Experiment 2

Objective:

To verify Bernoulli's equation and to calculate volumetric flow.

Discussion:

The Bernoulli equation relates the pressure, elevation, and velocity of a fluid along a stream-line. For an incompressible fluid, this equation is:

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2$$

For the following experiment, $z_1 = z_2$ and therefore cancels. Also, $\gamma = \rho g$, which means g can be canceled from the remaining equation. Thus:

$$\frac{P_1}{\rho} + \frac{v_1^2}{2} = \frac{P_2}{\rho} + \frac{v_2^2}{2}$$

Now if we assume that the air starts outside of the duct at zero velocity ($V_1 = 0$) and that density remains fairly constrant, we can calculate the velocity achieved inside the duct by the change in the pressure. Therefore:

$$\frac{P_1 - P_2}{\rho} = \frac{v_2^2}{2}$$

and in rearranging to get velocity:

$$v = \sqrt{\frac{2 \cdot \Delta P}{\rho}}$$

This allows us to find the velocity of the of the air inside the duct using the density of the air ($\rho = .075$ lb/ft^3 at 70°F, 29.92inHg). Also, since the manometers used are in W.C., we will substitute $\Delta P = \rho g h$. (note that this ρ is for H_2O)

$$v = \sqrt{\frac{2 \cdot \rho_{H20} g h}{\rho}} = \sqrt{\frac{2 \cdot 1.94(\text{slug}/\text{ft}^3) \cdot 32.174(\text{lb}/\text{slug}) \cdot 32.174(\text{ft}/\text{s}^2) \cdot h(\text{in}) \cdot 1/12(\text{ft}/\text{in})}{.075(\text{lb}/\text{ft}^3)}}$$

$v = 66.8\sqrt{h}(\text{ft}/\text{s})$ or $v = 4008.2\sqrt{h}(\text{ft}/\text{min})$ (where h is in inches)

Once we have the velocity, the volumetric flow rate can be calculated with

$$Q = v_{ave} A$$