Noise Synthetic Aperture Radar (SAR) Imagery Compressing and Reconstruction Based on Compressed Sensing

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Abstract: In this paper, a denoise approach is proposed to reduce the speckle noise in SAR images based on compress sensing. Through the skill of compressed sensing, we divide the image into some blocks, and propose an image reconstruction method based on block compressing sensing with Orthogonal Matching Pursuit. By adding some simulated speckle noise in the SAR image, the performance of the proposed approach is shown and compared with a conventional algorithm. the result has been shown that our method can get better result in terms of peak signal noise ratio (PSNR).

Keywords: SAR image, Block Compressed Sensing, Orthogonal Matching Pursuit

1. Introduction

Compressed Sensing (CS) is a sampling paradigm that provides the signal compression at a rate significantly below the Nyquist rate. Based on the CS theory, a sparse or compressible signal can be represented by the fewer number of bases than the one required by Nyquist theorem, when it is mapped to the space with bases incoherent to the sparse data space [1, 2]. Synthetic aperture radar (SAR) systems are all-weather, night and day, imaging systems. Due to the low computational resources of the acquisition platforms and the steadily increasing resolution of SAR systems, the data cannot generally be processed on board and must be stored or transmitted to the ground where the image formation process is performed [3]. The amount of image data produced is now constrained by on board storage capabilities and transmission links. However, in practical applications, the transform coefficients of SAR images usually have weak sparsity, especially when it includes speckle noise that arises from an imaging device and strongly hinders data interpretation. Exactly reconstructing these noise images is very challenging [4].

In this paper, we study noise SAR imagery data compressing and reconstruction based on CS. The contents of most references are about SAR imagery and raw data compressing and reconstruction based on CS theory. Reference [5] proposes a new method of fast encoding for SAR raw data by using the CS theory to complete SAR raw data compressing and reconstruction. Reference [6] gives a random sampling method for radar image compression. Reference [7] proposes an improved method by dividing the SAR imagery into several sub-imageries, and a modified Orthogonal Matching Pursuit (OMP) algorithm is proposed to perform better.

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In our study, we propose an approach to radar noise imaging based on the concept of CS. Block Compressed Sensing is applied to each signal in the ensemble to reconstruct noisy output.

1. Compressed Sensing

CS is based on the assumption of the sparse property of signal and incoherency between the bases of sparse domain and the bases of measurement vectors. CS has three major steps: the construction of *k*-sparse representation, the compression, and the reconstruction. The first step is the construction of *k*-sparse representation, where *k* is the number of the non-zero entries of sparse signal. Most natural signal can be made sparse by applying orthogonal transforms such as wavelet transform, Fast Fourier transform, and discrete cosine transform. This step is represented as [8]

$$s = \Psi^T x \tag{1}$$

Where x is an N-dimensional non-sparse signal; s is a weighted N-dimensional vector (sparse signal with k nonzero elements), and Ψ is an $N \times N$ orthogonal basis matrix. The second step is compression. In this step, the random measurement matrix is applied to the sparse signal according to the following equation

$$y = \Phi s = \Phi \Psi^T x \tag{2}$$

where Φ is an $M \times N$ random measurement matrix ($M < N$).

Let M be the number of measurements (the row dimension of **y**) sufficient for high probability of successful reconstruction, and M is determined by

$$M \geq C\mu^2(\Phi, \Psi) k \log N . \tag{3}$$

For some positive constant C, $\mu(\Phi, \Psi)$ is the coherence between Φ and Ψ , and defined by

$$\mu(\Phi, \Psi) = \sqrt{N} \max_{i,j} |< \phi_i, \psi_j >| \tag{4}$$

If the elements in ϕ and ψ are correlated, the coherence is large. Otherwise, it is small. From linear algebra, it is known that $\mu(\Phi, \Psi) \in [1, \sqrt{N}]$. In the measurement process, the noise may occur. The noise is added into the compressed measurement vector as follows

$$y = \Phi s + noise \tag{5}$$

where $noise$ is an M-dimensional noise vector.

2 Reconstruction method

The successful reconstruction depends on the measurement matrix Φ that complies with RIP (Restricted isometry property). RIP is defined as follows [9].

$$(1 - \delta_k) \| s \|_2^2 \leq \| \Phi s \|_2^2 \leq (1 + \delta_k) \| s \|_2^2 \tag{6}$$

where δ_k is the *k*-restricted isometry constant of a matrix Φ. RIP is used to ensure that all subsets of *k* columns taken from Φ are nearly orthogonal. It should be noted that Φ has more column than rows; thus, Φ cannot be exactly orthogonal.

The reconstruction is the optimization problem to solve (2). In (2), when Ψ is an identity matrix.The following equation is the reconstruction problem used in our study.

$$\arg\max_x \|x\|_0 \; s.t. y = \Phi x \tag{7}$$

3 Reconstruction Algorithms for Block Compressed Sensing

3.1 Block Compressed Sensing

An $N_1 \times N_2$ image is divided into small blocks with size of $n_1 \times n_2$ .Let f_i represent the vectorized signal of the i-th block through raster scanning, i =1, 2… *n*, and $n = N_1 N_2 / n_1 n_2$. We can get a m-dimensional sampled vector y_b through the following linear transformation [10]:

$$y_b = \Phi_B f_i + e \tag{8}$$

where f_i is an $n_1 n_2$-dimensional vector, Φ_B is an $m \times n_1 n_2$ measurement matrix, $m << n_1 n_2$, e is noise. Note that block CS is memory efficient as we just need to store a $m \times n_1 n_2$ Gaussian ensemble Φ_B, rather than a full $M \times N_1 N_2$ (i.e., $nm \times n_1 n_2$) one. Small requires less memory in storage and faster implementation, while large offers better reconstruction performance.

The main advantages of block-based CS can be summarized as follow: (1) Measurement operator can easily be stored and implemented through a random under-sampled filter bank; (2) Block-based measurement is more advantageous for real-time applications as the encoder does not need to send the sampled data until the whole image is measured; (3) Since each block is processed independently, the initial solution can be obtained and the reconstruction process is substantially speeded up[11].

3.2 Signal recovery algorithm

A prototype of the OMP algorithm first appeared in the statistics community in the 1950s. Later, the algorithm made a strong self-development in the signal processing, and approximation theory, etc. Let us now give a detailed description of the Orthogonal Matching Pursuit (OMP) algorithm [12].

Suppose that s is an arbitrary m-sparse signal in R^d, and let $\{x_1, \dots . x_N)$ be a family of N measurement vectors. Form an $N \times d$ matrix Φ whose rows are the measurement vectors, and observe that the N measurements of the signal can be collected in an N-dimensional data vector $v = \Phi s$. We refer to Φ as the measurement matrix and denote its columns by $\varphi_1, \dots, \varphi_d$.

As we mentioned, it is natural to think of signal recovery as a problem due to sparse approximation.

Since s has only m nonzero components, the data vector $v = \Phi s$ is a linear combination of m columns from Φ. In the language of sparse approximation, we say that v has an m-term representation over the dictionary Φ.

Therefore, sparse approximation algorithms can be used for recovering sparse signals. To identify the ideal signal s, we need to determine which columns of Φ participate in the measurement vector v. The idea behind the algorithm is to pick columns in a greedy fashion. At each iteration, we choose the column of Φ that is most strongly correlated with the remaining part of v. Then we subtract off its contribution to v and iterate on the residual. One hopes that, after m iterations, the algorithm will have identified the correct set of columns.

In our study, an improved OMP algorithm is used to signal recovery based on block compressing sensing, the detail algorithm is shown as follows.

Input:

(1)An $N \times d$ measurement matrix Φ

(2)An N-dimensional data vector v

(3)The sparsity level m of the ideal signal

(4)The small blocks size $n \times n$ and sample rate w ($w \in (0,1]$), $M = N * w$.

Output:

(1)An estimate $\hat{s}$ in $M \times d$ matrix R^d for the ideal signal

(2)A set Λ_m containing m elements from $\{1,\ldots,d\}$

(3) An N-dimensional approximation a_m of the data vector v.

(4)An N-dimensional residual $r_m = v - a_m$

Procedure

For each block $n_i \times n_i$ image procedure:

(1) Initialize the residual $r_0^{'} = v^{'}$, the index set $\Lambda_0^{'} = \varnothing$, and the iteration counter $t = 1$.

(2) Find the index $\lambda_t^{'}$ that solves the easy optimization problem

$$\lambda^{'}_t = \arg\max_{j=1,\ldots,d} \left| < r_{t-1}^{'}, \varphi_j^{'} > \right|$$

If the maximum occurs for multiple indices, break the tie deterministically.

(3)Augment the index set and the matrix of chosen atoms:

$\Lambda_t^{'} = \Lambda_{t-1}^{'} \bigcup \{\lambda_t^{'}\}$ and $\Phi_t^{'} = [\Phi_{t-1}^{'} \ \varphi^{'}\lambda_t^{'}]$. We use the convention that $\Phi_0^{'}$ is an empty matrix.

(4) Solve a least-squares problem to obtain a new signal estimate:

$$x_t' = \arg\min_x \left\| v' - \Phi_t' x' \right\|_2$$

(5) Calculate the new approximation of the data and the new residual:

$a_t = \Phi_t' x_t'$; $r_t' = v' - a_t'$

(6) Increment t, and return to Step 2 if $t < m$ or $r_{t-1} = 0$.

(7)The estimate $\hat{s}'$ for the ideal signal has nonzero indices at the components listed in Λ_m'. The value of the estimate $\hat{s}'$ in component λ_j' equals the j th component of x_t'.

(8) Each $\hat{s}'$ consist of $\hat{s}$, each Λ_m' consist of Λ_m.

End

4 Experiment and result

In order to evaluate the quality of the reconstructed results, the mean square error (MSE) and peak signal noise ratio (PSNR) can be utilized. They are defined as[13]

$$MSE = \frac{1}{M \times N} \sum_{i=1}^{M} \sum_{j=1}^{N} (\hat{f}(i,j) - f(i,j))^2 \tag{9}$$

$$\text{PSNR} = 10 \log_{10}\left(\frac{255^2}{MSE}\right) \tag{10}$$

Where M and N are the image dimensions, $\hat{f}$ is the denoised image, and f is the original noiseless image. In our study, the PSNR is used to compare the experiment result.

An original SAR image was used as a test image in figure 1(N=256). It was degraded by speckle noise 0.03 in figure 2. The denoise result based on conventional Compressed Sensing with matrix R's rows M=251 can be shown as figure 3, and the denoise result based on block Compressed Sensing with sample rate 0.98(M/N≈0.98) and blocks size 64×64 can be shown as figure 4.

Fig1 original image

Fig.2 speckle noise image

Fig.3 CS reconstruction (PSNR=20.98)   Fig.4 Block CS reconstruction (PSNR=22.48)

We can see from by comparing figures 3 and 4 that our method can reduce more noise result than the method based on conventional Compressed Sensing. The results for varying speckle noise, are summarized in table 1

Table 1: Quantization comparison of reconstructed results

method	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
CS (M=251)	23.73	22.27	20.98	20.19	19.47	18.93	18.39	17.83
BCS(rate=0.98, n=16)	26.34	24.26	22.88	21.67	20.99	20.22	19.58	19.10
BCS(rate=0.98,n=64)	26.18	23.81	22.48	21.44	20.63	19.87	19.34	18.85

From Table 1, we can see that the PSNR of the reconstructed results is improved. Our method can get better result than that based on conventional Compressed Sensing.

5 Conclusion

Removing speckle noise has become an essential step in SAR image analysis and improving image quality. The method of reducing speckle nose in SAR image is studied based on Compressed Sensing, and a novel denoise method is proposed based on block Compressed Sensing. The experiments have demonstrated that our approach had a better image denoising result.

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