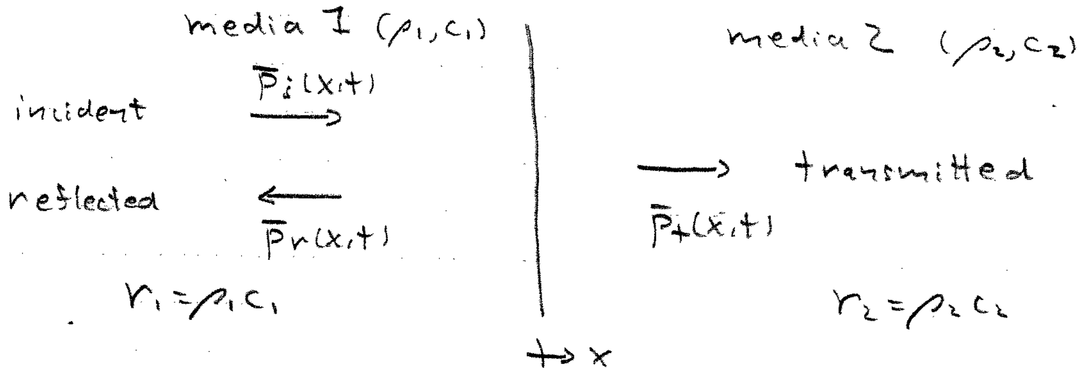
Chapter 6 Reflection and Transmission

<u>normal incidence</u>



plane harmonic

$$\bar{p}_i(x,t) = \bar{P}_i \, e^{j(\omega t - k_1 x)}$$
$$\bar{p}_r(x,t) = \bar{P}_r \, e^{j(\omega t + k_1 x)}$$
$$\bar{p}_t(x,t) = \bar{P}_t \, e^{j(\omega t - k_2 x)}$$

continuity of acoustic pressure @ $x = 0$

$$\bar{p}_i + \bar{p}_r = \bar{p}_t$$
$$\bar{P}_i + \bar{P}_r = \bar{P}_t$$

continuity of normal acoustic velocity

@ $x = 0$ $\quad \bar{u}_i + \bar{u}_r = \bar{u}_t$

$$\bar{P}_i - \bar{P}_r = (r_1/r_2)\,\bar{P}_t$$

pressure transmission coefficient

$$\bar{T} = \frac{\bar{P}_t}{\bar{P}_i} = \frac{2r_2}{(r_1 + r_2)}$$

pressure reflection coefficient

$$\bar{R} = \frac{\bar{P}_r}{\bar{P}_i} = \frac{(r_2 - r_1)}{(r_1 + r_2)}$$

$r_2 >> r_1 \Rightarrow$ rigid boundary

$r_2 << r_1 \Rightarrow$ pressure release boundary intensity transmission coefficient

$$T_I = I_t / I_i = (r_1/r_2)\,|\overline{T}|^2$$

intensity reflection coefficient

$$R_I = I_r / I_i = |\overline{R}|^2.$$

power reflection coefficient

$$R_\pi = \frac{A_r I_r}{A_i I_i} = \frac{I_r}{I_i} = R_I = |\overline{R}|^2 \quad \text{since } A_r = A_i$$

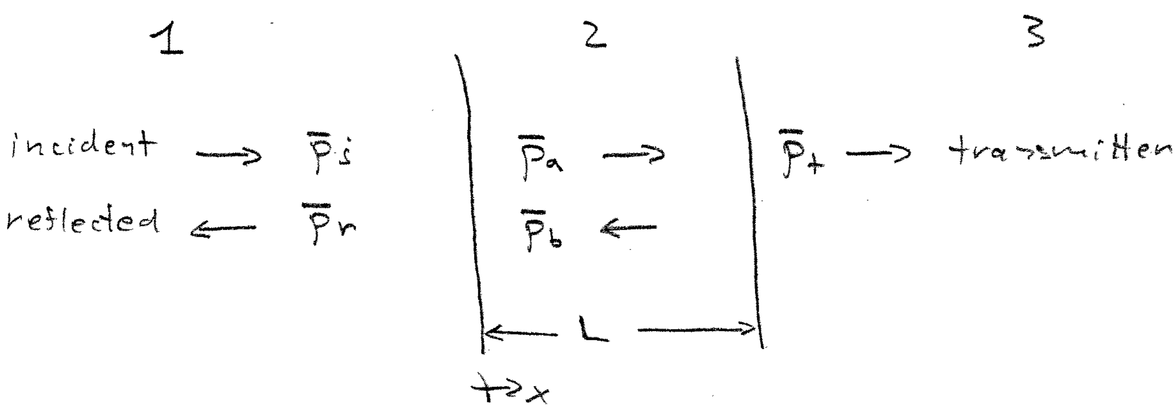
power transmission coefficient

$$T_\pi = \frac{A_t I_t}{A_i I_i} = \frac{A_t}{A_i} T_I = \left(\frac{A_t}{A_i}\right)\left(\frac{r_1}{r_2}\right)|\overline{T}|^2$$

note: for normal incidence $A_t = A_i$
for oblique incidence $A_t \neq A_i$

conservation of energy $R_\pi + T_\pi = 1$ (general)
$R_I + T_I = 1$ (normal incidence only)

<u>normal incidence, layer</u>



apply boundary conditions at $x = 0$ and $x = L$

four algebraic equations for $\overline{R}, \overline{T}, \overline{T}_a, \overline{R}_b$

$$\bar{R} = \frac{(1 - r_1/r_3)\cos(k_2 L) + j\,(r_2/r_3 - r_1/r_2)\sin(k_2 L)}{(1 + r_1/r_3)\cos(k_2 L) + j\,(r_2/r_3 + r_1/r_2)\sin(k_2 L)}$$

$$\bar{T} = \frac{2\, e^{+j k_3 L}}{(1 + r_1/r_3)\cos(k_2 L) + j\,(r_2/r_3 + r_1/r_2)\sin(k_2 L)}$$

$$T_I = (r_1/r_3)\,|\bar{T}|^2 = (r_1/r_3)\,\bar{T}\,\bar{T}^*$$

$$T_I = \frac{4}{2 + (r_3/r_1 + r_1/r_3)\cos^2(k_2 L) + \left(\frac{r_2^2}{r_1 r_3} + \frac{r_1 r_3}{r_2^2}\right)\sin^2(k_2 L)}$$

$$= 1 - R_I$$

special cases

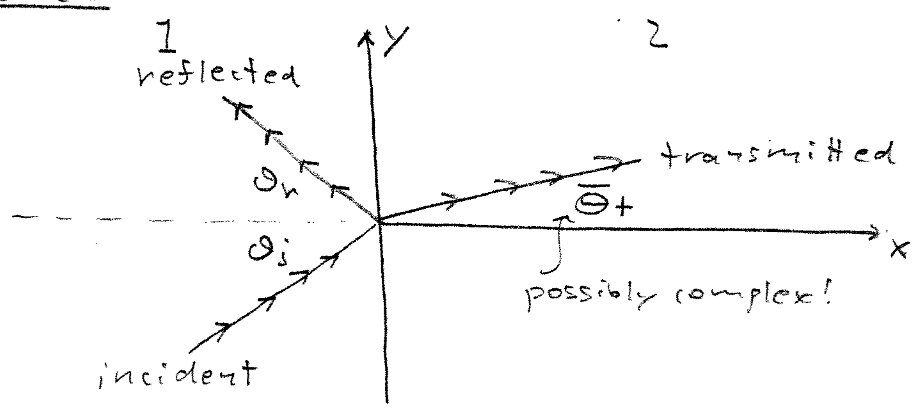
if $k_2 L = (n - \tfrac{1}{2})\pi$ and $r_2 = \sqrt{r_1 r_3}$ then $T_I = 100\%$
$R_I = 0\%$

if $k_2 L = n\pi$ and $r_1 = r_3$ then $T_I = 100\%$
$R_I = 0\%$

specific acoustic impedance of interface

$$\bar{Z}_{1\text{-}2} = \left.\frac{\bar{p}_i + \bar{p}_r}{\bar{u}_i + \bar{u}_r}\right)_{x=0} = r_1 \frac{(1 + \bar{R})}{(1 - \bar{R})}$$

<u>oblique incidence</u>



plane harmonic, apply boundary conditions

$$\theta_r = \theta_i \qquad \text{angle reflection = angle of incidence}$$

$$\frac{\sin\theta_i}{c_1} = \frac{\sin\bar{\theta}_t}{c_2} \qquad \left(\begin{array}{c}\text{acoustic} \\ \text{"Snell's law"}\end{array}\right)$$

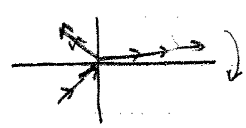
$$\bar{R} = \frac{r_2/r_1 - \cos\bar{\theta}_t/\cos\theta_i}{r_2/r_1 + \cos\bar{\theta}_t/\cos\theta_i}$$

$$\cos\bar{\theta}_t = [1 - \sin^2\bar{\theta}_t]^{1/2} = [1 - (c_2/c_1)^2 \sin^2\theta_i]^{1/2}$$

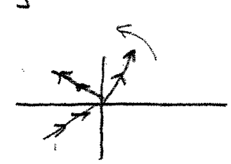
$$\bar{T} = 1 + \bar{R}$$

special cases

- if $c_1 > c_2$ (e.g. fast $\Rightarrow$ slow, water $\rightarrow$ air)
  then $\bar{\theta}_t \Rightarrow \theta_t$ real for all θ_i
  $\theta_t < \theta_i$ transmitted bent towards normal



- if $c_1 < c_2$ (e.g. slow $\Rightarrow$ fast, air $\rightarrow$ water)
  and $\theta_i < \theta_c$ then $\bar{\theta}_t \Rightarrow$ real and $> \theta_i$
  transmitted bent towards interface
  $\theta_c = \sin^{-1}(c_1/c_2)$



- if $c_1 < c_2$ and $\theta_i > \theta_c$ then

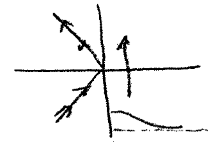
$$\bar{\theta}_t = -j\left[(c_2/c_1)^2 \sin^2\theta_i - 1\right]^{1/2} \Rightarrow \text{pure imaginary}$$

propagates parallel to interface $T_\pi = 0$, $R_\pi = 1$

"total internal reflection"

$$\bar{p}_t(x,y,t) = \bar{P}_t e^{-\gamma x} e^{j(\omega t - k_1 y \sin\theta_i)}$$

$$\gamma = k_2 \left[(c_2/c_1)^2 \sin^2\theta_i - 1 \right]^{1/2}$$



$$\bar{R} = e^{j\phi}$$

$$\phi = 2 \tan^{-1}\left\{ \left(\frac{\rho_1}{\rho_2}\right) \sqrt{(\cos\theta_c / \cos\theta_i)^2 - 1} \right\}$$

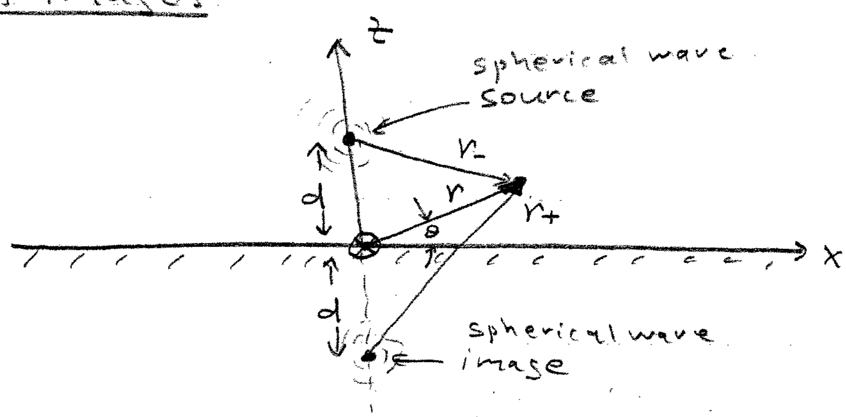
- if $r_1 < r_2$ and $c_1 > c_2$
  or $r_1 > r_2$ and $c_1 < c_2$

  then possibility of angle of intromission where

  $\bar{R} = 0$ and $T_\pi = 100\%$

$$\sin\theta_I = \left\{ \frac{(r_2/r_1)^2 - 1}{(r_2/r_1)^2 - (c_2/c_1)^2} \right\}^{1/2}$$

<u>method of images</u>



$$r_- = \sqrt{x^2 + y^2 + (z-d)^2}$$

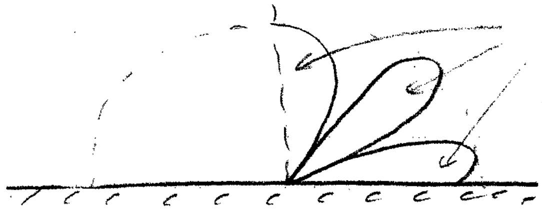
$$r_+ = \sqrt{x^2 + y^2 + (z+d)^2}$$

rigid boundary

$$\bar{p}(r,\theta,t) = A\left(\frac{1}{r_-} e^{-jkr_-} + \frac{1}{r_+} e^{-jkr_+} \right) e^{j\omega t}$$

in the far-field $r \gg d$

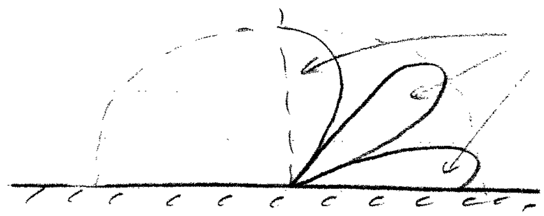
$$\bar{p}(r,\theta,t) = \frac{2A}{r} \cos(kd\sin\theta)\, e^{j(\omega t - kr)}$$



pressure release boundary

$$\bar{p}(r,\theta,t) = A\left(\frac{1}{r_-}e^{-jkr_-} - \frac{1}{r_+}e^{-jkr_+}\right)e^{j\omega t}$$

principle of acoustic reciprocity ⇒ if interchange location of source and receiver, the acoustic field at the receiver will be the same



pressure release boundary

$$\bar{p}(r,\theta,t) = A\left(\frac{1}{r_-}e^{-jkr_-} - \frac{1}{r_+}e^{-jkr_+}\right)e^{j\omega t}$$

principle of acoustic reciprocity $\Rightarrow$ if interchange location of source and receiver, the acoustic field at the receiver will be the same

Chapter 7 Radiation of Acoustic Waves

pulsating sphere

$$\vec{u}(a,t) = U_o e^{j\omega t}\,\hat{r}$$

$$\bar{p}(r,t) = \rho_o c\, U_o\,(a/r)\cos\theta_a\, e^{j[\omega t - k(r-a) + \theta_a]}$$

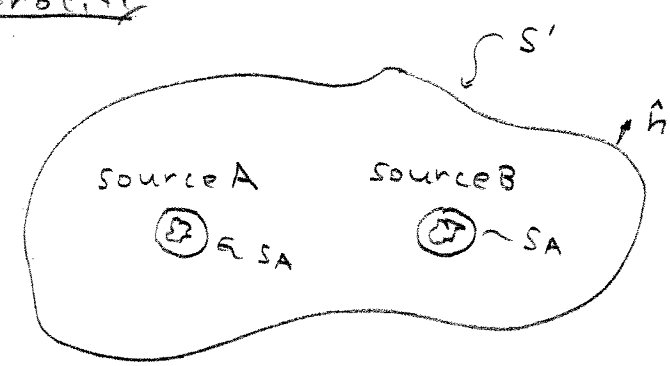
$$I(r) = \tfrac{1}{2}\rho_o c\, U_o^2\,(a/r)^2\cos^2\theta_a$$

$$\theta_a = \tan^{-1}(1/ka) \qquad \cos\theta_a = \frac{ka}{\sqrt{1+(ka)^2}}$$

if $ka \ll 1$ then $\cos\theta_a \simeq ka$, $\theta_a \simeq \pi/2$ so

$$ka \ll 1 \quad \left.\begin{aligned} \bar{p}(r,t) &\simeq j\rho_o c\, U_o (a/r)(ka)\, e^{j(\omega t - kr)} \\ I(r,t) &\simeq \tfrac{1}{2}\rho_o c\, U_o^2 (a/r)^2 (ka)^2 \end{aligned}\right\} \text{unbaffled simple source}$$

acoustic reciprocity



$$S = S' + S_A + S_B$$

Green's theorem

$$\int_S (\bar{\Phi}_1 \vec{\nabla} \bar{\Phi}_2 - \bar{\Phi}_2 \vec{\nabla} \bar{\Phi}_1) \cdot \hat{n}\, dS = \int_V (\bar{\Phi}_1 \nabla^2 \bar{\Phi}_2 - \bar{\Phi}_2 \nabla^2 \bar{\Phi}_1)\, dV$$

any $\bar{\Phi}_1, \bar{\Phi}_2$

$$\nabla^2 \bar{\Phi} + k^2 \bar{\Phi} = 0 \quad \vec{\bar{u}} = \vec{\nabla} \bar{\Phi}, \quad \bar{p} = -j\rho_0 \omega \bar{\Phi}$$

$$\int_S (\bar{p}_1 \vec{\bar{u}}_2 \cdot \hat{n} - \bar{p}_2 \vec{\bar{u}}_1 \cdot \hat{n})\, dS = 0$$

"simple source"
source size $\ll \lambda$

$$\frac{1}{\bar{p}_1} \int_{S_A} \vec{\bar{u}}_1 \cdot \hat{n}\, dS = \frac{1}{\bar{p}_2} \int_{S_B} \vec{\bar{u}}_2 \cdot \hat{n}\, dS$$

$$\bar{Q} e^{j\omega t} = \int_S \vec{\bar{u}} \cdot \hat{n}\, dS \qquad \bar{Q} = \text{source strength} \left[\frac{\text{volume}}{\text{time}} \right]$$

$$\boxed{\frac{\bar{Q}_1}{\bar{P}_1(r)} = \frac{\bar{Q}_2}{\bar{P}_2(r)}}$$

all simple sources radiate the same!

e.g. pulsating sphere with $ka \ll 1 \Rightarrow \lambda \gg 2\pi a$

$$Q = (4\pi a^2) U_0$$

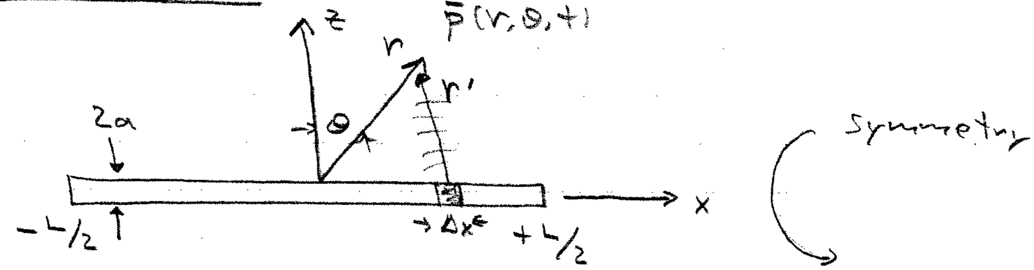
$$\bar{p}(r,t) = \tfrac{1}{2} j \rho_0 c \left(\bar{Q}/\lambda r \right) e^{j(\omega t - kr)}$$

unbaffled simple source

baffled simple source $\quad \bar{P}_{baffled} = 2\, \bar{p}_{unbaffled}$

for a large radiator $\Rightarrow$ divide surface into small elements, treat each element as a simple source, sum contributions of each simple source

continuous line source



$$\bar{p}(r,\theta,t) = \frac{1}{2} j \rho_0 c U_0 (ka) \int_{-L/2}^{L/2} \frac{1}{r'} e^{j(\omega t - kr')} \, dx$$

$$r' = \text{func}(x, r, \theta)$$

in the far-field $r \gg L$

angular dependence

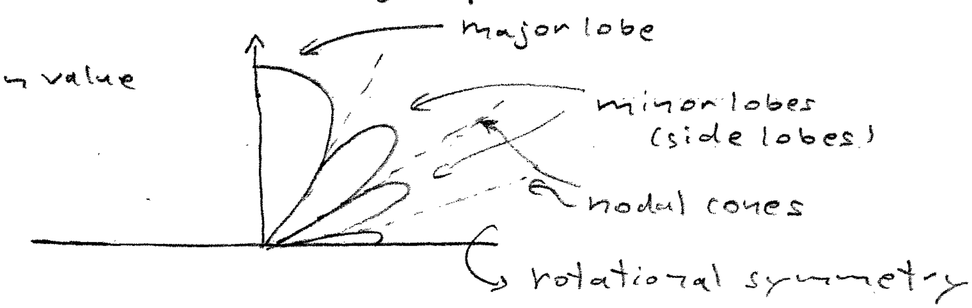
$$\bar{p}(r,\theta,t) = P_{ax}(r) \, H(\theta) \, e^{j(\omega t - kr)}$$

axial dependence ($\theta = 0$)

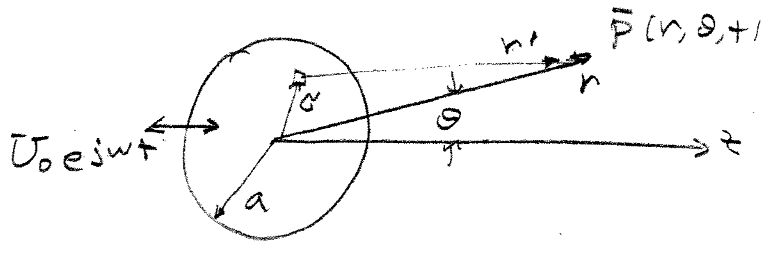
$$P_{ax}(r) = \frac{1}{2} \rho_0 c U_0 (a/r)(kL)$$

$$H(\theta) = |\text{sinc}(v)| = \left| \frac{\sin v}{v} \right| \qquad v = \frac{1}{2} kL \sin\theta$$

depending on value of kL



baffled plane circular piston



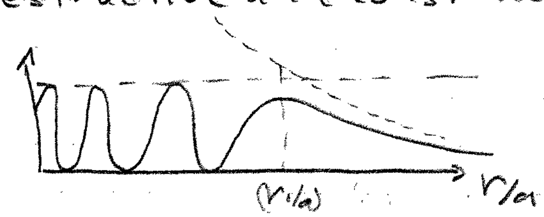
surface element $\Rightarrow$ baffled simple source

$$\bar{p}(r,\theta,t) = j\rho_0 c \frac{U_0}{\lambda} \int_0^{2\pi}\int_0^{a} \frac{1}{r'} e^{j(\omega t - kr')}\, \sigma\, d\sigma\, d\phi$$

along the axis $(\theta = 0)$

$$P(r,0) = 2\rho_0 c U_0 \sin\left[\tfrac{1}{2} kr \left(\sqrt{1+(a/r)^2} - 1\right)\right]$$

depending on value of ka, $P(r,0)$ may exhibit near-field destructive and constructive interference.



last local maximum $\quad r_1/a = a/\lambda - \lambda/4a$

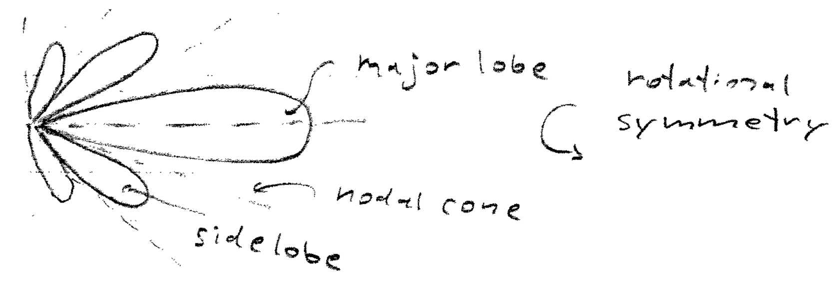
if $r > r_1$ then "far-field" and

$$P_{ax}(r) = \tfrac{1}{2}\rho_0 c U_0 (ka)(a/r) \sim 1/r$$

$$H(\theta) = \left|\frac{2J_1(v)}{v}\right| \qquad v = ka\sin\theta$$

$$|\bar{p}(r,\theta,t)| = P_{ax}(r)\, H(\theta)$$

depending on ka



<u>radiation impedance</u>

$$\bar{Z}_r = \bar{f}_s/\bar{u}_0 \qquad \bar{f}_s = \int_S \bar{p}\, dS'$$

circular piston $\quad \bar{Z}_r = \rho_0 c S\,[R_1(2ka) + jX_1(2ka)]$

$R_1(x),\ X_1(x) \rightarrow$ Fig 7.5.2 / $\rightarrow$ App A6

power $\Pi = \frac{1}{2} R_r U_0^2$

low frequency limit $ka \ll 1$

$$R_r = \tfrac{1}{2}\rho c S\,(ka)^2$$
$$X_r = (8/3\pi)\rho_0 c S\,(ka) \sim \omega$$

radiation mass loading $m_r = \dfrac{X_r}{\omega} = \rho_0 S\left(\dfrac{8a}{3\pi}\right)$

high frequency limit $ka \gg 1$ $\quad R_r \approx \rho_0 c S$

$$X_r = \rho_0 c S\,(2/\pi ka) \sim 1/\omega$$

pulsating sphere

$$\bar{Z}_r = \frac{\bar{P}_0 S}{\bar{u}_0} = \rho_0 c S \cos\theta_a\, e^{j\theta_a} \qquad \cos\theta_a = \frac{ka}{\sqrt{1+(ka)^2}}$$
$$\theta_a = \tan^{-1}(1/ka)$$

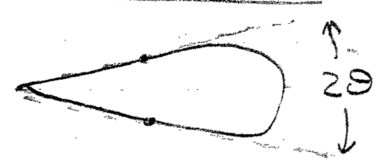
low frequency limit $ka \ll 1$ $\quad \bar{Z}_r \approx \rho_0 c S (ka)^2 + j\rho_0 c S\,(ka)$

$$m_r = X_r/\omega = \rho_0 S a$$

high frequency limit $ka \gg 1$ $\quad \bar{Z}_r \approx \rho_0 c S + j\rho_0 c S/(ka)$

<u>fundamental properties of radiators</u>

- beam width



2θ -3 dB, -6 dB, or -10 dB points

- directional factor $\quad P(r,\theta,\phi) = P_{ax}(r)\, H(\theta,\phi)$

  P_{ax}: major lobe axis $\sim 1/r$ $\qquad H$: direction factor ≤ 1

- source level $\quad SL = 20\log_{10}\left[\dfrac{P_e(r=1\,\mathrm{m})}{P_{ref}}\right]$ as if far-field

- directivity $D = \frac{P_{ax}^2(r)}{P_s^2(r)} = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} H^2(\theta,\phi)\sin\theta\, d\theta\, d\phi}$

S simple source of same power

$D \geq 1$

- directivity index $DI = 10 \log_{10} D \geq 0\,dB$
- beam pattern $b(\theta,\phi) = 20 \log_{10} H(\theta,\phi) \leq 0\,dB$
- estimates of radiation patterns

far-field $r_{min} \approx L^2/4\lambda$

beam width, $2\theta_1$ $\sin\theta_1 \approx \lambda/L$

Chap 8 Sound Absorption and Attenuation

sound absorption →
- viscous stresses (fluid friction)
- thermal conduction (heat transfer)
- molecular thermal relaxation

<u>viscous stress</u>

$$\tau_s = \left(\frac{4/3\,\eta + \eta_B}{\rho_0 c^2}\right)$$ relaxation time

$$\left(1 + \tau_s \frac{\partial}{\partial t}\right)\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$ "lossy" wave equation

spatial absorption coefficient

$$\bar{p}(x,t) = P_0 e^{-\alpha x} e^{j(\omega t - kx)}$$

low frequency limit $(\omega \tau_s) << 1$

$$\alpha_s = \frac{\omega^2}{2\rho_0 c^3}\left(4/3\,\eta + \eta_B\right) \quad [\text{Neper}/\text{length}]$$

$\sim \omega^2$

Chap 8 Sound Absorption and Attenuation (Summary)

sound absorption →
- viscous stresses (fluid friction)
- thermal conduction (heat transfer)
- molecular thermal relaxation

<u>spatial absorption coefficient</u>, α [Nepers/length]

complex wave number $\bar{k} = k - j\alpha$

plane wave

$$\bar{p}(x,t) = \bar{P} e^{j[\omega t - \bar{k}x]} = \bar{P} e^{-\alpha x} e^{j[\omega t - kx]}$$

$$I(x) = \frac{|\bar{p}|^2}{2\rho_0 c} = I(0)\, e^{-2\alpha x}$$

$$IL(0) - IL(x) = 10 \log_{10} \frac{IL(0)}{IL(x)} = 8.7\alpha x \ [dB]$$
$$= a x \ [dB]$$

$a = 8.7\alpha$ [dB/length]

<u>classical absorption coefficient</u> (viscous plus thermal) effects

$$\alpha_c = \frac{\omega^2}{2\rho_0 c^3}\left(\tfrac{4}{3}\eta + \frac{(\gamma-1)\kappa}{c_p}\right) \sim f^2$$

η = fluid viscosity, κ = fluid thermal conductivity

c_p = specific heat at constant pressure

γ = ratio of specific heats

good agreement for →
- noble gases (He, Ar, etc.)
- liquid metals (e.g. mercury)
- highly viscous liquids (e.g. glycerin)

mixed results for other gases and liquids

molecular thermal relaxation effects

graphical / curvefit results for → air, → freshwater, → seawater

wall (boundary layer) effects

within a pipe

$$\alpha_w = \frac{1}{ac}\left(\frac{\eta\omega}{2\rho_0}\right)^{1/2}\left(1 + \frac{\gamma - 1}{\sqrt{Pr}}\right) \sim \omega^{1/2}$$

$$Pr = \frac{\mu c_p}{\kappa} = \text{Prandtl number}$$

Chap 9 Cavities and Waveguides

rectangular cavity with rigid walls

$$\bar{p}(x,y,z,t) \sim e^{j\omega t}$$

$$\nabla^2 \bar{p} + k^2 \bar{p} = 0$$

separation of variables $\bar{p} = \bar{X}(x)\,\bar{Y}(y)\,\bar{Z}(z)\,e^{j\omega t}$

boundary conditions $\frac{\partial \bar{p}}{\partial n}$ at wall eigenfunctions (natural modes)

$$\bar{p}_{\ell mn}(x,y,z,t) = \bar{A}_{\ell mn} \cos\left(\frac{\ell\pi}{L_x}x\right)\cos\left(\frac{m\pi}{L_y}y\right)\cos\left(\frac{n\pi}{L_z}z\right) \times e^{j\omega_{\ell mn}t}$$

eigenfrequencies (natural frequencies)

$$\omega_{\ell mn} = c\left[\left(\frac{\ell\pi}{L_x}\right)^2 + \left(\frac{m\pi}{L_y}\right)^2 + \left(\frac{n\pi}{L_z}\right)^2\right]^{1/2}$$

$\ell, m, n \Rightarrow 0, 1, 2, 3 \cdots \infty$

general solution $\bar{p}(x,y,z,t) = \sum_{\ell,m,n} \bar{p}_{\ell mn}(x,y,z,t)$

$\bar{A}_{\ell mn}$ determined from initial conditions

<u>cylindrical cavity with rigid walls</u>

$$\bar{p}_{\ell mn}(r,\theta,z,t) = \bar{A}_{\ell mn} J_m(k_{mn}r)\cos(m\theta + \gamma_{\ell mn}) \times \cos\left(\frac{\ell\pi}{L_z}z\right) e^{j\omega_{\ell mn}t}$$

$$k_{mn} = \frac{j'_{mn}}{a}$$

$\ell = 0, 1, 2, 3 \cdots$
$m = 0, 1, 2, 3 \cdots$
$n = 1, 2, 3 \cdots$

$$\omega_{\ell mn} = c\sqrt{\left(\frac{j'_{mn}}{a}\right)^2 + \left(\frac{\ell\pi}{L_z}\right)^2}$$

<u>rectangular waveguide with rigid sidewalls</u>

$$\bar{p}_{\ell m}(x,y,z,t) = \bar{A}_{\ell m}\cos\left(\frac{\ell\pi}{L_x}x\right)\cos\left(\frac{m\pi}{L_y}\right) e^{j(\omega t - k_z z)}$$

$$k_z = \frac{1}{c}\left[\omega^2 - \omega_{\ell m}^2\right]^{1/2}$$

cut-off frequency for (l,m) mode

$$\omega_{lm} = c\left\{\left(\frac{l\pi}{L_x}\right)^2 + \left(\frac{m\pi}{L_y}\right)^2\right\}^{1/2}$$

- for $\omega > \omega_{lm}$ => propagating mode

phase speed $c_p = \dfrac{\omega}{k_z} = \dfrac{c}{[1-(\omega_{lm}/\omega)^2]^{1/2}}$

group speed $c_g = \dfrac{d\omega}{dk_z} = c\{1-(\omega_{lm}/\omega)^2\}^{1/2}$

- for $\omega < \omega_{lm}$ => evanescent mode (non propagating)

$$k_z = -j\,\frac{1}{c}\left[\omega_{lm}^2 - \omega^2\right]^{1/2}$$

$$\bar{p}_{lm}(x,y,z,t) = \bar{A}_{lm}\cos\left(\frac{l\pi}{L_x}x\right)\cos\left(\frac{m\pi}{L_y}y\right)e^{-\frac{1}{c}(\omega_{lm}^2-\omega^2)^{1/2}z} \times e^{j\omega t}$$

- plane wave mode, evanescent modes, "sloshing" modes

<u>circular waveguide with rigid sidewall</u>

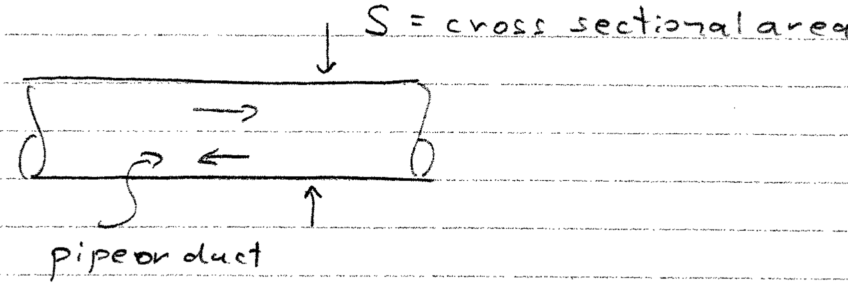
$$\bar{p}_{ml}(r,\theta,z,t) = \bar{A}_{ml}\,J_m(k_{ml}r)\cos(m\theta)\,e^{j(\omega t - k_z z)}$$

$$k_{ml} = \frac{j'_{ml}}{a} \qquad \begin{array}{l} m = 0,1,2\ldots \\ l = 1,2,3\ldots \end{array}$$

$$k_z = \frac{1}{c}\left[\omega^2 - \omega_{ml}^2\right]^{1/2}$$

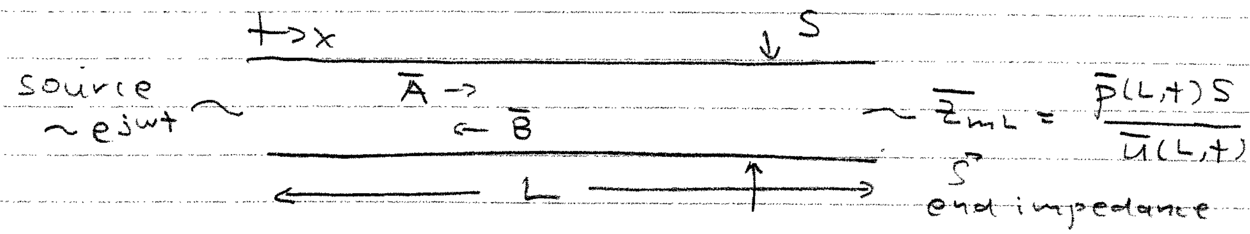
$\omega_{ml} = c\left(j'_{ml}/a\right)$ ~ cut-off frequencies

Chapter 10 Pipes, Resonators, and Filters



if $\lambda >> S^{1/2}$ => propagating plane waves only

Sec 10.2 resonance in pipes



$$\bar{p}(x,t) = \bar{A}\,e^{j[\omega t + k(L-x)]} + \bar{B}\,e^{j[\omega t - k(L-x)]}$$

$$\bar{Z}_{m0} = \frac{\bar{p}(0,t)S}{\bar{u}(0,t)}$$

inlet impedance

find
$$\boxed{\frac{\bar{Z}_{m0}}{(\rho_0 c S)} = \frac{(\bar{Z}_{mL}/\rho_0 c S) + j\tan(kL)}{1 + j\,(\bar{Z}_{mL}/\rho_0 c S)\tan(kL)} \quad Eq\,(10.2.4)}$$

$\rho_0 c S$ = char. mech. impedance of fluid in pipe

- resonance frequencies $\quad Im[\bar{Z}_{m0}(\omega)] = 0$

$$f_n = \left(\frac{c}{2L}\right)(n - 1/2) \quad \text{closed-end}$$

$$f_n = \frac{c}{L_{eff}}(n/2) \quad \text{open-end}$$

$$L_{eff} = L + 0.85a \quad \text{flanged}$$

$$= L + 0.6a \quad \text{unflanged}$$

Sec 10.3 power radiated from open-end pipes

- power transmission coefficient (open end)

$$T_\pi = \frac{\Pi_{out}}{\Pi_{incident}} \simeq 2(ka)^2 \text{ flanged}$$
$$\simeq (ka)^2 \text{ unflanged}$$

- power in $\displaystyle \Pi_{in} = \frac{1}{2}\frac{F^2 R_{mo}}{|\bar{Z}_{mo}|^2}$

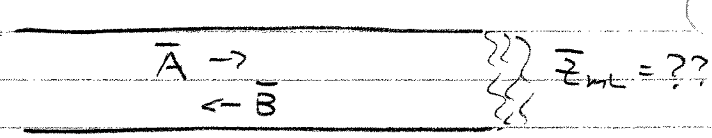
for unflanged

$$\Pi_n = \frac{2}{(n\pi)^2}\frac{F^2}{\rho_o c S}\left(\frac{L+0.6a}{a}\right)^2 \quad \text{Eq (10.3.9)}$$

$$\sim 1/n^2$$

Sec 10.4 standing wave patterns

standing wave tube



$$\boxed{\frac{\bar{Z}_{mL}}{\rho_o c S} = \frac{1+(B/A)e^{j\theta}}{1-(B/A)e^{j\theta}} \quad \text{Eq (10.4.2)} \quad \theta = \theta_B - \theta_A}$$

$$P(x) = \{(A+B)^2\cos^2[k(L-x) - \theta/2]$$
$$+ (A-B)^2 \sin^2\{k(L-x) - \theta/2\}\}^{1/2} \quad \text{Eq (10.4.3)}$$

nodes, anti-nodes

standing wave ratio $\displaystyle SWR = P_{max}/P_{min} = \frac{(A+B)}{(A-B)}$

$$B/A = \frac{SWR-1}{SWR+1}$$

$$\theta = 2k(L - x_1) - \pi$$

$(L - x_1)$ = distance of first node from end $(x = L)$

Sec 10.5 absorption of sound in pipes

same as before but now $\bar{k} = k - j\alpha$

standing wave tube with rigid termination

$\tilde{u}(L,t) = 0$

$$\frac{\bar{Z}_{mo}}{\rho_o c S} = \frac{\alpha L - j\cos(kL)\sin(kL)}{\sin^2(kL) + (\alpha L)^2\cos^2(kL)} \qquad \text{Eq } (10.5.4)$$

input power

$$\Pi = \frac{1}{2}\frac{F^2}{\rho_o c S}(\alpha L)\frac{[\sin^2(kL) + (\alpha L)^2\cos^2(kL)]}{(\alpha L)^2 + \cos^2(kL)\sin^2(kL)}$$

resonance $\qquad \Pi_r = \frac{1}{2}\frac{F^2}{\rho_o c S}\frac{1}{(\alpha L)} \Rightarrow$ "large"

antiresonance $\qquad \Pi_{ar} = \frac{1}{2}\frac{F^2}{\rho_o c S}(\alpha L) \Rightarrow$ "small"

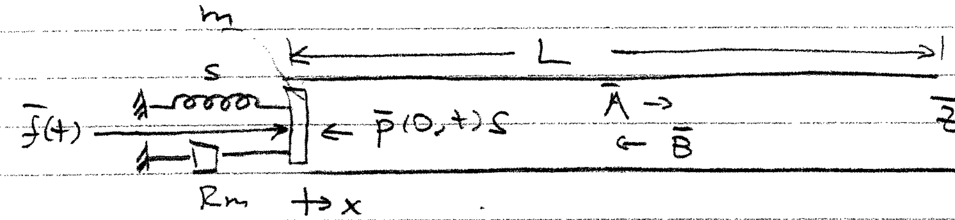
$$P(x) = 2P_L\{\cosh^2[\alpha(L-x)]\cos^2[k(L-x)] + \sinh^2[\alpha(L-x)]\sin^2[k(L-x)]\}^{1/2}$$

nodes $\qquad P_{min}/P_L = 2\sinh[\alpha(L-x)] \approx 2\alpha(L-x)$

anti-nodes $\qquad P_{max}/P_L = 2\cosh[\alpha(L-x)] \approx 2\left\{1 + \frac{\alpha^2(L-x)^2}{2}\right\}$

experimental determination of α

Sec 10.6 combined driver-pipe system



- @ $x = 0$ $\quad \bar{Z}_m = \dfrac{\bar{f}(t)}{\bar{u}(0,t)} = \bar{Z}_{md} + \bar{Z}_{mo}$

- resonance frequencies and peak pressure values affected by mechanical impedance of the driven

- for closed-end tube with absorption, resonance occurs when

$$\frac{\cos(kL)\sin(kL)}{\sin^2(kL) + (\alpha L)^2\cos^2(kL)} = a(kL) - b/(kL) \qquad Eq\ (10.6.8)$$

where $a = \dfrac{m}{\rho_0 SL} = \dfrac{m_{driver}}{m_{fluid\ in\ pipe}}$ and

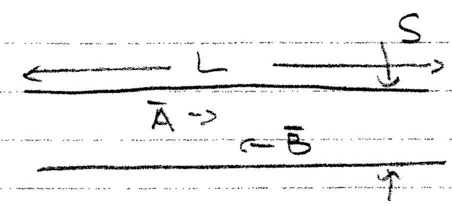
$$s = s/(\rho c^2 S/L) = \frac{s_{driver}}{s_{fluid\ in\ pipe}}$$

also follows

$$P(L) = \rho_0 c \frac{F}{|\bar{Z}_m|} \frac{1}{[\sin^2(kL) + (\alpha L)^2\cos^2(kL)]^{1/2}} \qquad Eq\ (10.6.11)$$

see Figs. 10.6.2 and 10.6.3

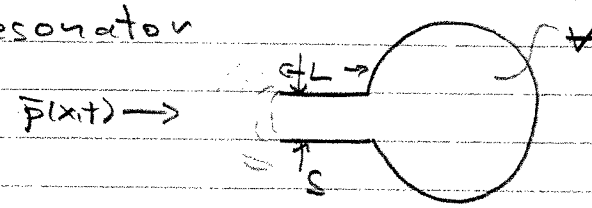
Sec 10.7 long wavelength limit and
Sec 10.8 the Helmholtz resonator



$\lambda >> S^{1/2} \Rightarrow$ plane wave only

$\lambda >> L \Rightarrow$ treat pipe section as <u>lumped acoustic element</u>

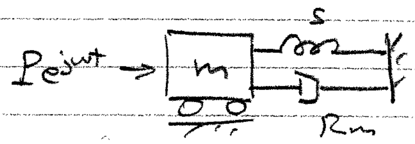
Helmholtz resonator



$\lambda >> S^{1/2}$

$\lambda >> L$

$\lambda >> V^{1/3}$



$m = \rho_0 S L'$

$L' = L + 1.7a$ outer flanged

$= L + 1.4a$ outer unflanged

$= 1.6a$ hole $(L = 0)$

$s = \rho_0 c^2 S^2 / V$

resonance $\omega_0 = \sqrt{s/m} = c\sqrt{\frac{S}{V L'}}$ Eq (10.8.8)

$R_m = R_r + R_w$ $R_w = 2mc\alpha_w$

$R_r = \rho_0 c k^2 S^2 / 2\pi$ outer flanged

$= \rho_0 c k^2 S^2 / 4\pi$ outer unflanged

$Q = \omega_0 m / R_m$

cavity

$P_c / P = Q$

applied

$Q = 2\pi \left[V (L'/S)^3 \right]^{1/2}$ Eq (10.8.12)

outer flanged, $R_w << R_r$

$Q_{unflanged} = 2 Q_{flanged}$

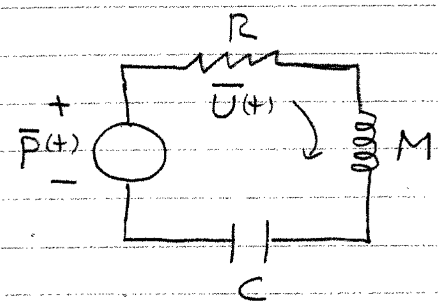
Sec 10.9 acoustic impedance

$$\bar{Z} = \frac{\bar{p}(x,t)}{\underbrace{\bar{U}(x,t)}_{\text{volume velocity}}} = \frac{\bar{p}(x,t)}{\bar{u}(x,t)S} = \frac{\bar{Z}_m}{S^2}$$

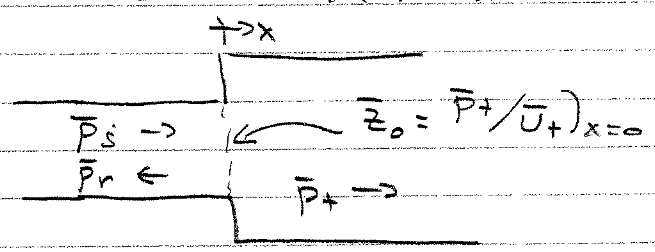
if $\bar{Z}_m = R_m + j(m\omega - s/\omega)$

then $\bar{Z} = \underbrace{R}_{\text{acoustic resistance}} + j(\underbrace{M}_{\text{acoustic inertance}}\omega - \frac{1}{C\omega})$ ← acoustic compliance

electric circuit analog



Sec 10.10 reflection and transmission in a pipe

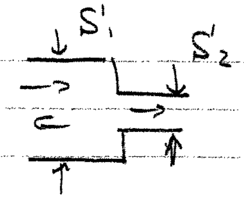


$$\bar{R} = \frac{\bar{Z}_0 - (\rho_0 c/S)}{\bar{Z}_0 + (\rho_0 c/S)} \qquad \text{Eq } (10.10.5)$$

$$R_\pi = \frac{[R_0 - (\rho_0 c/S)]^2 + X_0^2}{[R_0 + (\rho_0 c/S)]^2 + X_0^2} \qquad \text{Eq } (10.10.6)$$

$$T_\pi = 1 - R_\pi = \frac{4(\rho_0 c/S)R_0}{[R_0 + (\rho_0 c/S)]^2 + X_0^2} \qquad \text{Eq } (10.10.8)$$

– e.g. simple area change

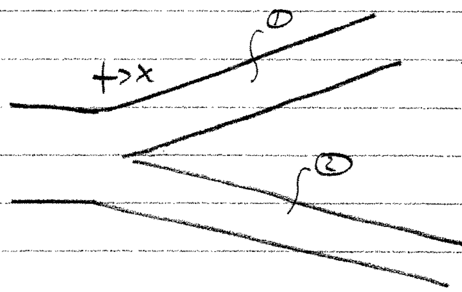


$\bar{Z}_0 = \rho_0 c / S_2'$

$$\bar{R} = \frac{S_1 - S_2}{S_1 + S_2}$$

$$R_\pi = \frac{(S_1 - S_2)^2}{(S_1 + S_2)^2} \qquad T_\pi = \frac{4 S_1 S_2}{(S_1 + S_2)^2}$$

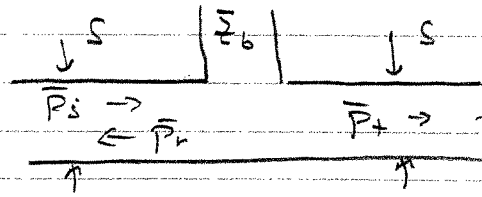
– branch



$$\frac{1}{\bar{Z}_0} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2}$$

acoustic admittance

– side branch

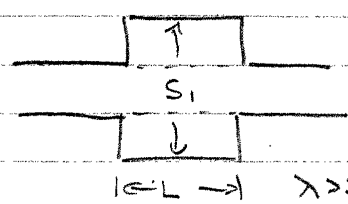


$\sim \bar{Z}_2 = \rho_0 c / S$

$\bar{R}, \bar{T}, R_\pi, T_\pi, T_{\pi b} \Rightarrow$ Eqs (10.10.15) → (10.10.17)

Sec 10.11 acoustic filters

a.) low pass



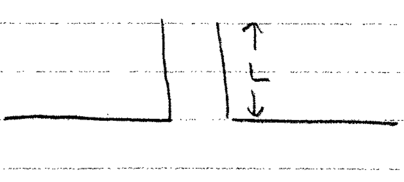
$\lambda \gg L$

Eq (10.11.12)

$$T_\pi \approx \frac{1}{1 + \left(\frac{S_1 - S}{2S} kL\right)^2}$$

$kL < 1$

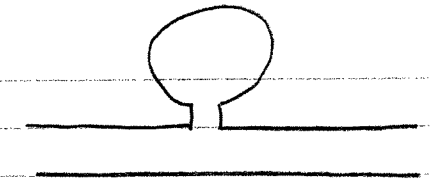
b.) high pass



Eq (10.11.5)

$$T_\pi \approx \frac{1}{1 + (\pi a^2 / 2 S L' k)^2}$$

c.) band stop



$$T_\pi = \frac{1}{1 + \left\{ \dfrac{c/2S}{(\omega L'/S_b) - (c^2/\omega V)} \right\}^2} \quad Eq(10.11.9)$$

$$T_\pi = 0 \;@\; \omega = c\left(S_b/L'V\right)^{1/2} = \omega_0\big)_{\text{Helmholtz}}$$