3.3 Cramer's Rule

Solving systems of equations

$$A\vec{X} = \vec{b}$$

using determinants provided

$A -\quad n \times n$ (square)

$A -$ invertible $(det A \neq 0)$

Let A_1 be the matrix A with column $\sharp 1$ replaced by $\vec{b}$ then

$$\begin{array}{lcl} det(A)X_1 & = & det(A_1) \\ X_1 & = & \frac{det(A_1)}{det(A)} \end{array}$$

Let A_2 be the matrix A with column 2 replaced by $\vec{b}$! Then

$$X_2 = \frac{det(A_2)}{det(A)}$$

A= $\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ $\quad A_2 = \begin{array}{c} \begin{array}{cc} & \vec{b} \\ & \downarrow \end{array} \\ \begin{bmatrix} 2 & 4 \\ 5 & 7 \end{bmatrix} \end{array}$

$$X_2 = \frac{det(A_2)}{det(A)} = \frac{14-20}{-2-15} = \frac{6}{17}$$

The solution to the system is $(\frac{25}{17}, \frac{6}{17})$

EX:

$$
\begin{array}{rcrcrcr}
X_1 & + & 2X_2 & + & 3X_3 & = & 4 \\
-X_1 & + & X_2 & & & = & 5 \\
& & X_2 & + & 4X_3 & = & 10
\end{array}
$$

$$
X_2 = \frac{\begin{vmatrix} 1 & 4 & 3 \\ -1 & 5 & 0 \\ 0 & 10 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 3 \\ -1 & 1 & 0 \\ 0 & 1 & 4 \end{vmatrix}} = \cdots
$$