5.6 Dynamical Systems

$\underline{EX}$ *a Preditu – Prey System*

$$\begin{array}{ccc} \uparrow & & \uparrow \\ \text{owls} & , & \text{Rats} \end{array}$$

Let Q_0, R_0 represent the number of owls + rats living in an area, counted at some fixed initial time. Suppose the number of owls + rats K months later is Q_k, R_k where from one month to the next,

$$\begin{aligned} Q_{k+1} &= (.5)Q_k + (.4)R_k \\ R_{k+1} &= (-.104)Q_k + (1.1)R_k \end{aligned}$$

In matrix form,

$$\begin{bmatrix} Q_{k+1} \\ R_{k+1} \end{bmatrix} = \begin{bmatrix} .5 & .4 \\ -.104 & 1.1 \end{bmatrix} \begin{bmatrix} Q_k \\ R_k \end{bmatrix}$$

We can find a formula for the populations of owls + rats after K months using eigenvalues + eigenvectors.

The eigenvalues and eigenvector for A

$$A = \begin{bmatrix} .5 & .4 \\ -.104 & 1.1 \end{bmatrix}$$

are

$$\lambda_1 = 1.02 \quad \vec{V_1} = \begin{bmatrix} 10 \\ 13 \end{bmatrix}$$
$$\lambda_2 = .58 \quad \vec{V_2} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

Suppose

$$\begin{bmatrix} Q_0 \\ R_0 \end{bmatrix} = c_1 \begin{bmatrix} 10 \\ 13 \end{bmatrix} + c_2 \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

Then

$$\begin{bmatrix} Q_k \\ R_k \end{bmatrix} = c_1 (1.02)^k \begin{bmatrix} 10 \\ 13 \end{bmatrix} + c_2 (.58)^k \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

As K increases, $\quad (.58)^k \longrightarrow 0$

$$\begin{bmatrix} Q_k \\ R_k \end{bmatrix} \approx c_1(1.02)^k \begin{bmatrix} 10 \\ 13 \end{bmatrix}$$

Both Population will grow by about 2% each month