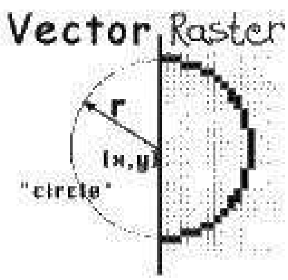
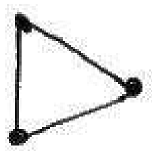
Vector Graphics	Raster or Bitmap Graphics
Draws the image through a sequence of commands and mathematics that draws lines, circles, and other basic shapes.	Treats an image as a grid of dots (pixels) in which each dot is given its own color.
PDF, PS, SVG	BMP, TIFF, GIF, JPEG
PowerPoint, Adobe Illustrator, Corel Draw	PhotoShop, PaintShop
Resolution independent: no loss of quality as image is resized.	Resolution dependent: loss of quality as image is enlarged.
Fonts, line drawings, charts and graphs	Scanners, digital cameras, photos



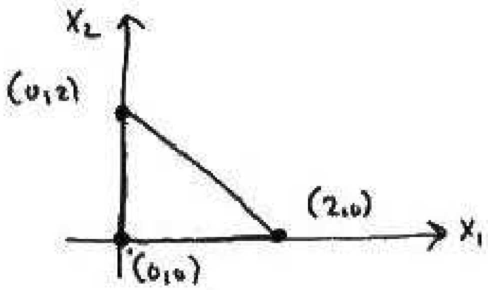
2.7 Computer Graphics

- An application of matrix multiplication $\underline{Vector \;\; \forall \;\; Graphics}$

Instructions on how to draw objects using line segments, circles, $\cdots$



A triangle can represented as a list of points (vertices) with the understanding that we draw straight line segments from one pt to another.



$$B = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

(coord of points ↓ ; close ↓)

To manipulate or transform the triangle, we can use matrix transformations:

$$A \quad - \quad 2 \times 2 \quad \text{(rotation, reflection, scaling, shear...)}$$

$$\begin{aligned} AB &= A \begin{bmatrix} \vec{p_1} & \vec{p_2} & \vec{p_3} & \vec{p_4} \end{bmatrix} \\ &= \begin{bmatrix} A\vec{p_1} & A\vec{p_2} & A\vec{p_3} & A\vec{p_4} \end{bmatrix} \\ & \quad \text{(A transforms each point)} \end{aligned}$$

<u>EX</u> (a) Scale the triangle by $\frac{1}{2}$

obj. $B = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$

Transform $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

↓

tran. $A = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$

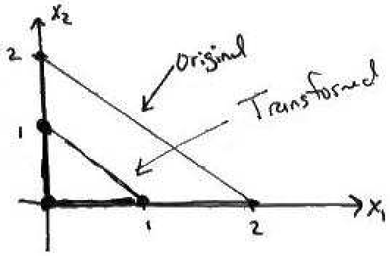
↑

Transform $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

tran.obj

$$\begin{aligned} AB &= \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{aligned}$$

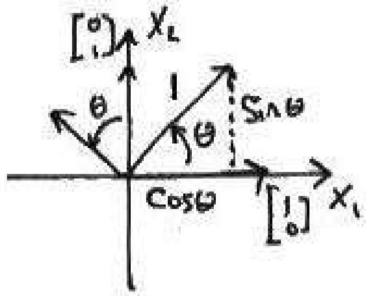
New obj.



(b) Rotate triangle by 60^o counter clockwise around the origin

obj. $B = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$

trans. $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad \theta = 60^o$



$$
\begin{aligned}
\underset{\text{trans.obj}}{AB} &= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 2\cos\theta & -2\sin\theta & 0 \\ 0 & 2\sin\theta & 2\cos\theta & 0 \end{bmatrix}
\end{aligned}
$$

$$
\cos 60^o = \tfrac{1}{2} \qquad \sin 60^o = \tfrac{\sqrt{3}}{2}
$$
$$
2\cos 60^o = 1 \qquad 2\sin 60^o = \sqrt{3}
$$

$$AB = \begin{bmatrix} 0 & 1 & -\sqrt{3} & 0 \\ 0 & \sqrt{3} & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -1.7 & 0 \\ 0 & 1.7 & 1 & 0 \end{bmatrix}$$

<u>Basic 2-D Graphics Transformations</u>

Rotation around the origin by θ

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Scalings

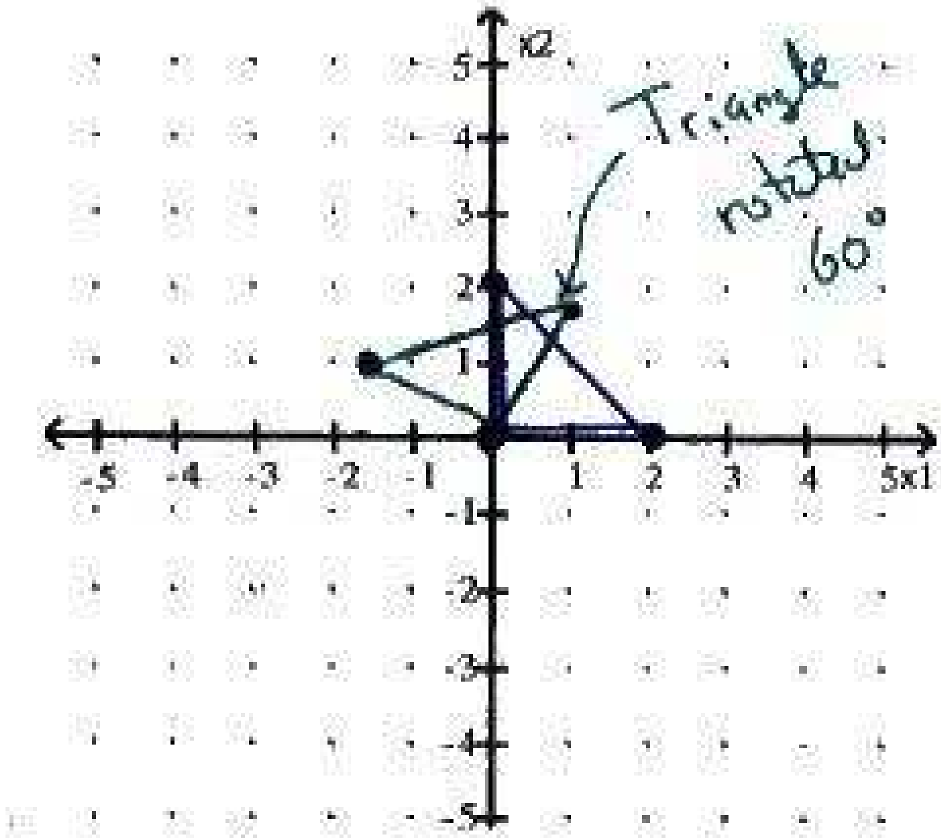
$$\begin{bmatrix} 1 & 0 \\ 0 & s \end{bmatrix} \qquad \begin{bmatrix} s & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}$$

Vertical scaling by factor s   Horizonal Scaling   scaling

Shear $\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$

Horizontal   vertical

Reflections


x2
5
4
3
2
1
-1
-2
-3
-4
-5
-5
-4
-3
-2
-1
1
2
3
4
5x1
Triangle rotated 60°

Composite Transformations

Combining 2 or more transformations

$a_1\vec{X}$	a_1 transforms	$\vec{X}$
$a_2(a_1\vec{X})$	a_2 transforms	$a_1\vec{X}$
$a_3(a_2a_1\vec{X})$	a_3 transforms	$a_2a_1\vec{X}$
$(a_3a_2a_1)\vec{X}$		

Matrices multiply to produce a single matrix A.

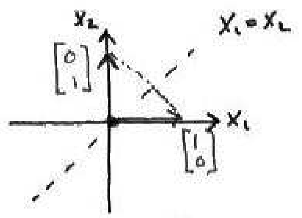
Ex. Reflect across $x_2 = x_1$ and then rotate counterclockwise by $\pi/4$ (45^o).

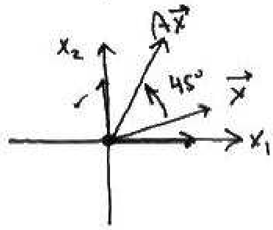
Method (1) Construct a matrix for this combined transformation by computing

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right), \quad T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right),$$

Method (2) Construct a matrix by matrix multiplication

$$\begin{aligned} a_1 &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ a_2 &= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \end{aligned}$$





Combine a_1, a_2 by matrix multiplication

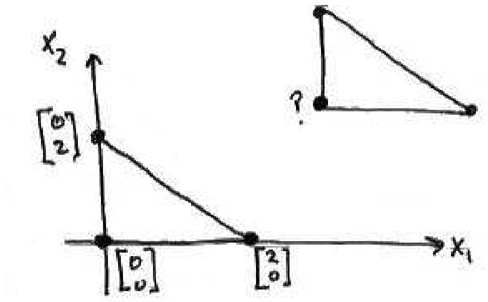
$a_1 a_2 \quad or \quad a_2 a_1$

$$\begin{array}{cc} a_2 & (a_1\vec{X}) \\ \nearrow & \nwarrow \\ 2^{nd} & 1^{st} \end{array}$$

$$a_2 a_1 = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/\sqrt{2} & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} -\sqrt{2}/2 & \sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

Translation + Homogeneous Coordinates



$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Mult. by 2×2 matrix will not translate a figure

Homogeneous coordinates - representing points in 2D as points in 3D.

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \longleftarrow \text{Add a row of 1}$$

Suppose we want to translate (x,y) h units right and k units up.

the translate

$$\begin{bmatrix} x+h \\ y+k \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

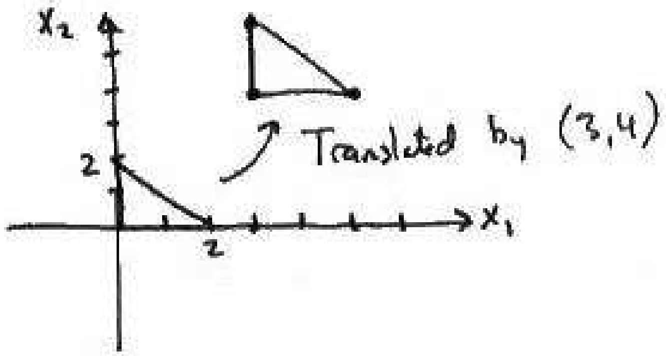
$\underline{\text{EX}}$ Triangle $B = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{matrix} \longleftarrow & x's \\ \longleftarrow & y's \\ \longleftarrow & 1's \end{matrix}$

Translate 3 right, 4 up Matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

Translate Triangle

$$AB = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

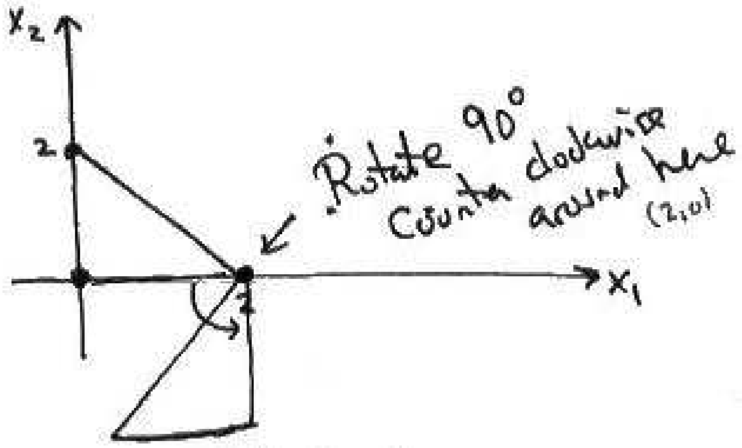
$$= \begin{bmatrix} 3 & 5 & 3 & 3 \\ 4 & 4 & 6 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{matrix} \longleftarrow & x's \\ \longleftarrow & y's \\ & \end{matrix}$$



Note: Any other 2×2 matrix transformation can be represented in homogeneous coordinates as a 3×3 matrix

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \longrightarrow \begin{bmatrix} \cos\theta & -\sin\theta & \vdots & 0 \\ \sin\theta & \cos\theta & \vdots & 0 \\ \cdots & \cdots & \vdots & \cdots \\ 0 & 0 & \vdots & 1 \end{bmatrix}$$

<u>EX</u>



(1)First translate the rotation pt(2,0)[the object] to (0,0)

(2)Then rotate 90^o CC around origin

(3)Finally, translate (0,0) back to (2,0)

(1) Translate 2 units left, 0 unit up

$$a_1 = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(2) Rotate 90^o CC around origin

$$a_2 = \begin{bmatrix} \cos 90^o & -\sin 90^o & 0 \\ \sin 90^o & \cos 90^o & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(3) Translate 2 units right, 0 units up

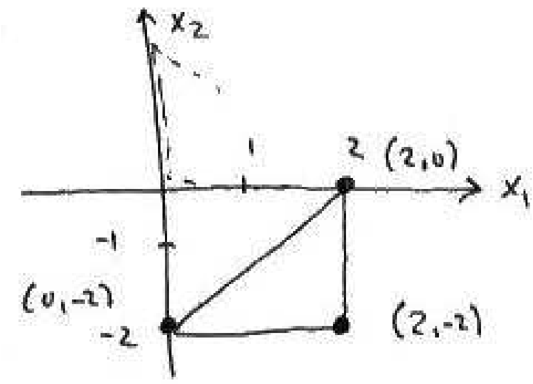
$$a_3 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then

$$\begin{aligned} A &= a_3 a_2 a_1 \\ &= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$
\begin{aligned}
A &= \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix} \\
B &= \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
AB &= \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 2 & 2 & 0 & 2 \\ -2 & 0 & -2 & -2 \\ 1 & 1 & 1 & 1 \end{bmatrix}
\end{aligned}
$$


x2
1
2 (2,0)
x1
-1
(0,-2)
-2
(2,-2)