- imaginary unit, i : defined such that $i^2 = -1$

- complex variable, z : an ordered pair of real variables (x, y) such that $z = x + iy$

- complex conjugate of z : $z^* = x - iy$

- $Re[z] = x$ and $Im[z] = y$

- equality: $z_1 = z_2$ if $x_1 = x_2$ and $y_1 = y_2$

- addition: $z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$

- subtraction: $z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

- multiplication: $z_1 z_2 = (x_1 + i y_1)(x_2 + i y_2)$
$$= (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$$

- division: $z_1/z_2 = z_1 z_2^* / z_2 z_2^*$
$$= \frac{(x_1 x_2 + y_1 y_2) + i(-x_1 y_2 + x_2 y_1)}{(x_2^2 + y_2^2)}$$

- any complex variable can be represented as a point on the complex plane
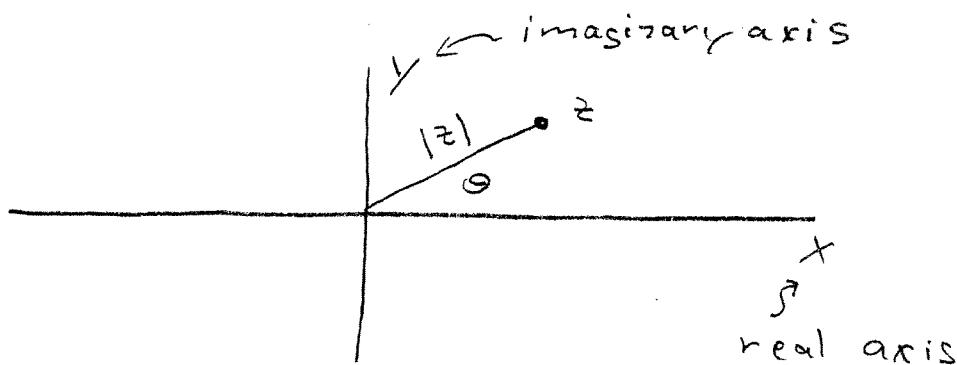
— polar form $z = |z|(\cos\theta + i\sin\theta)$

where $|z| = $ modulus $z = (x^2 + y^2)^{1/2}$

and $\theta = $ argument $z = \tan^{-1}\frac{y}{x} \pm \pi$ if $x < 0$

principal value $-\pi < \theta < \pi$

— Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \qquad \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

— exponential polar form $z = |z|e^{i\theta}$

where $|z| = (zz^*)^{1/2}$

— conjugate : $z^* = |z|e^{-i\theta}$

— multiplication : $z_1 z_2 = |z_1||z_2|e^{i(\theta_1 + \theta_2)}$

— division : $z_1/z_2 = \frac{|z_1|}{|z_2|}e^{i(\theta_1 - \theta_2)}$

— conjugate of a sum : $(z_1 + z_2)^* = z_1^* + z_2^*$

— conjugate of a product : $(z_1 z_2)^* = z_1^* z_2^*$

— conjugate of a quotient : $(z_1/z_2)^* = z_1^*/z_2^*$

— a complex function $f(z)$ is analytic in region R if the derivative of $f(z)$ exists for each point in R

— series expressions of common complex functions

$$e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \cdots \frac{z^n}{n!} + \cdots$$

$$\sin(z) = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \cdots$$

$$\cos(z) = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + \cdots$$

— derivatives of common complex functions

$$\frac{d[e^{az}]}{dz} = a e^{az}$$

$$\frac{d[\sin(az)]}{dz} = a\cos(az)$$

$$\frac{d[\cos(az)]}{dz} = -a\sin(az)$$

$$\frac{d(z^n)}{dz} = n z^{n-1}$$