a = 1 \text{ s}^{-2}$ and $b = 4 \text{ s}^{-1}$. Find the equation of the streamlines at any time t . Plot several streamlines at $t = 0 \text{ s}$, $t = 1 \text{ s}$, and $t = 20 \text{ s}$.


 **2.15** Verify that $x_p = -a\sin(\omega t)$, $y_p = a\cos(\omega t)$ is the equation for the pathlines of particles for the flow field of Problem 2.10. Find the frequency of motion ω as a function of the amplitude of motion, a , and K . Verify that $x_p = -a\sin(\omega t)$, $y_p = a\cos(\omega t)$ is also the equation for the pathlines of particles for the flow field of Problem 2.11, except that ω is now a function of M . Plot typical pathlines for both flow fields and discuss the difference.


 **2.16** Air flows downward toward an infinitely wide horizontal flat plate. The velocity field is given by $\vec{V} = (ax\hat{i} - ay\hat{j})(2 + \cos \omega t)$, where $a = 5 \text{ s}^{-1}$, $\omega = 2\pi \text{ s}^{-1}$, x and y (measured in meters) are horizontal and vertically upward, respectively, and t is in s. Obtain an algebraic equation for a streamline at $t = 0$. Plot the streamline that passes through point $(x, y) = (3, 3)$ at this instant. Will the streamline change with time? Explain briefly. Show the velocity vector on your plot at the same point and time. Is the velocity vector tangent to the streamline? Explain.

 **2.17** Consider the flow described by the velocity field $\vec{V} = Bx(1 + At)\hat{i} + Cy\hat{j}$, with $A = 0.5 \text{ s}^{-1}$, and $B = C = 1 \text{ s}^{-1}$. Coordinates are measured in meters. Plot the pathline traced out by the particle that passes through the point $(1, 1)$ at time $t = 0$. Compare with the streamlines plotted through the same point at the instants $t = 0, 1, \text{ and } 2 \text{ s}$.

 **2.18** Consider the flow field given in Eulerian description by the expression $\vec{V} = A\hat{i} + Bt\hat{j}$, where $A = 2 \text{ m/s}$, $B = 0.6 \text{ m/s}^2$, and the coordinates are measured in meters. Derive the Lagrangian position functions for the fluid particle that was located at the point $(x, y) = (1, 1)$ at the instant $t = 0$. Obtain an algebraic expression for the pathline followed by this particle. Plot the pathline and compare with the streamlines plotted through the same point at the instants $t = 0, 1, \text{ and } 2 \text{ s}$.


 **2.19** A velocity field is given by $\vec{V} = ax\hat{i} - by\hat{j}$, where $a = 0.1 \text{ s}^{-2}$ and $b = 1 \text{ s}^{-1}$. For the particle that passes through the point $(x, y) = (1, 1)$ at instant $t = 0 \text{ s}$, plot the pathline during the interval from $t = 0$ to $t = 3 \text{ s}$. Compare with the streamlines plotted through the same point at the instants $t = 0, 1, \text{ and } 2 \text{ s}$.


 **2.20** Consider the velocity field $V = ax\hat{i} + by(1 + ct)\hat{j}$, where $a = b = 2 \text{ s}^{-1}$, and $c = 0.4 \text{ s}^{-1}$. Coordinates are measured in meters. For the particle that passes through the point $(x, y) = (1, 1)$ at the instant $t = 0$, plot the pathline during the interval from $t = 0$ to 1.5 s . Compare this pathline with the streamlines plotted through the same point at the instants $t = 0, 1, \text{ and } 1.5 \text{ s}$.


 **2.21** Consider the flow field $\vec{V} = ax\hat{i} + b\hat{j}$, where $a = 0.1 \text{ s}^{-2}$ and $b = 4 \text{ m/s}$. Coordinates are measured in meters. For the particle that passes through the point $(x, y) = (3, 1)$ at the instant $t = 0$, plot the pathline during the interval from $t = 0$ to 3 s . Compare


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this pathline with the streamlines plotted through the same point at the instants $t = 1, 2,$ and 3 s.


 **2.22** Consider the garden hose of Fig. 2.5. Suppose the velocity field is given by $\vec{V} = u_0\hat{i} + v_0\sin[\omega(t - x/u_0)]\hat{j}$, where the x direction is horizontal and the origin is at the mean position of the hose, $u_0 = 10$ m/s, $v_0 = 2$ m/s, and $\omega = 5$ cycle/s. Find and plot on one graph the instantaneous streamlines that pass through the origin at $t = 0$ s, 0.05 s, 0.1 s, and 0.15 s. Also find and plot on one graph the pathlines of particles that left the origin at the same four times.

 **2.23** Using the data of Problem 2.22, find and plot the streakline shape produced after the first second of flow.

 **2.24** Consider the velocity field of Problem 2.17. Plot the streakline formed by particles that passed through the point $(1, 1)$ during the interval from $t = 0$ to $t = 3$ s. Compare with the streamlines plotted through the same point at the instants $t = 0, 1,$ and 2 s.

 **2.25** Streaklines are traced out by neutrally buoyant marker fluid injected into a flow field from a fixed point in space. A particle of the marker fluid that is at point (x, y) at time t must have passed through the injection point (x_0, y_0) at some earlier instant $t = \tau$. The time history of a marker particle may be found by solving the pathline equations for the initial conditions that $x = x_0, y = y_0$ when $t = \tau$. The present locations of particles on the streakline are obtained by setting τ equal to values in the range $0 \leq \tau \leq t$. Consider the flow field $\vec{V} = ax(1 + bt)\hat{i} + cy\hat{j}$, where $a = c = 1$ s⁻¹ and $b = 0.2$ s⁻¹. Coordinates are measured in meters. Plot the streakline that passes through the initial point $(x_0, y_0) = (1, 1)$, during the interval from $t = 0$ to $t = 3$ s. Compare with the streamline plotted through the same point at the instants $t = 0, 1,$ and 2 s.

 **2.26** Consider the flow field $\vec{V} = axt\hat{i} + bj\hat{j}$, where $a = 0.2$ s⁻² and $b = 1$ m/s. Coordinates are measured in meters. For the particle that passes through the point $(x, y) = (1, 2)$ at the instant $t = 0$, plot the pathline during the time interval from $t = 0$ to 3 s. Compare this pathline with the streakline through the same point at the instant $t = 3$ s.

 **2.27** Tiny hydrogen bubbles are being used as tracers to visualize a flow. All the bubbles are generated at the origin $(x = 0, y = 0)$. The velocity field is unsteady and obeys the equations:


$$\begin{array}{lll} u = 1 \text{ m/s} & v = 2 \text{ m/s} & 0 \leq t < 2 \text{ s} \\ u = 0 & v = -1 \text{ m/s} & 0 \leq t \leq 4 \text{ s} \end{array}$$


Plot the pathlines of bubbles that leave the origin at $t = 0, 1, 2, 3,$ and 4 s. Mark the locations of these five bubbles at $t = 4$ s. Use a dashed line to indicate the position of a streakline at $t = 4$ s.

2.28 A flow is described by velocity field, $\vec{V} = ay^2\hat{i} + b\hat{j}$, where $a = 1$ m⁻¹ s⁻¹ and $b = 2$ m/s. Coordinates are measured in meters. Obtain the equation for the streamline passing through point $(6, 6)$. At $t = 1$ s, what are the coordinates of the particle that passed through point $(1, 4)$ at $t = 0$? At $t = 3$ s, what are the coordinates of the particle that passed through point $(-3, 0)$ 2 s earlier? Show that pathlines, streamlines, and streaklines for this flow coincide.

2.29 A flow is described by velocity field, $\vec{V} = a\hat{i} + bx\hat{j}$, where $a = 2$ m/s and $b = 1$ s⁻¹. Coordinates are measured in meters. Obtain the equation for the streamline passing through point $(2, 5)$. At $t = 2$ s, what are the coordinates of the particle that

passed through point $(0, 4)$ at $t = 0$? At $t = 3$ s, what are the coordinates of the particle that passed through point $(1, 4.25)$ 2 s earlier? What conclusions can you draw about the pathline, streamline, and streakline for this flow?

 **2.30** A flow is described by velocity field, $\vec{V} = ay\hat{i} + bt\hat{j}$, where $a = 1$ s⁻¹ and $b = 0.5$ m/s². At $t = 2$ s, what are the coordinates of the particle that passed through point $(1, 2)$ at $t = 0$? At $t = 3$ s, what are the coordinates of the particle that passed through point $(1, 2)$ at $t = 2$ s? Plot the pathline and streakline through point $(1, 2)$ and compare with the streamlines through the same point at the instants $t = 0, 1,$ and 2 s.

 **2.31** A flow is described by velocity field, $\vec{V} = at\hat{i} + b\hat{j}$, where $a = 0.4$ m/s² and $b = 2$ m/s. At $t = 2$ s, what are the coordinates of the particle that passed through point $(2, 1)$ at $t = 0$? At $t = 3$ s, what are the coordinates of the particle that passed through point $(2, 1)$ at $t = 2$ s? Plot the pathline and streakline through point $(2, 1)$ and compare with the streamlines through the same point at the instants $t = 0, 1,$ and 2 s.

2.32 The variation with temperature of the viscosity of air is correlated well by the empirical Sutherland equation


$$\mu = \frac{bT^{1/2}}{1 + S/T}$$

Best-fit values of b and S are given in Appendix A for use with SI units. Use these values to develop an equation for calculating air viscosity in British Gravitational units as a function of absolute temperature in degrees Rankine. Check your result using data from Appendix A.

2.33 The variation with temperature of the viscosity of air is represented well by the empirical Sutherland correlation

$$\mu = \frac{bT^{1/2}}{1 + S/T}$$

Best-fit values of b and S are given in Appendix A. Develop an equation in SI units for kinematic viscosity versus temperature for air at atmospheric pressure. Assume ideal gas behavior. Check using data from Appendix A.

 **2.34** Some experimental data for the viscosity of helium at 1 atm are

$T, ^\circ\text{C}$	0	100	200	300	400
$\mu, \text{N} \cdot \text{s}/\text{m}^2 (\times 10^5)$	1.86	2.31	2.72	3.11	3.46

Using the approach described in Appendix A-3, correlate these data to the empirical Sutherland equation

$$\mu = \frac{bT^{1/2}}{1 + S/T}$$

(where T is in kelvin) and obtain values for constants b and S .

2.35 The velocity distribution for laminar flow between parallel plates is given by

$$\frac{u}{u_{\max}} = 1 - \left(\frac{2y}{h}\right)^2$$

where h is the distance separating the plates and the origin is placed midway between the plates. Consider a flow of water at 15°C, with $u_{\max} = 0.10$ m/s and $h = 0.1$ mm. Calculate the shear


stress on the upper plate and give its direction. Sketch the variation of shear stress across the channel.

2.36 The velocity distribution for laminar flow between parallel plates is given by

$$\frac{u}{u_{\max}} = 1 - \left(\frac{2y}{h}\right)^2$$

where h is the distance separating the plates and the origin is placed midway between the plates. Consider flow of water at 15°C with maximum speed of 0.05 m/s and $h = 0.1\text{ mm}$. Calculate the force on a 1 m^2 section of the lower plate and give its direction.

2.37 Explain how an ice skate interacts with the ice surface. What mechanism acts to reduce sliding friction between skate and ice?

 **2.38** Crude oil, with specific gravity $SG = 0.85$ and viscosity $\mu = 2.15 \times 10^{-3}\text{ lbf} \cdot \text{s}/\text{ft}^2$, flows steadily down a surface inclined $\theta = 45$ degrees below the horizontal in a film of thickness $h = 0.1$ in. The velocity profile is given by

$$u = \frac{\rho g}{\mu} \left(hy - \frac{y^2}{2} \right) \sin \theta$$

(Coordinate x is along the surface and y is normal to the surface.) Plot the velocity profile. Determine the magnitude and direction of the shear stress that acts on the surface.

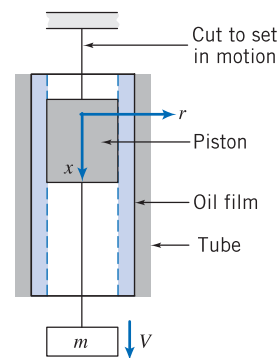
2.39 A female freestyle ice skater, weighing 100 lbf , glides on one skate at speed $V = 20\text{ ft/s}$. Her weight is supported by a thin film of liquid water melted from the ice by the pressure of the skate blade. Assume the blade is $L = 11.5\text{ in.}$ long and $w = 0.125\text{ in.}$ wide, and that the water film is $h = 0.0000575\text{ in.}$ thick. Estimate the deceleration of the skater that results from viscous shear in the water film, if end effects are neglected.

2.40 A block weighing 10 lbf and having dimensions 10 in. on each edge is pulled up an inclined surface on which there is a film of SAE 10W oil at 100°F . If the speed of the block is 2 ft/s and the oil film is 0.001 in. thick, find the force required to pull the block. Assume the velocity distribution in the oil film is linear. The surface is inclined at an angle of 25° from the horizontal.


2.41 Tape is to be coated on both sides with glue by drawing it through a narrow gap. The tape is 0.015 in. thick and 1.00 in. wide. It is centered in the gap with a clearance of 0.012 in. on each side. The glue, of viscosity $\mu = 0.02\text{ slug}/(\text{ft} \cdot \text{s})$, completely fills the space between the tape and gap. If the tape can withstand a maximum tensile force of 25 lbf , determine the maximum gap region through which it can be pulled at a speed of 3 ft/s .

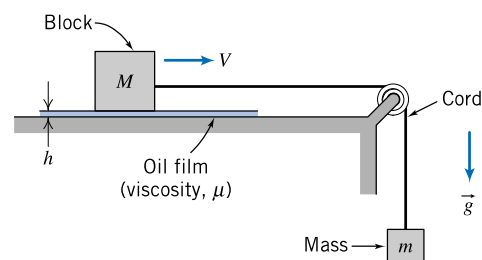
2.42 A 73-mm-diameter aluminum ($SG = 2.64$) piston of 100-mm length resides in a stationary $75\text{-mm-inner-diameter}$ steel tube lined with SAE 10W-30 oil at 25°C . A mass $m = 2\text{ kg}$ is suspended from the free end of the piston. The piston is set into motion by cutting a support cord. What is the terminal velocity of mass m ? Assume a linear velocity profile within the oil.

2.43 The piston in Problem 2.42 is traveling at terminal speed. The mass m now disconnects from the piston. Plot the piston speed vs. time. How long does it take the piston to come within 1 percent of its new terminal speed?




P 2.42, 2.43

 **2.44** A block of mass M slides on a thin film of oil. The film thickness is h and the area of the block is A . When released, mass m exerts tension on the cord, causing the block to accelerate. Neglect friction in the pulley and air resistance. Develop an algebraic expression for the viscous force that acts on the block when it moves at speed V . Derive a differential equation for the block speed as a function of time. Obtain an expression for the block speed as a function of time. The mass $M = 5\text{ kg}$, $m = 1\text{ kg}$, $A = 25\text{ cm}^2$, and $h = 0.5\text{ mm}$. If it takes 1 s for the speed to reach 1 m/s , find the oil viscosity μ . Plot the curve for $V(t)$.



P 2.44

 **2.45** A block 0.1 m square, with 5 kg mass, slides down a smooth incline, 30° below the horizontal, on a film of SAE 30 oil at 20°C that is 0.20 mm thick. If the block is released from rest at $t = 0$, what is its initial acceleration? Derive an expression for the speed of the block as a function of time. Plot the curve for $V(t)$. Find the speed after 0.1 s . If we want the mass to instead reach a speed of 0.3 m/s at this time, find the viscosity μ of the oil we would have to use.

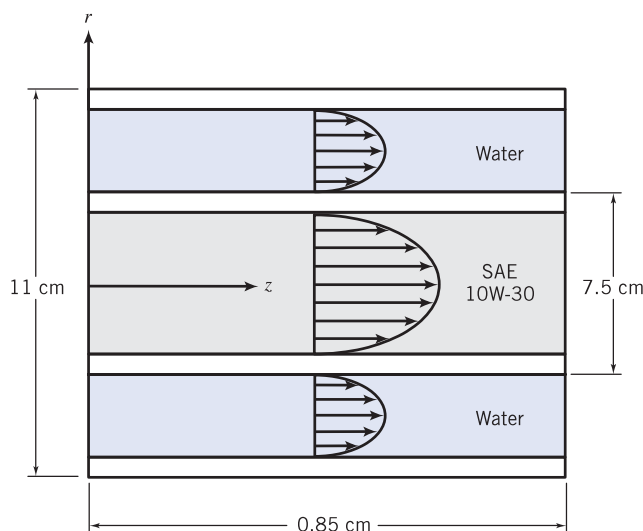
2.46 A block that is $a\text{ mm}$ square slides across a flat plate on a thin film of oil. The oil has viscosity μ and the film is $h\text{ mm}$ thick. The block of mass M moves at steady speed U under the influence of constant force F . Indicate the magnitude and direction of the shear stresses on the bottom of the block and the plate. If the force is removed suddenly and the block begins to slow, sketch the resulting speed versus time curve for the block. Obtain an expression for the time required for the block to lose 95 percent of its initial speed.

2.47 Magnet wire is to be coated with varnish for insulation by drawing it through a circular die of 1.0 mm diameter. The wire diameter is 0.9 mm and it is centered in the die. The varnish ($\mu = 20\text{ centipoise}$) completely fills the space between the wire and

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the die for a length of 50 mm. The wire is drawn through the die at a speed of 50 m/s. Determine the force required to pull the wire.

2.48 A double-pipe heat exchanger consists of two concentric fluid-carrying pipes used to transfer heat between nonmixing fluids. The figure shown below is a full-section view of a 0.85-m length of the double-pipe apparatus.



P 2.48

SAE10W-30 oil at 100°C flows through the 7.5-cm-outer-diameter inside pipe. Water at 10°C flows through the annulus between the inside pipe and the 11-cm-outer-diameter outside pipe. The wall thickness of each pipe is 3 mm. The theoretical velocity profiles for laminar flow through a pipe and annulus are:

$$\text{Inner pipe: } u_z(r) = u_{\max} \left[1 - \left(\frac{r}{R_{i, \text{inside}}} \right)^2 \right]$$

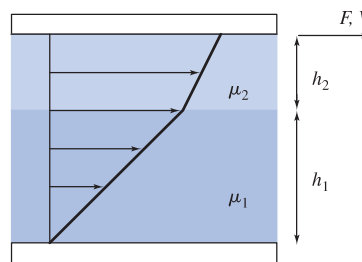
$$\text{where: } u_{\max} = \frac{R_{i, \text{inside}}^2 \Delta P}{4\mu L}$$

$$\text{Annulus: } u_z(r) = \frac{1}{4\mu} \left(\frac{\Delta P}{L} \right) \times \left[R_{i, \text{outside}}^2 - r^2 - \frac{R_o^2 - R_i^2}{\ln\left(\frac{R_{i, \text{outside}}}{R_{o, \text{inside}}}\right)} \cdot \ln\left(\frac{r}{R_{i, \text{outside}}}\right) \right]$$

Show that the no-slip condition is satisfied by these expressions. The pressure drop across the given length is 2.5 Pa and 8 Pa for the water and oil flows, respectively. If both flows are in the same direction (along the +z axis), what is the net viscous force acting on the inner pipe?

2.49 Repeat Problem 2.48 assuming a counterflow arrangement, where the oil flows in the +z direction and the water flows in the -z direction.

2.50 Fluids of viscosities $\mu_1 = 0.1 \text{ N}\cdot\text{s}/\text{m}^2$ and $\mu_2 = 0.15 \text{ N}\cdot\text{s}/\text{m}^2$ are contained between two plates (each plate is 1 m^2 in area). The thicknesses are $h_1 = 0.5 \text{ mm}$ and $h_2 = 0.3 \text{ mm}$, respectively. Find the force F to make the upper plate move at a speed of 1 m/s. What is the fluid velocity at the interface between the two fluids?



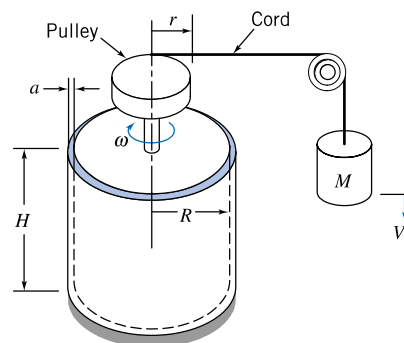
P 2.50

2.51 A concentric cylinder viscometer may be formed by rotating the inner member of a pair of closely fitting cylinders (see Fig. P2.53). For small clearances, a linear velocity profile may be assumed in the liquid filling the annular clearance gap. A viscometer has an inner cylinder of 75 mm diameter and 150 mm height, with a clearance gap width of 0.02 mm. A torque of 0.021 N·m is required to turn the inner cylinder at 100 rpm. Determine the viscosity of the liquid in the clearance gap of the viscometer.

2.52 A concentric cylinder viscometer may be formed by rotating the inner member of a pair of closely fitting cylinders (see Fig. P2.53). The annular gap is small so that a linear velocity profile will exist in the liquid sample. Consider a viscometer with an inner cylinder of 4 in. diameter and 8 in. height, and a clearance gap width of 0.001 in., filled with castor oil at 90°F. Determine the torque required to turn the inner cylinder at 400 rpm.

2.53 A concentric cylinder viscometer is driven by a falling mass M connected by a cord and pulley to the inner cylinder, as shown. The liquid to be tested fills the annular gap of width a and height H . After a brief starting transient, the mass falls at constant speed V_m . Develop an algebraic expression for the viscosity of the liquid in the device in terms of M , g , V_m , r , R , a , and H . Evaluate the viscosity of the liquid using:

- $M = 0.10 \text{ kg}$ $r = 25 \text{ mm}$
- $R = 50 \text{ mm}$ $a = 0.20 \text{ mm}$
- $H = 80 \text{ mm}$ $V_m = 30 \text{ mm/s}$



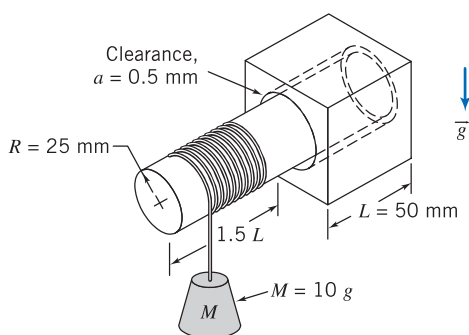
P2.53, 2.55

2.54 A shaft with outside diameter of 18 mm turns at 20 revolutions per second inside a stationary journal bearing 60 mm long. A thin film of oil 0.2 mm thick fills the concentric annulus between the shaft and journal. The torque needed to turn the shaft is 0.0036 N·m. Estimate the viscosity of the oil that fills the gap.

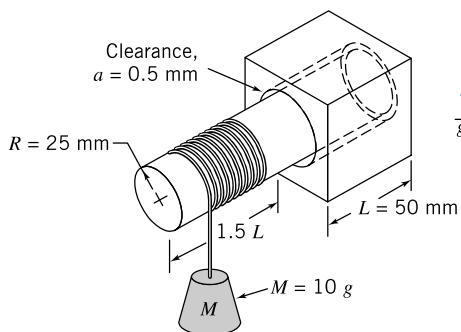
2.55 The viscometer of Problem 2.53 is being used to verify that the viscosity of a particular fluid is $\mu = 0.1 \text{ N}\cdot\text{s}/\text{m}^2$. Unfortunately

the cord snaps during the experiment. How long will it take the cylinder to lose 99% of its speed? The moment of inertia of the cylinder/pulley system is $0.0273 \text{ kg} \cdot \text{m}^2$.

2.56 The thin outer cylinder (mass m_2 and radius R) of a small portable concentric cylinder viscometer is driven by a falling mass, m_1 , attached to a cord. The inner cylinder is stationary. The clearance between the cylinders is a . Neglect bearing friction, air resistance, and the mass of liquid in the viscometer. Obtain an algebraic expression for the torque due to viscous shear that acts on the cylinder at angular speed ω . Derive and solve a differential equation for the angular speed of the outer cylinder as a function of time. Obtain an expression for the maximum angular speed of the cylinder.



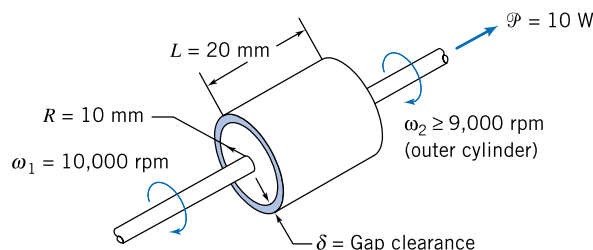
P 2.56



P 2.57

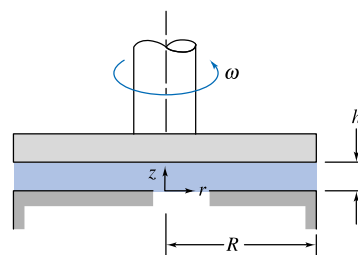
2.57 A circular aluminum shaft mounted in a journal is shown. The symmetric clearance gap between the shaft and journal is filled with SAE 10W-30 oil at $T = 30^\circ\text{C}$. The shaft is caused to turn by the attached mass and cord. Develop and solve a differential equation for the angular speed of the shaft as a function of time. Calculate the maximum angular speed of the shaft and the time required to reach 95 percent of this speed.

2.58 A shock-free coupling for a low-power mechanical drive is to be made from a pair of concentric cylinders. The annular space between the cylinders is to be filled with oil. The drive must transmit power, $\mathcal{P} = 10 \text{ W}$. Other dimensions and properties are as shown. Neglect any bearing friction and end effects. Assume the minimum practical gap clearance δ for the device is $\delta = 0.25 \text{ mm}$. Dow manufactures silicone fluids with viscosities as high as 10^6 centipoise. Determine the viscosity that should be specified to satisfy the requirement for this device.



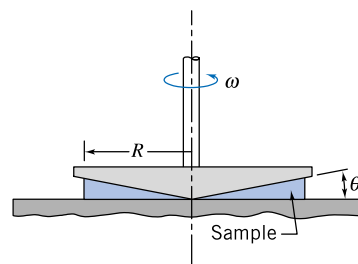
P 2.58

2.59 A proposal has been made to use a pair of parallel disks to measure the viscosity of a liquid sample. The upper disk rotates at height h above the lower disk. The viscosity of the liquid in the gap is to be calculated from measurements of the torque needed to turn the upper disk steadily. Obtain an algebraic expression for the torque needed to turn the disk. Could we use this device to measure the viscosity of a non-Newtonian fluid? Explain.



P 2.59

2.60 The cone and plate viscometer shown is an instrument used frequently to characterize non-Newtonian fluids. It consists of a flat plate and a rotating cone with a very obtuse angle (typically θ is less than 0.5 degrees). The apex of the cone just touches the plate surface and the liquid to be tested fills the narrow gap formed by the cone and plate. Derive an expression for the shear rate in the liquid that fills the gap in terms of the geometry of the system. Evaluate the torque on the driven cone in terms of the shear stress and geometry of the system.



P2.60, 2.61

2.61 The viscometer of Problem 2.60 is used to measure the apparent viscosity of a fluid. The data below are obtained. What kind of non-Newtonian fluid is this? Find the values of k and n used in Eqs. 2.16 and 2.17 in defining the apparent viscosity of a fluid. (Assume θ is 0.5 degrees.) Predict the viscosity at 90 and 100 rpm.

Speed (rpm)	10	20	30	40	50	60	70	80
$\mu \text{ (N} \cdot \text{s/m}^2\text{)}$	0.121	0.139	0.153	0.159	0.172	0.172	0.183	0.185