Relevant Equations, Formulas, Tables and Figures

Conservation of Mass (Continuity Equation)

$$\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$$ **(Integral Form)**

$$\nabla \cdot \rho \vec{V} + \frac{\partial \rho}{\partial t} = 0$$ **(Differential Form)**

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} + \frac{\partial \rho}{\partial t} = 0$$ **(Rectangular Coordinates)**

$$\frac{1}{r}\frac{\partial (r \rho V_r)}{\partial r} + \frac{1}{r}\frac{\partial (\rho V_\theta)}{\partial \theta} + \frac{\partial (\rho V_z)}{\partial z} + \frac{\partial \rho}{\partial t} = 0$$ **(Cylindrical Coordinates)**

Stream Function for Two-Dimensional Incompressible Flow (Rectangular Coordinates)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$ **(2-D Continuity Equation, Constant Density)**

$$u \equiv \frac{\partial \psi}{\partial y} \quad \text{and} \quad v \equiv -\frac{\partial \psi}{\partial x}$$ **(Stream Function *ψ*)**

Stream Function for Two-Dimensional Incompressible Flow (Cylindrical Coordinates)

$$\frac{\partial (r V_r)}{\partial r} + \frac{\partial V_\theta}{\partial \theta} = 0$$ **(2-D Continuity Equation, Constant Density)**

$$V_r \equiv \frac{1}{r}\frac{\partial \psi}{\partial \theta} \quad \text{and} \quad V_\theta \equiv -\frac{\partial \psi}{\partial r}$$ **(Stream Function *ψ*)**

Momentum Equation for Control Volume Moving with *Constant Velocity*

$$\vec{F} = \vec{F}_s + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}_{xyz} \, \rho \, d\forall + \int_{CS} \vec{V}_{xyz} \, \rho \vec{V}_{xyz} \cdot d\vec{A}$$

Momentum Equation for Inertial Control Volume with *Rectilinear Acceleration*

$$\vec{F}_S + \vec{F}_B - \int_{CV} \vec{a}_{rf} \, \rho \, d\forall = \frac{\partial}{\partial t}\int_{CV} \vec{V}_{xyz} \, \rho \, d\forall + \int_{CS} \vec{V}_{xyz} \, \rho \vec{V}_{xyz} \cdot d\vec{A}$$

For a *Newtonian* fluid: Shear Stress = $\tau_{yx} = \mu \frac{du}{dy}$

Where μ is the dynamic viscosity of the fluid.

For a Plane Submerged Surface:

$$F_R = p_{c_{\text{gage}}} A \qquad y' = y_c + \frac{I_{\hat{x}\hat{x}}}{A y_c} \qquad x' = x_c + \frac{I_{\hat{x}\hat{y}}}{A y_c}$$

where $I_{\hat{x}\hat{x}} = \frac{width \times height^3}{12}$

Pressure Variation in a Static Fluid

$$p_{gage} = p_{abs} - p_{atm} = \rho g h$$

where ρ = density of the fluid ; g = gravitational acceleration (9.81 m/s^2 or 32.2 ft/s^2) ; h = height of fluid column

Absolute pressure = atmospheric pressure + gauge pressure reading

Absolute pressure = atmospheric pressure – vacuum pressure reading

Specific Gravity = SG = $\frac{\rho_s}{\rho_{ref}}$; where ρ_s = density of substance

and ρ_{ref} = density of reference liquid which is water at 4°C (39°F) = 1000 kg/m^3 (1.94 slug/ft^3)

Fluid Translation: Acceleration of a Fluid Particle in a Velocity Field, $\vec{a}_p$

$$\underbrace{\vec{a}_p = \frac{D\vec{V}}{Dt}}_{\text{total acceleration of a particle}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\text{convective acceleration}} + \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\text{local acceleration}}$$

Fluid Deformation: Linear Deformation

$$\text{Volume dilation rate} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \nabla \cdot \vec{V}$$

Fluid Rotation:

$$\bar{\boldsymbol{\omega}} = \frac{1}{2}\nabla \times \bar{\boldsymbol{V}}$$; If $\nabla \times \bar{\boldsymbol{V}} = \boldsymbol{0}$ then flow is <u>irrotational</u>

<u>The Bernoulli Equation</u> is derived when the energy equation is applied to one-dimensional flows. Assuming no friction losses and that no pump or turbine exists between sections 1 and 2 in the system,

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1,$$ where

p_1, p_2 = pressure at sections 1 and 2,
V_1, V_2 = average velocity of the fluid at the sections,
z_1, z_2 = the vertical distance from a datum to the sections (the potential energy),
γ = the specific weight of the fluid (ρg), and
g = the acceleration of gravity.

Internal Pipe Flow

Energy Equation:

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} = \Sigma h_l + \Sigma h_{l_m}$$

$$\alpha = \begin{cases} 2.0 \;\; for\ Laminar\ flow \\ 1.0\ for\ turbulent\ flow \end{cases}$$

Minor losses: $h_{l_m} = K\dfrac{\bar{V}^2}{2}$ **or** $h_{l_m} = f\dfrac{L_e}{D}\dfrac{\bar{V}^2}{2}$

Major Losses: $h_l = f\dfrac{L}{D}\dfrac{\bar{V}^2}{2}$

Average Velocity: $\bar{V} = \dfrac{Q}{A} = \dfrac{1}{A}\displaystyle\int u\,dA$

Reynolds Number: $\mathrm{Re} = \dfrac{\rho \bar{V} D}{\mu} = \dfrac{\bar{V} D}{\nu}$

Laminar Friction Factor: $f = \dfrac{64}{\mathrm{Re}}$ → **Re < 2300**

Turbulent Friction Factor - Implicit Relation:

$$\frac{1}{f^{0.5}} = -2.0\log\left(\frac{e/D}{3.7} + \frac{2.51}{Re\, f^{0.5}}\right) \rightarrow \mathbf{Re \geq 2300}$$

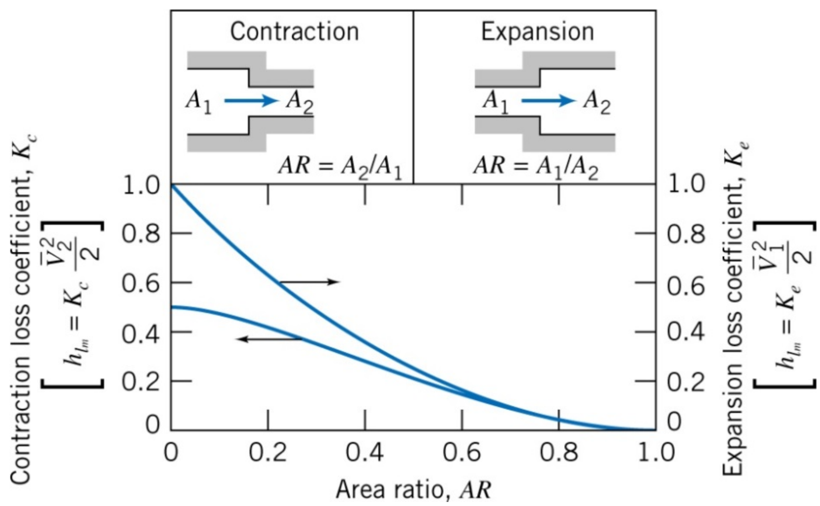
Turbulent Friction Factor Estimate (within 1% of actual value) – Explicit Relation:

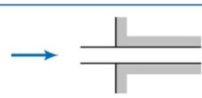
$$\boldsymbol{f_o = 0.25\left[log\left(\frac{e/D}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^{-2}} \rightarrow \mathbf{Re \geq 2300}$$

Pipe	Roughness, *e* Feet	Roughness, *e* Millimeters
Riveted steel	10.003–0.03	0.9–9
Concrete	0.001–0.01	0.3–3
Wood stave	0.0006–0.003	0.2–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Asphalted cast iron	0.0004	0.12
Commercial steel or wrought iron	0.00015	0.046
Drawn tubing	0.000005	0.0015

Nominal Pipe Size (in.)	Inside Diameter (in.)	Nominal Pipe Size (in.)	Inside Diameter (in.)
$\frac{1}{8}$	0.269	$2\frac{1}{2}$	2.469
$\frac{1}{4}$	0.364	3	3.068
$\frac{3}{8}$	0.493	4	4.026
$\frac{1}{2}$	0.622	5	5.047
$\frac{3}{4}$	0.824	6	6.065
1	1.049	8	7.981
$1\frac{1}{2}$	1.610	10	10.020
2	2.067	12	12.000

Fitting Type	Equivalent Length,[a] L_e/D
Valves (fully open)	
Gate valve	8
Globe valve	340
Angle valve	150
Ball valve	3
Lift check valve: globe lift	600
angle lift	55
Foot valve with strainer: poppet disk	420
hinged disk	75
Standard elbow: 90°	30
45°	16
Return bend, close pattern	50
Standard tee: flow through run	20
flow through branch	60



Entrance Type		Minor Loss Coefficient, K[a]
Reentrant		0.78
Square-edged		0.5
Rounded		r/D: 0.02, 0.06, ≥ 0.15; K: 0.28, 0.15, 0.04

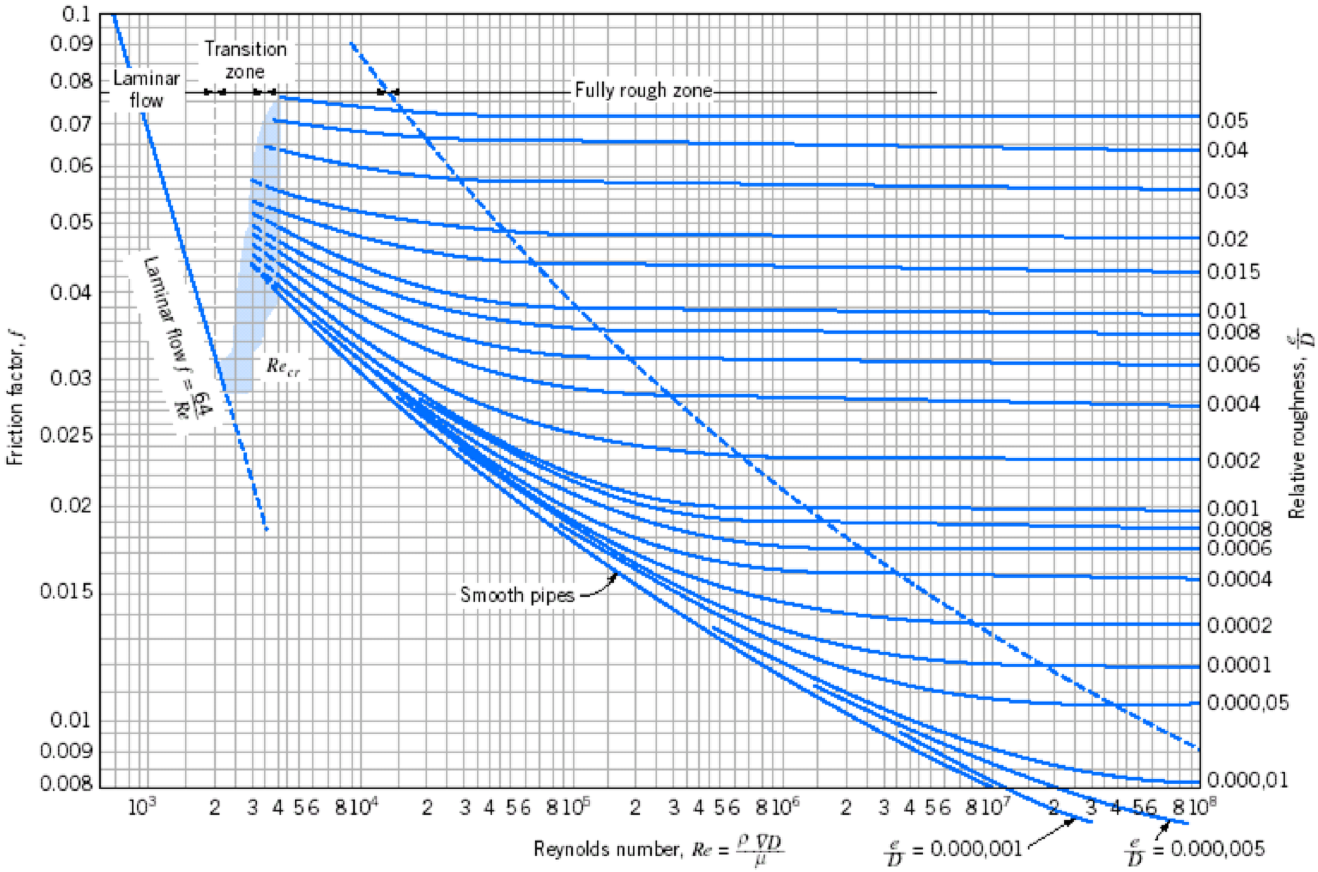


Fig. 8.12 Friction factor for fully developed flow in circular pipes. (Data from [8], used by permission.)

Pumps, Fans, and Blowers (Energy Equation and Other Useful Relations):

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2\right) = h_{l_T} - \Delta h_{\text{pump}}$$

$$\Delta h_{\text{pump}} = \frac{\Delta p_{\text{pump}}}{\rho} \qquad \dot{W}_{\text{pump}} = Q \Delta p_{\text{pump}} \qquad \eta = \frac{\dot{W}_{\text{pump}}}{\dot{W}_{\text{in}}}$$

Mass Flow Rate: $\dot{m} = \rho Q = \rho V A$ **; where** Q **= Volume Flow Rate**

Ideal Gas Law: $P = \rho R T$ **; where** T **= Absolute Temperature**

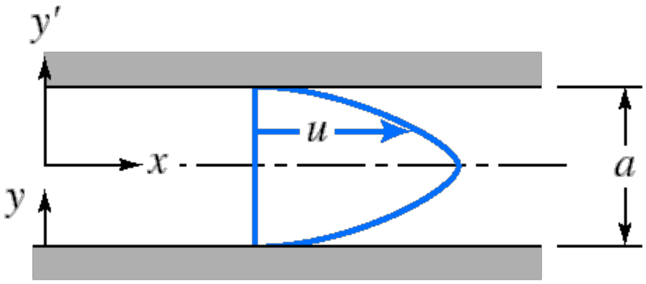
Hydraulic Gradient (Grade Line)

The hydraulic gradient (grade line) is defined as an imaginary line above a pipe so that the vertical distance from the pipe axis to the line represents the *pressure head* at that point. If a row of piezometers were placed at intervals along the pipe, the grade line would join the water levels in the piezometer water columns.

Energy Line (Bernoulli Equation)

The Bernoulli equation states that the sum of the pressure, velocity, and elevation heads is constant. The energy line is the sum or the "total head line" above a horizontal datum. The difference between the hydraulic grade line and the energy line is the ($V^2/2g$) term.

Fully Developed *Laminar* Flow between Infinite Parallel Plates: (Both Plates Stationary)



$$u = \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{y}{a}\right)^2 - \left(\frac{y}{a}\right)\right]$$

Shear Stress Distribution: $\tau_{yx} = a\left(\frac{\partial p}{\partial x}\right)\left[\frac{y}{a} - \frac{1}{2}\right]$

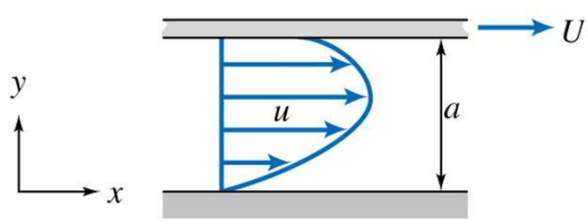
Volume Flow Rate: $\frac{Q}{l} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)a^3$

Volume Flow Rate as a Function of Pressure Drop: $\frac{Q}{l} = \frac{a^3 \Delta p}{12\mu L}$

Average Velocity: $\bar{V} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)a^2$

Maximum Velocity: $u_{\max} = -\frac{1}{8\mu}\left(\frac{\partial p}{\partial x}\right)a^2 = \frac{3}{2}\bar{V}$

Fully Developed *Laminar* Flow between Infinite Parallel Plates: (Upper Plate Moving with a Constant Speed *U* and Lower Plate Stationary)



$$u = \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{y}{a}\right)^2 - \left(\frac{y}{a}\right)\right] + U\left(\frac{y}{a}\right)$$

Shear Stress Distribution: $\tau_{yx} = a\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{y}{a}\right) - \frac{1}{2}\right] + \mu\left(\frac{U}{a}\right)$

Volume Flow Rate: $\frac{Q}{l} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)a^3 + \frac{Ua}{2}$

Average Velocity: $\frac{Q}{l} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)a^3 + \frac{Ua}{2}$

Point of Maximum Velocity: Set $\frac{du}{dy} = 0$ and Solve for $y \longrightarrow y = \frac{a}{2} - \frac{\mu U/a}{\left(\partial p/\partial x\right)}$

Fully Developed *Laminar* Flow in a Pipe:

Velocity Distribution: $u = -\frac{R^2}{4\mu}\left(\frac{\partial p}{\partial x}\right)\left[1-\left(\frac{r}{R}\right)^2\right]$

Shear Stress Distribution: $\tau_{rx} = \frac{r}{2}\left(\frac{\partial p}{\partial x}\right)$

Volume Flow Rate: $Q = -\frac{\pi R^4}{8\mu}\left(\frac{\partial p}{\partial x}\right)$

Volume Flow Rate as a Function of Pressure Drop: $Q = \frac{\pi \Delta p D^4}{128\mu L}$

Average Velocity: $\bar{V} = -\frac{R^2}{8\mu}\left(\frac{\partial p}{\partial x}\right)$

Maximum Velocity: $u_{\max} = -\frac{R^2}{4\mu}\left(\frac{\partial p}{\partial x}\right) = 2\bar{V}$

For Non-Circular Ducts: Replace the Diameter with the Hydraulic Diameter $D_h = \frac{4A}{P}$ where A = Cross-Sectional Area and P = Perimeter

Liquid	Isentropic Bulk Modulus[a] (GN/m^2)	Specific Gravity (—)
Benzene	1.48	0.879
Carbon tetrachloride	1.36	1.595
Castor oil	2.11	0.969
Crude oil	—	0.82–0.92
Ethanol	—	0.789
Gasoline	—	0.72
Glycerin	4.59	1.26
Heptane	0.886	0.684
Kerosene	1.43	0.82
Lubricating oil	1.44	0.88
Methanol	—	0.796
Mercury	28.5	13.55
Octane	0.963	0.702
Seawater[b]	2.42	1.025
SAE 10W oil	—	0.92
Water	2.24	0.998

Properties of *Water* in US and SI Units:

Temperature, T (°F)	Density, ρ (slug/ft^3)	Dynamic Viscosity, μ (lbf · s/ft^2)	Kinematic Viscosity, ν (ft^2/s)	Surface Tension, σ (lbf/ft)	Vapor Pressure, p_v (psia)	Bulk Modulus, E_v (psi)
32	1.94	3.68E-05	1.90E-05	0.00519	0.0886	2.92E + 05
40	1.94	3.20E-05	1.65E-05	0.00514	0.122	
50	1.94	2.73E-05	1.41E-05	0.00509	0.178	
59	1.94	2.38E-05	1.23E-05	0.00504	0.247	
60	1.94	2.35E-05	1.21E-05	0.00503	0.256	
68	1.94	2.10E-05	1.08E-05	0.00499	0.339	
70	1.93	2.05E-05	1.06E-05	0.00498	0.363	3.20E + 05
80	1.93	1.80E-05	9.32E-06	0.00492	0.507	
90	1.93	1.59E-05	8.26E-06	0.00486	0.699	
100	1.93	1.43E-05	7.38E-06	0.00480	0.950	
110	1.92	1.28E-05	6.68E-06	0.00474	1.28	
120	1.92	1.16E-05	6.05E-06	0.00467	1.70	3.32E + 05
130	1.91	1.06E-05	5.54E-06	0.00461	2.23	
140	1.91	9.70E-06	5.08E-06	0.00454	2.89	
150	1.90	8.93E-06	4.70E-06	0.00448	3.72	
160	1.89	8.26E-06	4.37E-06	0.00441	4.75	
170	1.89	7.67E-06	4.06E-06	0.00434	6.00	
180	1.88	7.15E-06	3.80E-06	0.00427	7.52	
190	1.87	6.69E-06	3.58E-06	0.00420	9.34	
200	1.87	6.28E-06	3.36E-06	0.00413	11.5	3.08E + 05
212	1.86	5.84E-06	3.14E-06	0.00404	14.7	

Temperature, T (°C)	Density, ρ (kg/m^3)	Dynamic Viscosity, μ (N · s/m^2)	Kinematic Viscosity, ν (m^2/s)	Surface Tension, σ (N/m)	Vapor Pressure, p_v (kPa)	Bulk Modulus, E_v (GPa)
0	1000	1.76E-03	1.76E-06	0.0757	0.661	2.01
5	1000	1.51E-03	1.51E-06	0.0749	0.872	
10	1000	1.30E-03	1.30E-06	0.0742	1.23	
15	999	1.14E-03	1.14E-06	0.0735	1.71	
20	998	1.01E-03	1.01E-06	0.0727	2.34	2.21
25	997	8.93E-04	8.96E-07	0.0720	3.17	
30	996	8.00E-04	8.03E-07	0.0712	4.25	
35	994	7.21E-04	7.25E-07	0.0704	5.63	
40	992	6.53E-04	6.59E-07	0.0696	7.38	
45	990	5.95E-04	6.02E-07	0.0688	9.59	
50	988	5.46E-04	5.52E-07	0.0679	12.4	2.29
55	986	5.02E-04	5.09E-07	0.0671	15.8	
60	983	4.64E-04	4.72E-07	0.0662	19.9	
65	980	4.31E-04	4.40E-07	0.0654	25.0	
70	978	4.01E-04	4.10E-07	0.0645	31.2	
75	975	3.75E-04	3.85E-07	0.0636	38.6	
80	972	3.52E-04	3.62E-07	0.0627	47.4	
85	969	3.31E-04	3.41E-07	0.0618	57.8	
90	965	3.12E-04	3.23E-07	0.0608	70.1	2.12
95	962	2.95E-04	3.06E-07	0.0599	84.6	
100	958	2.79E-04	2.92E-07	0.0589	101	

Properties of *Air* in US and SI Units:

Temperature, T (°F)	Density, ρ (slug/ft^3)	Dynamic Viscosity, μ (lbf · s/ft^2)	Kinematic Viscosity, ν (ft^2/s)
40	0.00247	3.63E-07	1.47E-04
50	0.00242	3.69E-07	1.52E-04
59	0.00238	3.74E-07	1.57E-04
60	0.00237	3.74E-07	1.58E-04
68	0.00234	3.79E-07	1.62E-04
70	0.00233	3.80E-07	1.63E-04
80	0.00229	3.85E-07	1.68E-04
90	0.00225	3.91E-07	1.74E-04
100	0.00221	3.96E-07	1.79E-04
110	0.00217	4.02E-07	1.86E-04
120	0.00213	4.07E-07	1.91E-04
130	0.00209	4.12E-07	1.97E-04
140	0.00206	4.18E-07	2.03E-04
150	0.00202	4.23E-07	2.09E-04
160	0.00199	4.28E-07	2.15E-04
170	0.00196	4.33E-07	2.21E-04
180	0.00193	4.38E-07	2.27E-04
190	0.00190	4.43E-07	2.33E-04
200	0.00187	4.48E-07	2.40E-04

Temperature, T (°C)	Density, ρ (kg/m^3)	Dynamic Viscosity, μ (N · s/m^2)	Kinematic Viscosity, ν (m^2/s)
0	1.29	1.72E-05	1.33E-05
5	1.27	1.74E-05	1.37E-05
10	1.25	1.76E-05	1.41E-05
15	1.23	1.79E-05	1.45E-05
20	1.21	1.81E-05	1.50E-05
25	1.19	1.84E-05	1.54E-05
30	1.17	1.86E-05	1.59E-05
35	1.15	1.88E-05	1.64E-05
40	1.13	1.91E-05	1.69E-05
45	1.11	1.93E-05	1.74E-05
50	1.09	1.95E-05	1.79E-05
55	1.08	1.98E-05	1.83E-05
60	1.06	2.00E-05	1.89E-05
65	1.04	2.02E-05	1.94E-05
70	1.03	2.04E-05	1.98E-05
75	1.01	2.06E-05	2.04E-05
80	1.00	2.09E-05	2.09E-05
85	0.987	2.11E-05	2.14E-05
90	0.973	2.13E-05	2.19E-05
95	0.960	2.15E-05	2.24E-05
100	0.947	2.17E-05	2.29E-05

DIMENSIONAL HOMOGENEITY AND DIMENSIONAL ANALYSIS

Equations that are in a form that do not depend on the fundamental units of measurement are called *dimensionally homogeneous* equations. A special form of the dimensionally homogeneous equation is one that involves only *dimensionless groups* of terms.

Buckingham's Theorem: The *number of independent dimensionless groups* that may be employed to describe a phenomenon known to involve *n* variables is equal to the number $(n-\bar{r})$, where $\bar{r}$ is the number of basic dimensions (*i.e.*, M, L, T) needed to express the variables dimensionally.

SIMILTUDE

In order to use a model to simulate the conditions of the prototype, the model must be *geometrically, kinematically,* and *dynamically similar* to the prototype system.

To obtain dynamic similarity between two flow pictures, all independent force ratios that can be written must be the same in both the model and the prototype. Thus, dynamic similarity between two flow pictures (when all possible forces are acting) is expressed in the five simultaneous equations below.

$$\left[\frac{F_L}{F_p}\right]_p = \left[\frac{F_L}{F_p}\right]_m = \left[\frac{\rho V^2}{p}\right]_p = \left[\frac{\rho V^2}{p}\right]_m$$

$$\left[\frac{F_L}{F_V}\right]_p = \left[\frac{F_L}{F_V}\right]_m = \left[\frac{\rho VL}{\mu}\right]_p = \left[\frac{\rho VL}{\mu}\right]_m = [\mathrm{Re}]_p = [\mathrm{Re}]_m$$

$$\left[\frac{F_L}{F_G}\right]_p = \left[\frac{F_L}{F_G}\right]_m = \left[\frac{V^2}{Lg}\right]_p = \left[\frac{V^2}{Lg}\right]_m = [Fr]_p = [Fr]_m$$

$$\left[\frac{F_L}{F_E}\right]_p = \left[\frac{F_L}{F_E}\right]_m = \left[\frac{\rho V^2}{E_v}\right]_p = \left[\frac{\rho V^2}{E_v}\right]_m = [Ca]_p = [Ca]_m$$

$$\left[\frac{F_L}{F_T}\right]_p = \left[\frac{F_L}{F_T}\right]_m = \left[\frac{\rho LV^2}{\sigma}\right]_p = \left[\frac{\rho LV^2}{\sigma}\right]_m = [We]_p = [We]_m$$

where the subscripts *p* and *m* stand for *prototype* and *model* respectively, and

F_L = inertia force → $\rho V^2 L^2$

F_P = pressure force → $\Delta p\, A \propto \Delta p\, L^2$

F_V = viscous force → $\tau A = \mu \dfrac{du}{dy} A \propto \mu \dfrac{V}{L} L^2 = \mu VL$

F_G = gravity force → $mg \propto g\rho L^3$

F_E = elastic force → $E_v A \propto E_v L^2$

F_T = surface tension force → σL

Re = Reynolds number → $\mathrm{Re} = \dfrac{\rho VD}{\mu} = \dfrac{VD}{\nu} = \dfrac{inertia\ forces}{viscous\ forces}$

We = Weber number → $We = \dfrac{\rho V^2 L}{\sigma} = \dfrac{inertia\ forces}{surface\ tension\ forces}$

Ca = Cavitation number $\rightarrow Ca = \dfrac{p - p_v}{\frac{1}{2}\rho V^2} = \dfrac{pressure\ forces}{inertia\ forces}$

Eu = Euler number (pressure coefficient C_p) $\rightarrow Eu = \dfrac{\Delta p}{\frac{1}{2}\rho V^2} = \dfrac{pressure\ forces}{inertia\ forces}$

Fr = Froude number $\rightarrow Fr = \dfrac{V}{\sqrt{gL}} = \dfrac{inertia\ forces}{gravity\ forces}$

M = Mach number $\rightarrow M = \dfrac{V}{c} = \dfrac{inertia\ forces}{compressibility\ forces}$

L = characteristic length,
V = velocity,
ρ = density,
σ = surface tension,
E_V = bulk modulus,
μ = dynamic viscosity,
p = pressure,
p_v = liquid vapor pressure
g = acceleration of gravity, and
c = local sonic speed