Lecture #19 Intro. to Acoustics of Fluids
H.W. #13 Prob 5.6.1, 5.6.3, 5.7.3
Read Secs 5.10 → 5.12

last time

linearized continuity equation $\quad \frac{\partial s}{\partial t} + \vec{\nabla} \cdot \vec{u} = 0 \quad$ Eq 5.3.5

linearized conserv. of mom. eqn. $\quad \rho_0 \frac{\partial \vec{u}}{\partial t} = -\vec{\nabla} p \quad$ Eq (5.4.10)

$$p = \mathcal{B} s$$

combine, eliminate $\vec{u}, s \Rightarrow p$

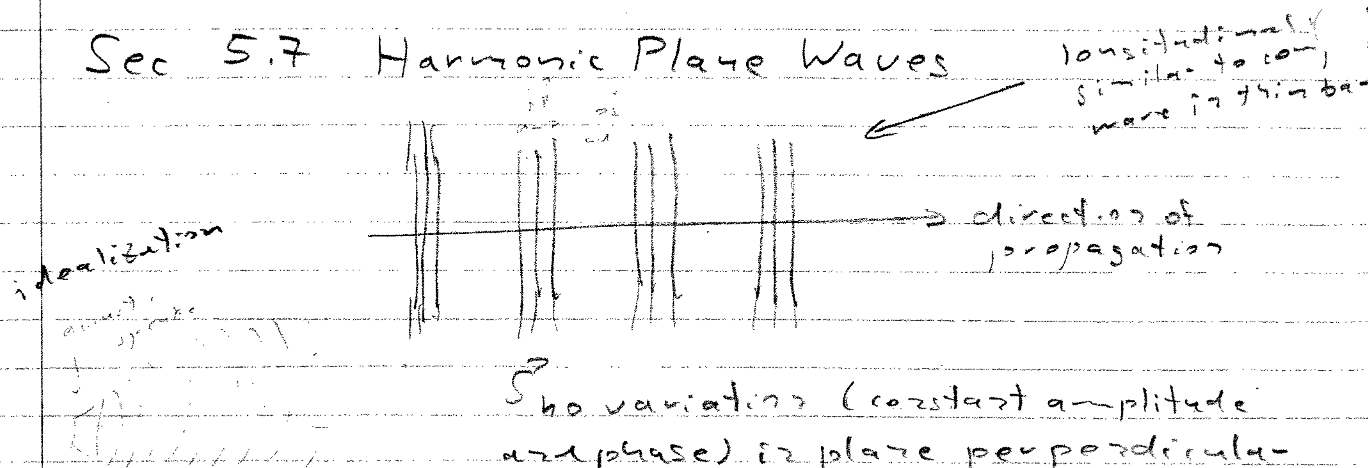
$$\boxed{\frac{\partial^2 p}{\partial t^2} = c^2 \nabla^2 p \quad \text{Eq}(5.5.4)}$$

$$c = \sqrt{\frac{\mathcal{B}}{\rho_0}} \qquad c_{\substack{\text{ideal}\\\text{gas}}} = \sqrt{\gamma \frac{R}{M} T_K}$$

also for ideal gas found $\mathcal{B} = \gamma \mathcal{P} = \gamma \mathcal{B}_T \quad \mathcal{B}_T = \mathcal{P}$

$\int$

Sec 5.7 Harmonic Plane Waves



no variation (constant amplitude and phase) in plane perpendicular to direction of propagation

if direction of propagation is in the x-direction then (analogous to compression waves along axis of a bar, Chap 3)

mathematical expression for acoustic pressure for a harmonic plane wave would be

$$\bar{p}(x,t) = \bar{A} e^{j(\omega t - kx)} + \bar{B} e^{j(\omega t + kx)}$$

+x prop. $\bar{p}_+(x,t)$ Eq (5.7.2) −x prop. $\bar{p}_-(x,t)$

(physical part is real part)

now from linearized momentum equation

$$\rho_0 \frac{\partial \bar{u}}{\partial t} = -\nabla p$$

$$j\omega \rho_0 \bar{u} \cancel{\hat{x}} = -\frac{\partial \bar{p}}{\partial x} \cancel{\hat{x}} \qquad \bar{u} = u\hat{x}$$

$$j\omega \rho_0 \bar{u}_\pm = -(\mp jk)\, \bar{p}_\pm$$

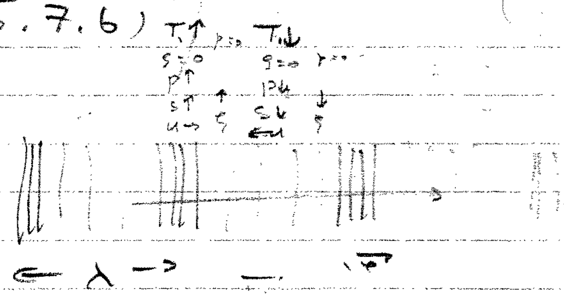
$$\bar{u}_\pm = \pm \frac{\bar{p}_\pm}{\rho_0 c} \quad \text{Eq (5.7.5)}$$

$$s = \frac{p}{\mathcal{B}} \times \frac{\rho_0}{\rho_0} = \frac{p}{\rho_0 c^2}$$

Lecture # 20 Intro. to Acoustics of Fluids
H.W. #14 Prob 5.7.4, 5.9.1, 5.9.2, 5.9.3

last time harmonic plane wave prop. $\pm x$ direction

now plane wave prop. in arbitrary direction so $\quad \overline{S}_{\pm} = \dfrac{\overline{P}_{\pm}}{\rho_o c^2} \quad$ Eq (5.7.6)



$\overline{p}, \overline{u}, \overline{s} \Rightarrow$ all in phase

$$p = -\rho_o \frac{\partial \Phi}{\partial t}$$

$$\overline{p}_{\pm} = -j\rho_o \omega \overline{\Phi}_{\pm}$$

$$\boxed{\overline{\Phi} = \frac{-\overline{p}_{\pm}}{j\rho_o \omega} \quad \text{Eq (5.7.7)}}$$

Φ 90° out of phase with p

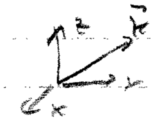
also $\quad u = \dfrac{\partial \xi}{\partial t} \Rightarrow \overline{u} = j\omega \overline{\xi}$

$$\boxed{\overline{\xi}_{\pm} = \frac{\overline{u}_{\pm}}{j\omega} = \frac{\pm \overline{p}_{\pm}}{j\omega\rho_o c}}$$

$\overline{\xi}$ 90° out of phase with $\overline{p}$

also $\overline{\xi} \sim 1/\omega$

for plane wave propagating in arbitrary direction



propagation vector (wavenumber has direction as well as magnitude)

$$\vec{k} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z} = k\hat{k}$$

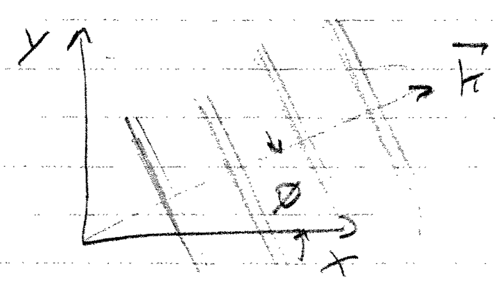
$$k = \sqrt{k_x^2 + k_y^2 + k_z^2} = (\omega/c)$$

$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$ = spatial position vector

then

$$\overline{p}(\vec{r}, t) = \overline{A} e^{j(\omega t - \vec{k}\cdot\vec{r})} \quad \text{Eq (5.7.12)}$$

Consider plane wave propagating parallel to the x-y plane



$$\bar{k} = k_x \hat{x} + k_y \hat{y}$$

$$= k\cos\phi\,\hat{x} + k\sin\phi\,\hat{y}$$

$$\bar{r} = x\hat{x} + y\hat{y} + z\hat{z}$$ position vector

$$\bar{k}\cdot\bar{r} = k\cos\phi\, x + k\sin\phi\, y$$

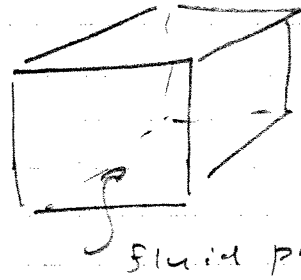
thus

$$\bar{p} = \bar{A}e^{j(\omega t - k\cos\phi x - k\sin\phi y)}$$

(note for $\phi \Rightarrow 0$, reduces to $+x$-dir prop. plane wave)

$\phi = 180°, +90°, -90°$

Sec 5.8 acoustic energy density



$\mathcal{V}$ = volume of fluid particle $\neq$ constant

fluid particle (moves with fluid)

acoustic kinetic energy $E_k = \frac{1}{2} m_p u^2$ ← acoustic velocity

$$= \frac{1}{2}(\rho_0 \mathcal{V}_0) u^2$$

acoustic potential energy due to compression

(p d$\mathcal{V}$ work of thermo.)

$$E_p = -\int_{\mathcal{V}_0}^{\mathcal{V}} p \, d\mathcal{V} \quad \leftarrow \text{work in for compression!}$$

consv. of mass $\rho \mathcal{V} = \rho_0 \mathcal{V}_0$ = constant $\Rightarrow \mathcal{V} = \frac{\rho_0 \mathcal{V}_0}{\rho}$

$$\mathcal{V} = \frac{\rho_0 \mathcal{V}_0}{\rho_0(1+s)} = \mathcal{V}_0(1-s) = \mathcal{V}_0 - \frac{\mathcal{V}_0 p}{\mathcal{B}_0} \times \frac{\rho_0}{\rho_0} = \mathcal{V}_0 - \frac{\mathcal{V}_0}{\rho c^2} p$$

$$d\mathcal{V} = -\frac{\mathcal{V}_0}{\rho_0 c^2} dp$$

change variables

$$E_p = + \frac{\mathcal{V}_0}{\rho_0 c^2} \int_0^p p' \, dp' \Rightarrow E_p = \frac{\mathcal{V}_0}{\rho_0 c^2} \frac{p^2}{2}$$

capital script E

$$E_p = \frac{1}{2} \frac{p^2}{\rho_0 c^2} \mathcal{V}_0$$

instantaneous energy density

$$\mathcal{E}_i(\vec{r}, t) = \frac{E_p + E_k}{\mathcal{V}_0} = \frac{1}{2} \rho_0 \left[u^2 + \left(\frac{p}{\rho_0 c} \right)^2 \right] \quad \text{Eq } (5.8.7)$$

nonlinear, use Real part!

if plane harmonic traveling-wave then

$$\overline{u} = \frac{\overline{p}}{\rho_o c}$$

or

$$u = \frac{p}{\rho_o c}$$

thus, for plane harmonic traveling-wave

$$\mathcal{E}_i = \rho_o u^2 = \frac{p^2}{\rho_o c^2} \qquad Eq\ (5.8.9)$$

$u = U\cos(\omega t - \vec{k}\cdot\vec{r} + \phi)$ $\quad p = P\cos(\omega t - \vec{k}\cdot\vec{r} + \phi)$ $\quad U$ = acoustic speed amplitude

in the time average then $\quad U = \frac{P}{\rho_o c}$ $\quad P$ = acoustic pressure amplitude

$\langle u^2\rangle = U^2/2 \quad \langle p^2\rangle = P^2/2$

$$\langle \mathcal{E}_i(\vec{r},t)\rangle = \mathcal{E} = \frac{1}{2}\rho_o U^2 = \frac{1}{2}\frac{P^2}{\rho_o c^2} = \frac{PU}{2c} \qquad Eq\ (5.8.10)$$

Sec 5.9 acoustic intensity

acoustic intensity => acoustic energy "flux"

=> rate of acoustic energy per unit area crossing a surface

instantaneous intensity $\quad \vec{I}(\vec{r},t) = p\,\vec{u}$ force per area, distance/time

acoustic wave

for a plane harmonic wave $\quad \vec{u} = p/\rho_o c\,\hat{k}$ so

$$\vec{I}(\vec{r},t) = \frac{p^2}{\rho_o c}\hat{k}$$

$$= \rho_o c u^2 \hat{k}$$

in the time average then

$$I = \langle I(\vec{r},t)\rangle = \frac{P^2}{2\rho_o c} = \frac{\rho_o c U^2}{2} = \frac{PU}{2}$$

or introducing the root-mean-square (effective) value

$$P_e = \left\{\frac{1}{T}\int_0^T p^2(t)\,dt\right\}^{1/2} = P/\sqrt{2} \qquad \frac{1}{\sqrt{2}} = 0.707$$

$$U_e = \left\{\frac{1}{T}\int_0^T u^2(t)\,dt\right\}^{1/2} = U/\sqrt{2}$$

$$I = \frac{P_e^2}{\rho_0 c} = \rho_0 c\, U_e^2 = P_e U_e$$

and $\mathcal{E} = \rho_0 U_e^2 = \dfrac{P_e^2}{\rho_0 c^2} = \dfrac{P_e U_e}{c}$ $\qquad p = P\cos(\omega t - kx)$, $P_e = P/\sqrt{2}$.

Sec 5.10 specific acoustic impedance

analogous to earlier development, can define specific (per unit area) acoustic impedance

$$\bar{z} = \bar{P}/\bar{u}$$

for plane waves $\quad \bar{u} = \bar{P}/\rho_0 c$

$$z = \underbrace{\rho_0 c}_{\text{purely real}} \qquad \text{Eq (5.10.2)}$$

legend
floating down the Nile boat
"Theory of Sound"
≈1877
Lord Rayleigh

$$\{\rho_0 c\} = \left\{\frac{F/L^2}{L/T}\right\} = \left\{\frac{\text{Pa}\cdot\text{sec}}{m}\right\} = \{\text{rayl}\}$$

$\rho_0 c \Rightarrow$ characteristic specific impedance of medium
$\Rightarrow$ see App A10 for values for various fluids

in general $\quad \bar{z} = r + jx$

r = specific acoustic resistance
x = specific acoustic reactance

(e.g. in pipes and ducts)

unless stated otherwise,
assume air @ 20°C

Lecture #21 Intro. to Acoustics of Fluids
H.W. #15 Prob 5.10.3, 5.11.3, 5.11.5, 5.12.3
Read Secs 6.1 → 6.2

last time

root-mean-square
$= P/\sqrt{2}$ for harmon

sound pressure level $SPL = 20 \log_{10}\left(\frac{P_e}{P_{ref}}\right)$ dB before discussing spherical waves, skip on to Sec 5.12 Decibel Scales

in air, human audible intensities extend over an incredible range

$$\approx 10^{-12}\ W/m^2 \quad \text{to} \quad \approx 100\ W/m^2 \leftarrow$$ 14 orders of magnitude!!

$\approx P_e \approx 20\,\mu Pa$ — threshold of hearing, $\approx 1000\,Hz$ — ≈ 3 billionths of a psi!

$\approx P_e \approx 200\ Pa\ (\approx 0.03\ psi)$ — threshold of hearing pain

human ear can only resolve differences in intensity $\approx \times 10$

to compress range, logrithmic scale devised

intensity level $\quad IL = 10 \log_{10} \left(\frac{I}{I_{ref}}\right)$ dB

but ordinarily $\quad I \sim P_e^2 \quad$ so equivalently

$$\boxed{\text{sound pressure level} \quad SPL = 20 \log_{10}\left(\frac{P_e}{P_{ref}}\right)\ dB}$$

in gases, ord. use $\quad P_{ref} = 20\,\mu Pa$

limit of linear acoustics $\quad \approx 20{,}000\,Pa \quad 180\,dB$

so sound	P_e	SPL re $20\,\mu Pa$
threshold of hearing	$\approx 20\,\mu Pa$	0 dB
conversational speech	$\approx 0.020\,Pa$	60 dB
OSHA 8 hr limit for noise exposure	$\approx 0.63\,Pa$	90 dB
threshold of pain	$\approx 200\,Pa$	140 dB

reverse

underwater acoustics, $P_{ref} = 1\,\mu Pa$ standard

EXAMPLE

say have $f = 1000\ Hz$, $P = 1\ Pa$ plane harmonic wave in air @ $T_o = 20°C$, $\mathcal{P}_o = 1 atm = 101.3\ kPa$

so
$$\bar{p}(x,t) = \bar{P} e^{j(\omega t - kx)}$$
$$= P e^{j\phi} e^{j(\omega t - kx)}$$
$$= P e^{j(\omega t - kx + \phi)}$$

$$p(x,t) = \mathrm{Re}\{\bar{p}(x,t)\} = P \cos(\omega t - kx + \phi)$$

calculate associated parameters

$$\underline{P_e} = \left\{ \frac{1}{T} \int_0^T p^2(x,t)\, dt \right\}^{1/2} = P/\sqrt{2} = \underline{0.707\ Pa}$$

$$\underline{SPL} = 20 \log_{10}\left(\frac{0.707\ Pa}{20\mu Pa}\right) = \underline{91.0\ dB\ re\ 20\mu Pa}$$

$c = \sqrt{\gamma r T_K}$ App A10 $\gamma = 1.402$
$r = 287\ J/kg \cdot K$
$T_K = (273 + 20)K = 293K$

$$\underline{c} = \sqrt{(1.402)(287\ J/kg \cdot K)(293K)} = \underline{343.4\ m/s}$$ 343 App 10

$= 1126.5\ ft/sec = 0.21\ \frac{mile}{sec} \approx \frac{1\ mile}{5\ sec}$ lightning flash sound of thunder

$\omega = 2\pi f = 6283\ rad/sec$ $k = \omega/c = 18.30/m = 2\pi/\lambda$

$\lambda = 2\pi/k = c/f = 0.3434\ m \approx 13.5\ inches \approx 1\ foot$

100 Hz $\Rightarrow \approx$ 10 ft (large speakers
10,000 Hz $\approx$ 1 inch — directional)

acoustic particle speed

$$\overline{u}_+ = \frac{\overline{P}_+}{\rho_o c}$$

$$\overline{u}(x,t) = \frac{\overline{P}}{\rho_o c} e^{j(\omega t - kx)} = \frac{P}{\rho_o c} e^{j(\omega t - kx + \phi)}$$

$$u(x,t) = \underbrace{\left(\frac{P}{\rho_o c}\right)}_{U} \cos(\omega t - kx + \phi)$$

$$U = P/\rho_o c$$

$$\rho_o = \frac{\mathcal{P}_o}{r T_K} = \frac{101.3 \times 10^3 \, N/m^2}{(287 \, J/kg \cdot K)(293 K)} ; \frac{J}{N \cdot m} = 1.205 \, kg/m^3$$

$$\underset{\sim}{z} = \rho_o c = (1.205 \, kg/m^3)(343.4 \, m/s) \times \frac{N \cdot s^2}{kg \cdot m}$$

$$= \underset{\sim}{413.7} \, \frac{Pa \cdot s}{m} \qquad (\text{App A10}, \ z = 415 \, \frac{Pa \cdot s}{m})$$

$$\underset{\sim}{U} = \frac{1 \, Pa}{413.7 \, \frac{Pa \cdot s}{m}} = 0.00242 \, m/s = \underline{2.42 \, mm/s}$$

acc. amplitude $A = \omega U = 15.2 \, m/s^2 \approx 1.5 g's$

condensation $\overline{s}_+ = \overline{P}_+/\mathcal{B} \Rightarrow s = P/\mathcal{B}$

$$c^2 = \mathcal{B}/\rho_o \qquad \mathcal{B} = \rho_o c^2 = \rho_o \left(\gamma \frac{\mathcal{P}_o}{\rho_o}\right) = \gamma \mathcal{P}_o$$

$$s = (P/\gamma \mathcal{P}_o) \cos(\omega t - kx + \phi)$$

$$S = P/\gamma \mathcal{P}_o = \frac{1 \, Pa}{(1.402)(101.3 \times 10^3 \, Pa)}$$

$$\underline{S = 7.04 \times 10^{-6}} \ll 1 \ !!$$

$$\bar{u} = \frac{\partial \bar{\xi}}{\partial t} = j\omega\bar{\xi}$$

$$\bar{\xi} = \frac{\bar{u}}{j\omega} = \frac{\bar{p}}{j\omega\rho_0 c} = -j\,\frac{\bar{p}}{\omega\rho_0 c} = \frac{-j\left(P e^{j(\omega t - kx + \phi)}\right)}{\omega\rho_0 c}$$

$$\xi = \mathrm{Re}\{\bar{\xi}\} = \underbrace{\left(\frac{P}{\omega\rho_0 c}\right)}_{\xi)_{amp}} \sin(\omega t - kx + \phi)$$

90° out-of-phase

$$\underline{\xi)}_{amp} = \frac{P}{\omega\rho_0 c} = \frac{U}{\omega} = \frac{2.42 \times 10^{-3}\ m/s}{2\pi(1000/sec)} = 3.85 \times 10^{-7}\ m$$

$= \underline{0.385}\ \mu m = 385\ nm$

$\approx \lambda_{\text{visible light}}$ (violet)

$\approx \frac{1}{100{,}000}$ th of an inch

(time-averaged) energy density $\quad \underline{\mathcal{E}} = \frac{1}{2}\frac{P^2}{\rho_0 c^2} = \frac{1}{2}\frac{P^2}{\gamma P_0} = 3.52 \times 10^{-6}\ J/m^3$

(time averaged) intensity $\quad \underline{I} = \frac{1}{2}\frac{P^2}{\rho_0 c} = c\,\mathcal{E} = 1.21 \times 10^{-3}\ W/m^2$

solar constant $\gg$ $I_{solar} = 1353\ W/m^2$

acoustic power of supersonic Concord at take-off $\approx$ acoustic power of world's population screaming at the same time

total acoustic energy of Concord at take-off

$\approx$ energy to fry one egg

H.W. Prob 5.9.2

$(T - T_0) = T' =$ acoustic temp $= ??$

ideal gas law $\quad \mathscr{P} = \rho r T \qquad T/T_0 = (\mathscr{P}/\mathscr{P}_0)^{\frac{\gamma-1}{\gamma}}$

$\mathscr{P} = \mathscr{P}_0 + p$

$\rho = \rho_0 + (\rho - \rho_0) = \rho_0 + \rho_0 s = \rho_0(1+s)$

$T = T_0 + T'$

$(\mathscr{P}_0 + p) = \rho_0(1+s) r (T_0 + T')$

$\cancel{\mathscr{P}_0} + p = \cancel{\rho_0 r T_0} + \rho_0 s r T_0 + \rho_0 r T' + \underbrace{\cancel{\rho_0 s r T'}}_{\text{2nd order}}$

$\frac{\rho_0 r T'}{\rho_0 r T_0} = \frac{p}{\mathscr{P}_0} - \frac{\rho_0 s r T_0}{\rho_0 r T_0} \Rightarrow T' = T_0\left\{ \frac{p}{\mathscr{P}_0} - s \right\}$

$T' = T_0\{ p/\mathscr{P}_0 - \underbrace{p/\gamma\mathscr{P}_0}_{\beta} \} = T_0\{1 - 1/\gamma\} p/\mathscr{P}_0 = T_0\left\{\frac{\gamma-1}{\gamma}\right\} p/\mathscr{P}_0$

5-19B

$$T'(x,t) = \underbrace{(293\,K)\left(\frac{0.402}{1.402}\right)\frac{1\ Pa}{101.3\times10^3\ Pa}}_{T')_{amp}} \cos(\omega t - kx + \phi)$$

$T')_{amp} = 8.30 \times 10^{-4}$ °C on adiabatic assumption!

Graduate Credit Handout!
Sound/noise demo??

Lecture #22 Intro. to Acoustics of Fluids
Read Sec. 6.3
no homework!

- return graded H.W., collect H.W.

- last time EXAMPLE

harmonic plane wave in air (20°C, 1 atm)
$f = 1000$ Hz, $P = 1.0$ Pa => SPL = 91.0 dB

now energy density, intensity

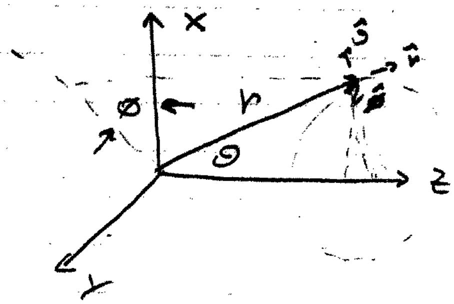
Sec 5-11 Spherical Waves

general wave equation

$$\frac{\partial^2 p}{\partial t^2} = c^2 \nabla^2 p$$

valid for <u>any</u> orthogonal coordinate system

e.g. spherical coordinates



$x = r \sin\theta \cos\phi$
$y = r \sin\theta \sin\phi$
$z = r \cos\theta$

from App A7 fig 1

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial}{\partial\theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

if assume spherical symmetry such that

$p(r,t)$ only (i.e., no θ, ϕ dep, spherical waves)

then

$$\nabla^2 p = \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right) p = \frac{1}{r}\frac{\partial^2 rp}{\partial r^2}$$

$$\frac{1}{r}\frac{\partial^2 rp}{\partial r^2} = \frac{1}{r}\frac{\partial}{\partial r}\left(p + r\frac{\partial p}{\partial r}\right) = \frac{1}{r}\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial r^2}$$

thus
$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{1}{r}\frac{\partial^2 (rp)}{\partial r^2}$$

$$\frac{\partial^2 (rp)}{\partial t^2} = c^2 \frac{\partial^2 (rp)}{\partial r^2}$$

so $\frac{\partial^2 (rp)}{\partial t^2} = c^2 \frac{\partial^2 (rp)}{\partial r^2}$ ← rp sat. rec. coord ordinary wave equation!!

$$rp = f_1(ct-r) + f_2(ct+r)$$

so expect $\quad p(r,t) = \underbrace{\frac{1}{r} f_1(ct-r)}_{\text{outgoing}} + \underbrace{\frac{1}{r} f_2(ct+r)}_{\text{incoming}}$

outgoing harmonic spherical wave

$$\boxed{\bar{p}(r,t) = \frac{\bar{A}}{r} e^{j(\omega t - kr)} \qquad \text{Eq } (5.11.6)}$$

pressure amplitude $\sim 1/r$ - !

$$\bar{\nabla} = \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\hat{\phi}$$

$$\rho_0 \frac{\partial \vec{u}}{\partial t} = -\bar{\nabla} p = -(\bar{\nabla} p)_r \hat{r}$$

acoustic velocity? $\quad \vec{u} = u(r,t)\,\hat{r}$

$$\rho_0 \frac{\partial u}{\partial t} = -(\bar{\nabla} p)_r = -\frac{\partial p}{\partial r}$$

$$\rho_0 \frac{\partial \bar{u}}{\partial t} = -\frac{\partial \bar{p}}{\partial r}$$

$$j\underbrace{\rho_0 \omega}_{k=\omega/c} \bar{u} = -\left[-\frac{1}{r} - jk\right] \bar{p}$$

$$\bar{u} = \frac{1}{j\rho_0 c}\left\{j + \frac{1}{kr}\right\} \bar{p} \qquad kr = 2\pi(r/\lambda)$$

$$\boxed{\bar{u} = \frac{1}{\rho_0 c}\left[1 - j/kr\right] \bar{p} \qquad \text{Eq } (5.11.8)}$$

$\bar{p}$ and $\bar{u}$ not in phase!

$$\bar{z} = \bar{p}/\bar{u} = \frac{\rho_0 c}{[1 - j/kr]} = \frac{(kr)\rho_0 c}{[(kr) - j]} \times \frac{[(kr) + j]}{[(kr) + j]}$$

note $\bar{z} \Rightarrow \rho_0 c$ as $(kr) \Rightarrow \infty$

$$= \rho_o c \frac{(kr)}{1+(kr)^2}\{(kr)+j\}$$

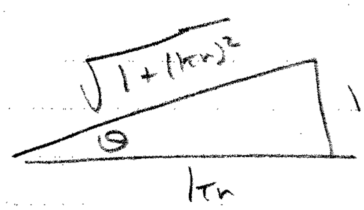
$$\boxed{\bar{Z} = \underbrace{\rho_o c \frac{(kr)^2}{1+(kr)^2}}_{\text{resistive}} + \underbrace{j\rho_o c \frac{(kr)}{1+(kr)^2}}_{\text{reactive}} \quad Eq\ (5.11.12)}$$

$$\bar{Z} = |\bar{Z}|\, e^{j\theta} \qquad |\bar{Z}| = (\bar{Z}\bar{Z}^*)^{1/2} = \frac{\rho_o c}{[1+(1/kr)^2]^{1/2}}$$

$$|\bar{Z}| = \rho_o c \frac{(kr)}{[1+(kr)^2]^{1/2}} \cancel{[(kr)^2+1]^{1/2}} = \rho_o c \frac{(kr)}{[1+(kr)^2]^{1/2}}$$

$$\theta = \tan^{-1}\left(\frac{1}{kr}\right) = \cot^{-1}(kr)$$

$$\bar{Z} = \rho_o c \frac{(kr)}{[1+(kr)^2]^{1/2}}\, e^{j\theta} \quad Eq\ (5.11.11)$$



$$\frac{(kr)}{\sqrt{1+(kr)^2}} = \cos\theta$$

$$\boxed{\bar{Z} = \rho_o c \cos\theta\, e^{j\theta} \quad Eq\ (5.11.10)}$$

if $\bar{p}(r,t) = \frac{\bar{A}}{r} e^{j(\omega t - kx)}$ then complex $\bar{U}$ = speed amplitude

$$\bar{u} = \bar{p}/\bar{Z} = \frac{(\bar{A}/r)e^{-j\theta}}{\rho_o c\cos\theta} e^{j(\omega t - kx)} \qquad \theta = \text{func}(r)!$$

$$\bar{U} = \frac{(A/r)}{\rho_o c\cos\theta}$$

intensity $I_s^r(t) = pu$

$$I = \langle I_s^r(t)\rangle = \frac{1}{2}\text{Real}[\bar{p}\,\bar{u}^*] = \frac{1}{2}\text{Real}\left\{\frac{|\bar{A}|^2/r^2}{\rho_o c\cos\theta} e^{j\theta}\right\}$$

$$I = \frac{|\bar{A}|^2/r^2}{2\rho_o c} = \frac{1}{2} P^2/\rho_o c \leftarrow \text{same as for plane wave!}$$

average power crossing spherical surface of radius r

$$\Pi = (4\pi r^2) I = 4\pi r^2 \frac{|\bar{A}|^2/\cancel{r^2}}{2\rho_0 c}$$

$$\Pi = 2\pi |\bar{A}|^2/\rho_0 c$$ ← ind. of r (as expected!)

SOUND DEMO!

Lecture #23 Acoustic Reflection and Transmission
Read Sec 6.4
H.W. #16 Prob 6.2.1, 6.3.2, 6.3.3

EXAM #2 Friday, March 25 (after S.B.)
Chap 5, 6, 7??

- tenative advanced notice on EXAM #2

- last time finished up Chap. 5, now begin
  Chapter 6 => Reflection and Transmission