# PROOF

If $\bar{f}(t) = \bar{F} e^{j\omega t}$ and $\bar{g}(t) = \bar{G} e^{j\omega t}$ then

$$\langle \mathrm{Re}\{\bar{f}(t)\}\,\mathrm{Re}\{\bar{g}(t)\}\rangle = \frac{1}{T}\int_0^T \mathrm{Re}\{\bar{f}(t)\}\,\mathrm{Re}\{\bar{g}(t)\}\,dt$$

$$= \frac{1}{2}\mathrm{Re}\{\bar{F}\bar{G}^*\} = \frac{1}{2}\mathrm{Re}\{\bar{F}^*\bar{G}\}$$

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$$\frac{1}{T}\int_0^T \mathrm{Re}\{\bar{f}(t)\}\,\mathrm{Re}\{\bar{g}(t)\}\,dt = \frac{1}{T}\int_0^T \left[\frac{\bar{f}(t)+\bar{f}^*(t)}{2}\right]\left\{\frac{\bar{g}(t)+\bar{g}^*(t)}{2}\right\}dt$$

$$= \frac{1}{4T}\int_0^T \left[\bar{f}(t)\bar{g}(t) + \bar{f}(t)\bar{g}^*(t) + \bar{f}^*(t)\bar{g}(t) + \bar{f}^*(t)\bar{g}^*(t)\right]dt$$

$$= \frac{1}{4T}\int_0^T \left[\bar{F}\bar{G}e^{j2\omega t} + \bar{F}\bar{G}^* + \bar{F}^*\bar{G} + \bar{F}^*\bar{G}^*e^{-j2\omega t}\right]dt$$

note: $\underline{\int_0^T e^{\pm j2\omega t}\,dt} = \left.\frac{e^{\pm j2\omega t}}{(\pm j2\omega)}\right|_0^T = \left.\frac{e^{\pm j4\pi(t/T)}}{(\pm j2\omega)}\right|_0^T$

$$= \frac{e^{\pm j4\pi}-1}{(\pm j2\omega)} = \frac{1-1}{(\pm j2\omega)} = \underline{0}$$

so

$$\langle \mathrm{Re}\{\bar{f}(t)\}\,\mathrm{Re}\{\bar{g}(t)\}\rangle = \frac{1}{4\cancel{T}}\left[\bar{F}\bar{G}^* + \bar{F}^*\bar{G}\right]\cancel{T}$$

$$= \frac{2\,\mathrm{Re}\{\bar{F}\bar{G}^*\}}{4} = \frac{1}{2}\mathrm{Re}\{\bar{F}\bar{G}^*\}$$

$$= \frac{1}{2}\mathrm{Re}\{\bar{F}^*\bar{G}\} \quad \therefore$$