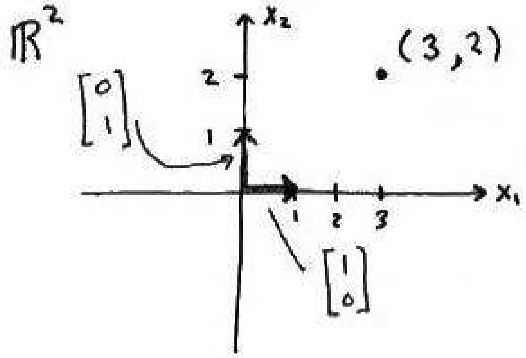
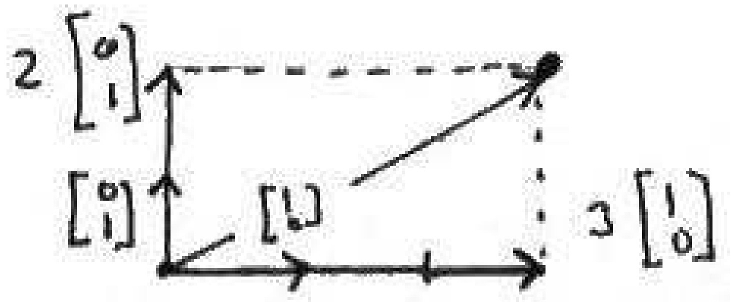
2.9

Basis, Coordinates, Dimension of Subspaces

Preliminary Ideas: The usual coordinate system for $\Re^2$



Is related to combination of vector?

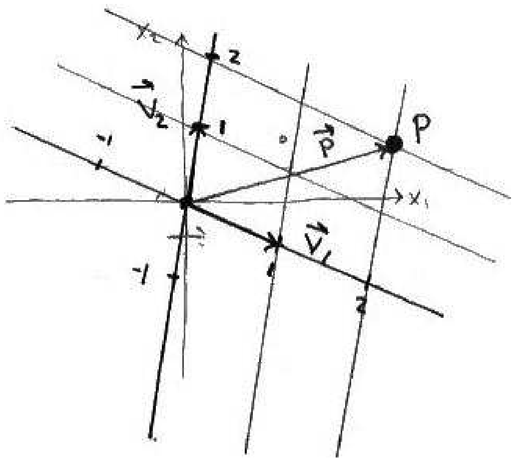


$$\begin{bmatrix}3\\2\end{bmatrix} = 3\begin{bmatrix}1\\0\end{bmatrix} + 2\begin{bmatrix}0\\1\end{bmatrix}$$

$\uparrow$ $\uparrow$

the weights (3,2) are coordinates

Another Coordinate System for $\Re^2$:



P has coordinates (2, 2)

$\vec{P} = 2\vec{v_1} + 2\vec{v_2}$

in terms of $\vec{v_1}, \vec{v_2}$

Definition A basis for a coordinate system on a subspace H of $\Re^n$ is a set of vectors

$$\{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_p\}$$

in H that are

(a) linearly independent, and

(b) span H.

Note Suppose $\{\vec{v}_1, \cdots, \vec{v}_p\}$ is a basis for H.

- Span $\{\vec{v}_1, \cdots, \vec{v}_p\} = H$ is important;

Every $\vec{u}$ in H is some combination of the basil vectors.

$\vec{u} = c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_p\vec{v}_p$

• Independence is important :

the combinations are unique.

$$\begin{array}{ccccccccc} & \vec{u} & = & c_1\vec{c}_1 & + & \cdots & + & c_p\vec{v}_p \\ - & \vec{u} & = & d_1\vec{v}_1 & + & \cdots & + & d_p\vec{v}_p \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & \vec{0} & = & (c_1 - d_1)\vec{v}_1 & + & \cdots & + & (c_p - d_p)\vec{v}_p \end{array}$$

Independent $\Rightarrow c_1 = d_1, \cdots, c_p = d_p$

• The weights $c_1, \cdots, c_p$ of

$$\vec{u} = c_1\vec{v}_1 + \cdots + c_p\vec{v}_p$$

are called the <u>coordinates</u> of $\vec{u}$

relative to the basis $\{\vec{v}_1, \cdots, \vec{v}_p\}$

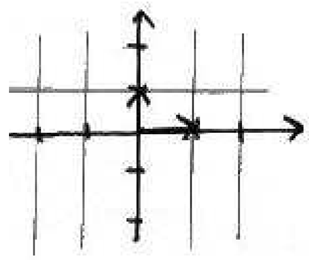
• If H has a basis with P vectors then all other bases of H have P vectors.

• The <u>dimension</u> of H is the number of vectors in any basis used to create a coordinate system for H.

$\underline{\text{EX}}$ (a) Bases for $\Re^2$

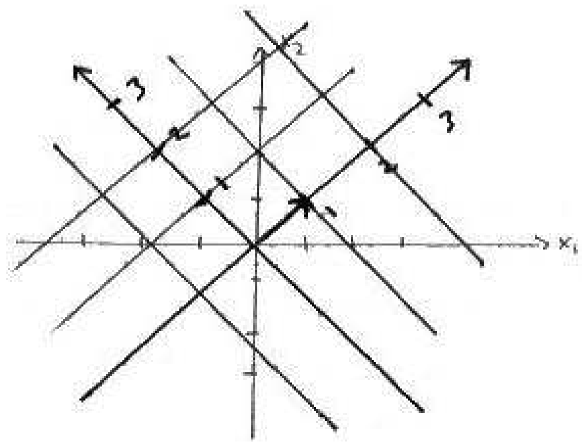
Two linearly independent vectors.

- $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ Standard basis for $\Re^2$



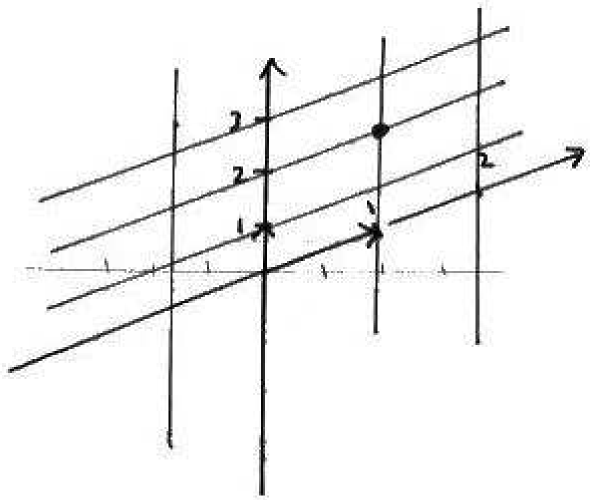
- $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$ Another basis

The coordinate system generated by this basis



• $\left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ basis

The coordinate system generated by this basis



(There are many other bases for $\Re^2$)