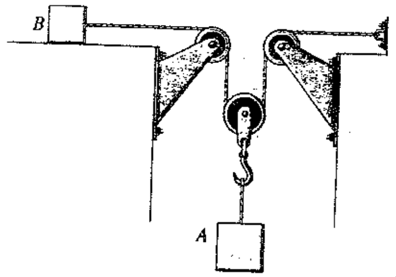
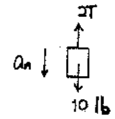
*13-24. At a given instant the 10-lb block A is moving downward with a speed of 6 ft/s. Determine its speed 2 s later. Block B has a weight of 4 lb, and the coefficient of kinetic friction between it and the horizontal plane is $\mu_k = 0.2$. Neglect the mass of the pulleys and cord.



Block A :

$+\downarrow \Sigma F_y = ma_y;\quad 10 - 2T = \frac{10}{32.2}a_A$



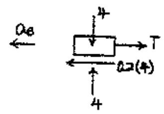
Block B :

$\overset{+}{\leftarrow} \Sigma F_x = ma_x;\quad -T + 0.2(4) = \frac{4}{32.2}a_B$

$2s_A + s_B = l$

$2a_A = -a_B$

Solving;



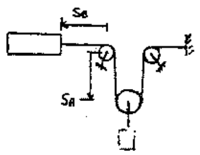
$T = 3.38$ lb

$a_A = 10.403$ ft/s^2

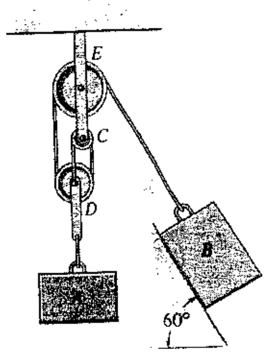
$a_B = -20.81$ ft/s^2

$(+\downarrow)\ v_A = (v_A)_0 + a_A t$

$v_A = 6 + 10.403(2) = 26.8$ ft/s **Ans**

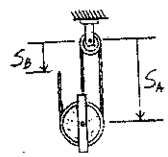


13-25. Determine the required mass of block A so that when it is released from rest it moves the 5-kg block B 0.75 m up along the smooth inclined plane in $t = 2$ s. Neglect the mass of the pulleys and cords.



Kinematic : Applying equation $s = s_0 + v_0 t + \frac{1}{2}a_c t^2$, we have

$$(\ +)\quad 0.75 = 0 + 0 + \frac{1}{2}a_B\left(2^2\right)\qquad a_B = 0.375\ \text{m/s}^2$$

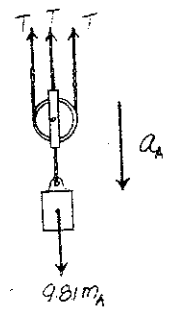


Establish the position - coordinate equation, we have

$$2s_A + (s_A - s_B) = l\qquad 3s_A - s_B = l$$

Taking time derivative twice yields

$$3a_A - a_B = 0 \qquad [1]$$



(a)

From Eq. [1],

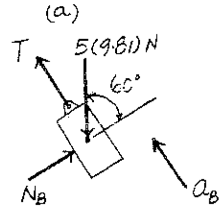
$$3a_A - 0.375 = 0\qquad a_A = 0.125\ \text{m/s}^2$$

Equation of Motion : The tension T developed in the cord is the same throughout the entire cord since the cord passes over the smooth pulleys. From FBD(b),

$$+\Sigma F_{y'} = ma_{y'};\quad T - 5(9.81)\sin 60° = 5(0.375)$$
$$T = 44.35\ \text{N}$$

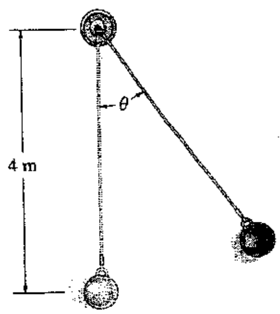
From FBD(a),

$$+\uparrow \Sigma F_y = ma_y;\quad 3(44.35) - 9.81m_A = m_A(-0.125)$$
$$m_A = 13.7\ \text{kg}\qquad \textbf{Ans}$$



(b)

13-69. The ball has a mass of 30 kg and a speed $v = 4\ \text{m/s}$ at the instant it is at its lowest point, $\theta = 0°$. Determine the tension in the cord and the rate at which the ball's speed is decreasing at the instant $\theta = 20°$. Neglect the size of the ball.



$+\nwarrow \Sigma F_n = ma_n: \quad T - 30(9.81)\cos\theta = 30\left(\frac{v^2}{4}\right)$

$+\nearrow \Sigma F_t = ma_t: \quad -30(9.81)\sin\theta = 30a_t$

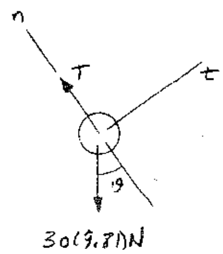
$a_t = -9.81\sin\theta$

$a_t\, ds = v\, dv$ Since $ds = 4\, d\theta$, then

$-9.81\int_0^\theta \sin\theta\,(4\, d\theta) = \int_4^v v\, dv$

$9.81(4)\cos\theta\Big|_0^\theta = \frac{1}{2}(v)^2 - \frac{1}{2}(4)^2$

$39.24(\cos\theta - 1) + 8 = \frac{1}{2}v^2$



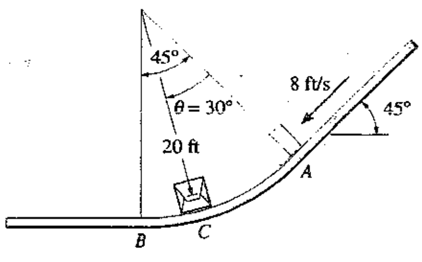
At $\theta = 20°$

$v = 3.357\ \text{m/s}$

$a_t = -3.36\ \text{m/s}^2 = 3.36\ \text{m/s}^2 \swarrow$ **Ans**

$T = 361\ \text{N}$ **Ans**

13-70. The package has a weight of 5 lb and slides down the chute. When it reaches the curved portion AB, it is traveling at 8 ft/s ($\theta = 0°$). If the chute is smooth, determine the speed of the package when it reaches the intermediate point C ($\theta = 30°$) and when it reaches the horizontal plane ($\theta = 45°$). Also, find the normal force on the package at C.



$+\swarrow \Sigma F_t = ma_t; \quad 5\cos\phi = \frac{5}{32.2}a_t$

$a_t = 32.2\cos\phi$

$+\nwarrow \Sigma F_n = ma_n; \quad N - 5\sin\phi = \frac{5}{32.2}\left(\frac{v^2}{20}\right)$

$v\, dv = a_t\, ds$

$\int_8^v v\, dv = \int_{45°}^\phi 32.2\cos\phi\,(20\, d\phi)$

$\frac{1}{2}v^2 - \frac{1}{2}(8)^2 = 644(\sin\phi - \sin 45°)$

At $\phi = 45° + 30° = 75°$,

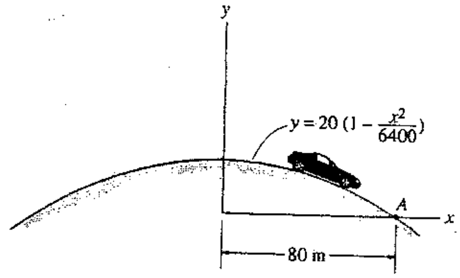
$v_C = 19.933\ \text{ft/s} = 19.9\ \text{ft/s}$ **Ans**

$N_C = 7.91\ \text{lb}$ **Ans**

At $\phi = 45° + 45° = 90°$

$v_B = 21.0\ \text{ft/s}$ **Ans**

13-73. The 0.8-Mg car is traveling over the hill having the shape of a parabola. If the driver maintains a constant speed of 9 m/s, determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at the instant it reaches point A. Neglect the size of the car.

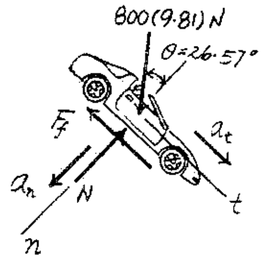


Geometry : Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\,\text{m}} = -0.00625(80) \qquad \theta = -26.57^\circ$$

and the radius of curvature at point A is

$$\rho = \frac{[1+(dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1+(-0.00625x)^2]^{3/2}}{|-0.00625|}\right|_{x=8\,\text{m}} = 223.61\ \text{m}$$



Equation of Motion : Here, $a_t = 0$. Applying Eq. 13 – 8 with $\theta = 26.57^\circ$ and $\rho = 223.61$ m, we have

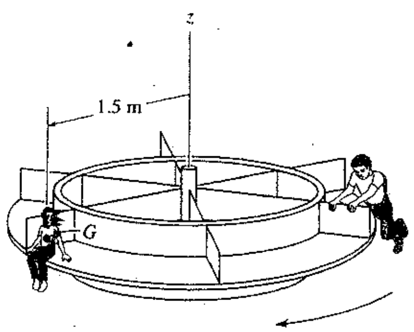
$\Sigma F_t = ma_t$; $\qquad 800(9.81)\sin 26.57^\circ - F_f = 800(0)$

$F_f = 3509.73\ \text{N} = 3.51\ \text{kN}$ **Ans**

$\Sigma F_n = ma_n$; $\qquad 800(9.81)\cos 26.57^\circ - N = 800\left(\frac{9^2}{223.61}\right)$

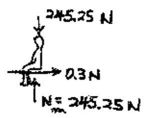
$N = 6729.67\ \text{N} = 6.73\ \text{kN}$ **Ans**

13-74. A girl having a mass of 25 kg sits at the edge of the merry-go-round so her center of mass G is at a distance of 1.5 m from the axis of rotation. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which she can have before she begins to slip off the merry-go-round. The coefficient of static friction between the girl and the merry-go-round is $\mu_s = 0.3$.

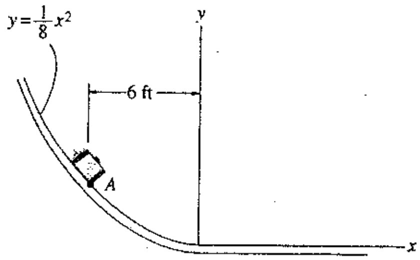


$\xrightarrow{+} \Sigma F_n = m a_n$; $\qquad 0.3(245.25) = 25\left(\frac{v^2}{1.5}\right)$

$v = 2.10$ m/s **Ans**



13-75. The 10-lb suitcase slides down the curved ramp for which the coefficient of kinetic friction is $\mu_k = 0.2$. If at the instant it reaches point A it has a speed of 5 ft/s, determine the normal force on the suitcase and the rate of increase of its speed.

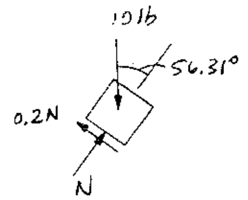


$$y = \frac{1}{8}x^2$$

$$\frac{dy}{dx} = \tan\theta = \frac{1}{4}x \Big|_{x=-6} = -1.5 \qquad \theta = -56.31^\circ$$

$$\frac{d^2y}{dx^2} = \frac{1}{4}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (-1.5)^2\right]^{\frac{3}{2}}}{\left|\frac{1}{4}\right|} = 23.436 \text{ ft}$$



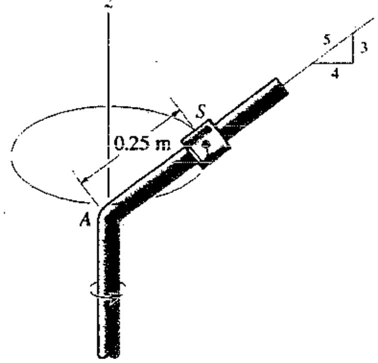
$+\nearrow \Sigma F_n = ma_n;$ $\quad N - 10\cos 56.31^\circ = \left(\frac{10}{32.2}\right)\left(\frac{(5)^2}{23.436}\right)$

$N = 5.8783 = 5.88$ lb **Ans**

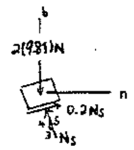
$+\searrow \Sigma F_t = ma_t;$ $\quad -0.2(5.8783) + 10\sin 56.31^\circ = \left(\frac{10}{32.2}\right)a_t$

$a_t = 23.0$ ft/s^2 **Ans**

***13-76.** The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the minimum constant speed the spool can have so that it does not slip down the rod.



$\rho = 0.25\left(\frac{4}{5}\right) = 0.2$ m



$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n;$ $\quad N_s\left(\frac{3}{5}\right) - 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$

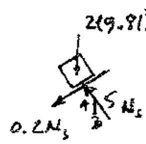
$+\uparrow \Sigma F_b = m\,a_b;$ $\quad N_s\left(\frac{4}{5}\right) + 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$

$N_s = 21.3$ N

$v = 0.969$ m/s **Ans**

13-77. The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the maximum constant speed the spool can have so that it does not slip up the rod.

$\rho = 0.25\left(\frac{4}{5}\right) = 0.2$ m



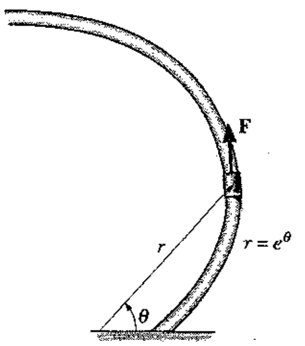
$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n;$ $\quad N_s\left(\frac{3}{5}\right) + 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$

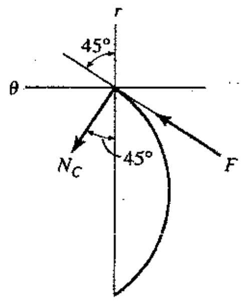
$+\uparrow \Sigma F_b = m\,a_b;$ $\quad N_s\left(\frac{4}{5}\right) - 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$

$N_s = 28.85$ N

$v = 1.48$ m/s **Ans**

13-103. The collar has a mass of 2 kg and travels along the smooth horizontal rod defined by the equiangular spiral $r = (e^{\theta})$ m, where θ is in radians. Determine the tangential force F and the normal force N acting on the collar when $\theta = 90°$, if the force F maintains a constant angular motion $\dot{\theta} = 2$ rad/s.





$r = e^{\theta}$

$\dot{r} = e^{\theta}\dot{\theta}$

$\ddot{r} = e^{\theta}(\dot{\theta})^2 + e^{\theta}\ddot{\theta}$

At $\theta = 90°$

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = 0$

$r = 4.8105$

$\dot{r} = 9.6210$

$\ddot{r} = 19.242$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 19.242 - 4.8105(2)^2 = 0$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(9.6210)(2) = 38.4838\ \text{m/s}^2$

$\tan\psi = \dfrac{r}{\left(\frac{dr}{d\theta}\right)} = e^{\theta}/e^{\theta} = 1$

$\psi = 45°$

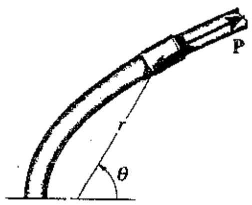
$+\uparrow \sum F_r = ma_r;\quad -N_C\cos 45° + F\cos 45° = 2(0)$

$\overset{+}{\leftarrow} \sum F_\theta = ma_\theta;\quad F\sin 45° + N_C \sin 45° = 2(38.4838)$

$N_C = 54.4N$ **Ans**

$F = 54.4$ N **Ans**

13-109. The collar, which has a weight of 3 lb, slides along the smooth rod lying in the *horizontal plane* and having the shape of a parabola $r = 4/(1 - \cos\theta)$, where θ is in radians and r is in feet. If the collar's angular rate is constant and equals $\dot{\theta} = 4$ rad/s, determine the tangential retarding force P needed to cause the motion and the normal force that the collar exerts on the rod at the instant $\theta = 90°$.



$$r = \frac{4}{1-\cos\theta}$$

$$\dot{r} = \frac{-4\sin\theta\ \dot{\theta}}{(1-\cos\theta)^2}$$

$$\ddot{r} = \frac{-4\sin\theta\ \ddot{\theta}}{(1-\cos\theta)^2} + \frac{-4\cos\theta\ (\dot{\theta})^2}{(1-\cos\theta)^2} + \frac{8\sin^2\theta\ \dot{\theta}^2}{(1-\cos\theta)^3}$$

At $\theta = 90°$, $\quad \dot{\theta} = 4, \quad \ddot{\theta} = 0$

$$r = 4$$
$$\dot{r} = -16$$
$$\ddot{r} = 128$$

$$a_r = \ddot{r} - r(\dot{\theta})^2 = 128 - 4(4)^2 = 64$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-16)(4) = -128$$

$$r = \frac{4}{1-\cos\theta}$$

$$\frac{dr}{d\theta} = \frac{-4\sin\theta}{(1-\cos\theta)^2}$$

$$\tan\psi = \frac{r}{\left(\frac{dr}{d\theta}\right)} = \left.\frac{\frac{4}{1-\cos\theta}}{\frac{-4\sin\theta}{(1-\cos\theta)^2}}\right|_{\theta=90°} = \frac{4}{-4} = -1$$

$$\psi = -45° = 135°$$

$+\uparrow \Sigma F_r = m\,a_r;\quad P\sin 45° - N\cos 45° = \frac{3}{32.2}(64)$

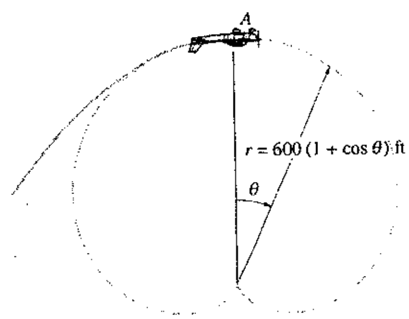
$\overset{+}{\leftarrow} \Sigma F_\theta = m a_\theta;\quad -P\cos 45° - N\sin 45° = \frac{3}{32.2}(-128)$

Solving,

$P = 12.6$ lb **Ans**

$N = 4.22$ lb **Ans**

13-110. The pilot of an airplane executes a vertical loop which in part follows the path of a cardioid, $r = 600(1 + \cos\theta)$ ft, where θ is in radians. If his speed at A ($\theta = 0°$) is a constant $v_P = 80$ ft/s, determine the vertical force the belt of his seat must exert on him to hold him to his seat when the plane is upside down at A. He weighs 150 lb. See hint related to Prob. 13–108.



$r = 600(1+\cos\theta)|_{\theta=0°} = 1200$ ft

$\dot{r} = -600\sin\theta\dot{\theta}|_{\theta=0°} = 0$

$\ddot{r} = -600\sin\theta\ddot{\theta} - 600\cos\theta\dot{\theta}^2|_{\theta=0°} = -600\dot{\theta}^2$

$v_p^2 = \dot{r}^2 + \left(r\dot{\theta}\right)^2$

$(80)^2 = 0 + \left(1200\dot{\theta}\right)^2 \qquad \dot{\theta} = 0.06667$

$2v_p\dot{v}_p = 2\dot{r}\ddot{r} + 2\left(r\dot{\theta}\right)\left(\dot{r}\dot{\theta} + r\ddot{\theta}\right)$

$0 = 0 + 0 + 2r^2\dot{\theta}\ddot{\theta} \qquad \ddot{\theta} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = -600(0.06667)^2 - 1200(0.06667)^2 = -8$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 0 = 0$

$+\uparrow \Sigma F_r = ma_r;\quad N - 150 = \left(\frac{150}{32.2}\right)(-8) \quad N = 113$ lb **Ans**