

Super-Resolution Reconstruction of Compressed Sensing Mammogram based on Contourlet Transform¹

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ABSTRACT

Calcification detection in mammogram is important in breast cancer diagnosis. A super-resolution reconstruction method is proposed to reconstruct mammogram image from one single low resolution mammogram based on the compressed sensing by the contourlet transform. The initial estimation of the super-resolution mammogram is obtained by the interpolation method of the low resolution mammogram reconstructed by compressed sensing, then contourlet transform is applied respectively to the initial estimation and the reconstructed low resolution mammogram. From the statistical characteristics of the mutiscale frequency bands between the initial estimation and the reconstructed low resolution mammogram, the thresholds are estimated to integrate the high frequency of the initial estimation and the low frequency of the reconstructed low resolution mammogram. The super-resolution mammogram is achieved through the reconstruction of contourlet inverse transform. The proposed method can retrieve some details of the low resolution images. The calcification in mammogram can be detected efficiently.

Keywords: Mammogram, compressed sensing, super-resolution, reconstruction, calcification

1. INTRODUCTION

Super-resolution (SR) mammogram is important for calcification detection and analysis. It is helpful for breast cancer detection in a large scale, which demands the automatic machine perception and processing [1-3]. Super-resolution is a method that reconstructs super-resolution images from observed low-resolution images. The SR mammogram contains more detailed information of some diseases than low resolution (LR) mammogram. SR process increases the high-frequency components, therefore the detail information of the mammogram could be obtained [4, 5]. There are many SR methods including interpolation method [6], learning based method [7, 8], and statistical method [9, 10]. However, the interpolation method does not use the geometry information of images, so the reconstructed SR mammogram can't keep the edge detail information. The learning based algorithms need more images to get the relation between low and high resolution mammograms. The statistical algorithm requires the statistical model of different mammograms and increases the computation.

In recent years, the theory of compressed sensing (CS) [11, 12] has attracted much more attention in the fields of signal processing, medical imaging, radar imaging, and inverse problems. CS is a better alternative way to the Nyquist

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rate for the sampling of “sparse” signals. The CS theory, instead of uniformly sampling of the signal at Nyquist rate, reconstructs signals from measurements fewer than Nyquist sampling with little information loss. Therefore, it is a better way to introduce the CS into mammogram with less sampling to realize the reconstruction of mammogram.

The contourlet transform is the representation of directional multi-resolution image [13] that can efficiently capture boundaries and detect the singularities of the 2-D mammogram. The contourlet transform includes two steps: Laplacian pyramid (LP) [14] and the directional filter banks (DFB) [15]. The transform sketch is shown in Figure 1. The Laplacian pyramid decomposes a 2-D image iteratively into low-pass and high-pass sub frequency bands, and the DFBs are applied to the high-pass sub-bands to further decompose the frequency spectrum into different directions. Compared with wavelet transform, contourlet transform has a similar multi-scale structure, but there will be multi-directions in each scale.

A method of SR with the contourlet transform, based on compressed sensing, is proposed to overcome the limitation of interpolation and statistical SR methods. The new method will use the multi-scale and multi-direction of contourlet transform to capture the calcification geometry structures. The proposed method firstly uses compressed sensing to reconstruct the low resolution mammogram with the contourlet transform to obtain the low frequency bands. Secondly, the interpolation method is applied to the reconstructed mammogram to obtain the initial super-resolution estimate, then the contourlet transform is applied to get the high frequency bands of different directions. Thirdly, the low frequency bands of the reconstructed mammogram is integrated with the high frequency bands of the initial mammogram estimation. The thresholds of different scales are estimated by statistical characteristics of contourlet domain. Finally the contourlet inverse transform is applied to the obtain the super-resolution mammogram.

2. SUPER-RESOLUTION RECONSRUCTION METHOD

2.1 Mathematical model

During the process of mammogram imaging, the mammogram is effected by the noise, imaging system blur and digitalization, which result in the low resolution and degradation of mammograms. The mathematical model of the mammogram imaging is expressed in equation (1)

$$x_L = f * x_H + n \quad (1)$$

Where x_H is the high resolution image without pollution, f is the imaging system blur, n is the noise. x_L is the low resolution image after the process of regression. $*$ represents the convolution process. x_L can be sparsely represented in transform Ψ domain and s is the coefficients with respect to Ψ , y can be presented as

$$y = \Phi x_L = \Phi \Psi s \quad (2)$$

Where y is the sampled vector by the sensing matrix Φ of size $M*N$, Φ can be random Gaussian matrix, hadamard matrix and fourier matrix. The sparsity is often expressed as

$$\|s\|_0 = \|\Psi^* x_L\|_0 \leq N \quad (3)$$

Where $\|s\|_0$ denotes the l_0 norm and means the nonzero entries in s . Ψ and Ψ^* mean the inverse and forward sparse transforms respectively. The undersampled measurements y of the compressed sensing can be used to reconstruct image. The l_0 norm optimization is

$$\hat{s} = \min_s \|s\|_0, \quad \text{s.t.} \quad y = \Phi\Psi s \quad (4)$$

However, solving equation (4) is both numerically unstable and NP complete, so l_1 is commonly used to recover x_L

$$\hat{s} = \min_s \|s\|_1, \quad \text{s.t.} \quad y = \Phi\Psi s \quad (5)$$

The CS reconstructed mammogram is

$$\hat{x}_L = \Psi\hat{s} \quad (6)$$

The wavelet transform is widely used in image sparsity transform. The point singularities can be efficiently represented by wavelet transform, but the curves of different directions in the image can not be efficiently represented as contourlet transform does. Contourlet transform provides a flexible number of directions at each scale and thus can capture the intrinsic geometrical structure of images, as is shown Fig.2. Therefore, contourlet transform is adopted in this paper to better present the curves of mammogram, such as the different shape of calcifications and tumors. Therefore, Ψ is chosen as the contourlet transform.

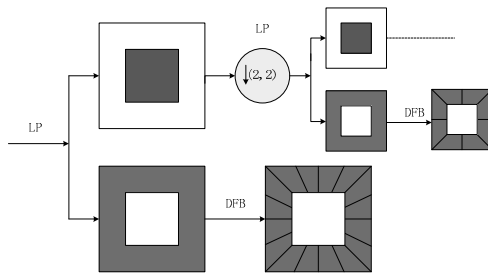


Fig. 1 Contourlet decomposition flow chart

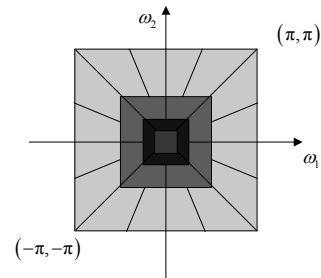


Fig. 2 Frequency domain decomposition

Fig.1 and Fig.2 have given the contourlet transform decomposition flow chart and the frequency domain structure. In Fig.1, the image is decomposed as the low frequency and high frequency bands by the Laplacian pyramid, then the high frequency is decomposed into different direction sub frequency bands by directional filter bank (DFB), each sub band contains the features of the detail information of image curves. The obtained low frequency band is decomposed again by LP and DFB filters, the next scale low frequency and high frequency band are obtained. This process will go on until the required scale is reached.

2.2 Proposed super-resolution reconstruction method

The traditional interpolation super-resolution reconstruct method uses the low resolution \hat{x}_L by interpolation method to get the high resolution image \hat{x}_H , however, the process will introduce noise and blur into the mammogram, including the low frequency bands and high frequency bands in contourlet domain. Assume the desired super-resolution image without pollution is x_H , the statistical characteristics of high frequency bands after contourlet transform is similar with that of \hat{x}_L , as is shown in Fig.3, but the positions and sizes of the coefficients are different, therefore, some detail information is lost. The initial estimation of the super-resolution image \hat{x}_H contains much detail high frequency information than \hat{x}_L , but not the same as x_H . The statistical characteristics of \hat{x}_L , \hat{x}_H and x_H are similar. The high frequency bands of \hat{x}_H is much similar with x_H although there are still noise in \hat{x}_H . \hat{x}_L has the basic information of x_H , so the low frequency bands of \hat{x}_L in contourlet domain can be considered as the basic information of x_H , the high frequency bands of \hat{x}_H in contourlet domain can be considered as the detail estimation of x_H . The integration of low

frequency bands of \hat{x}_L and high frequency bands of \hat{x}_H can generate the better reconstructed super-resolution mammogram than \hat{x}_H .

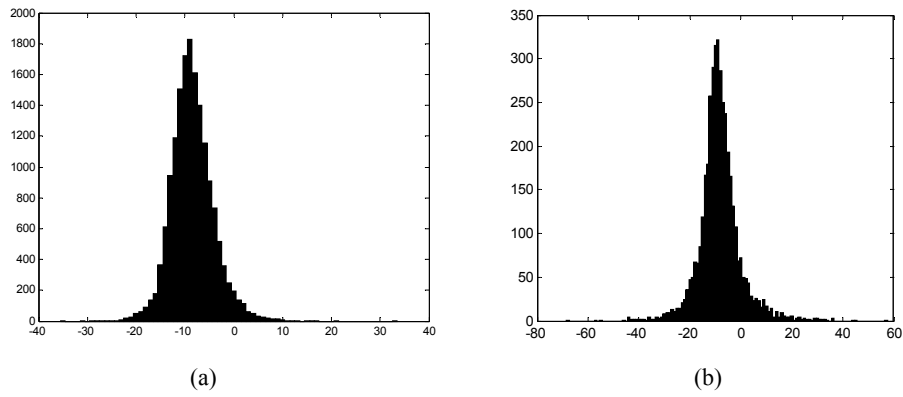


Fig.3 The statistical histogram of (a) x_H and (b) \hat{x}_L

The proposed super-resolution reconstruction of mammogram could be implemented by the following steps.

- 1) From equation (2), the sparsity measurements are obtained by contourlet transform Ψ . The sensing matrix Φ is chosen as the random Gaussian matrix. For example, this step is with decomposition level of three [2, 2, 3], the highest frequency subband is divided into 2^3 directions, the coefficients of which is written as γ .
- 2) Solve equation (5), the estimated contourlet coefficients \hat{s} are obtained by compressed sensing l_1 norm optimization.
- 3) The low resolution mammogram is reconstructed by contourlet inverse transform, $\hat{x}_L = \Psi\hat{s}$.
- 4) Interpolate \hat{x}_L , the initial high resolution estimation \hat{x}_H is obtained.
- 5) Contourlet transform is applied to \hat{x}_H to obtain the high frequency bands, this step is with decomposition level [2, 2, 3, 4], which means four decomposition levels and $2^2, 2^2, 2^3, 2^4$ directional subbands. The contourlet coefficients of level 4 is represented as β , this level has 16 different direction subbands. This decomposition will have high decomposition level than step 1). From the statistical of β and γ , the coefficients α of level 4 can be estimated.
- 6) \hat{s} from step 2) and α from step 5) are integrated to generate the SR contourlet coefficients and applied with the contourlet inverse transform, then the super-resolution mammogram \hat{x}_H is obtained.

3. EXPERIMENTAL RESULTS AND ANALYSIS

The experimental data are the lena image and the calcification mammogram. The ideal super-resolution lena image x_H is of the size 512×512 , the image is sampled into the low resolution image x_L with the size 256×256 . The experimental mammogram is provided by the Peking university people's hospital, the mammogram is a low resolution image of the size 256×256 . The computer is Intel P4 2.4GHz with 8G RAM.

The results of the proposed method is shown in Fig.4 and Fig.5.

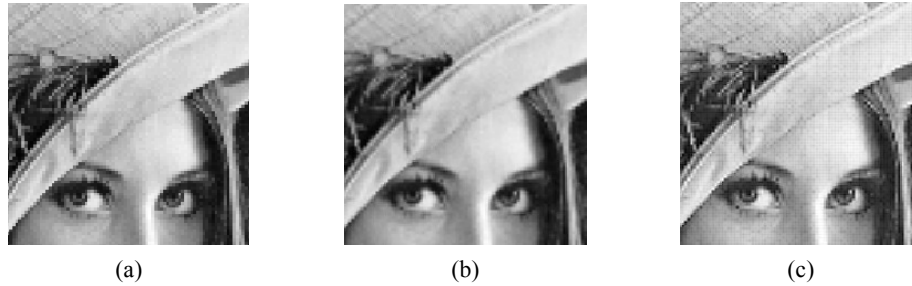


Fig.4 Comparison of the super-resolution reconstructed images, (a) The low resolution image (b) The reconstructed image by interpolate method (c) the reconstructed image by the proposed method.

Fig.4(a) is the low resolution image of size 256×256 . From Fig.4(c), the proposed method can keep the clear edges of the region around eyes and eyebrows, the braid on the hat has much clear edges than Fig4(b).

The proposed method is applied to calcification mammogram, the results are shown in Fig.5

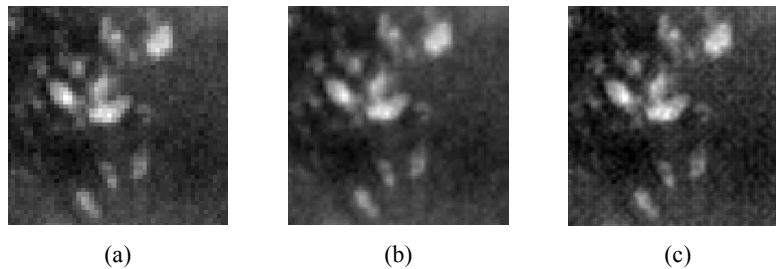


Fig.5 Comparison of the super-resolution reconstructed mammogram, (a) The low resolution mammogram (b) The reconstructed mammogram by interpolate method (c) the reconstructed mammogram by the proposed method.

Fig.5(a) is the low resolution mammogram, white area are the calcifications, the shape of the calcification is distorted. From Fig.5(b) and (c), we can see that the reconstructed mammogram by the proposed method has much clearer edges than the interpolation method and the shape of calcification is also reserved, which improves the accuracy extraction of the calcification in the successive process.

The advantage of this method is to avoid the training process of multiple images and the iterative process that adopted by many others SR reconstructed methods, which results in the smooth of the image and the blur of calcification edges, even the reconstruction accuracy will be dependent on many control parameters. The proposed method is much convenient to implement and the calcification geometry the mammogram are reconstructed clearly.

4. CONCLUSIONS

The super-resolution reconstruction of compressed sensing mammogram is important to accurately extract the calcification of mammogram, which is the key factor to detect the breast cancer. To obtain the features of the calcification shape, the contourlet transform is adopted to capture the curves of different directions of the mammogram. The compressed sensing is used to reduce the sampling rate during the imaging of the mammogram. The proposed method uses the low frequency of the compressed sensing reconstructed mammogram and the high frequency of the initial estimated super-resolution mammogram to generate the super-resolution reconstructed mammogram by contourlet transform. This reconstruction is under the direction of the statistical characteristics between the low resolution mammogram and the estimated high resolution mammogram. The experimental results show that the proposed method is effective in recovering the geometry of different shapes of calcifications in mammogram.

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