2.3 <u>The Invertible Matrix Theorem</u>

[SEE section 2.3; make the connections from one part to the next.]

One particular connections: the following are either both true or both false:

(a) A is invertible

(b) The columns of A are independent

<u>EX(a)</u> $A = \begin{bmatrix} 1 & 4 & 0 \\ 2 & 5 & 0 \\ 3 & 6 & 0 \end{bmatrix}$

$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$ contains $\vec{0}$, so the columns of A are <u>dependent</u>

So A is <u>not</u> invertible.

(b) $B = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 5 & 4 \\ 3 & 6 & 6 \end{bmatrix}$

$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \right\}$ is dependent because the

3^{rd} vector is a multiple of the 1^{st} so B is <u>not</u> inventible.

(c) $C = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

C has 3 pivot columns and is a 3×3 matrix, so the columns will be <u>dependent</u>.

So C <u>is</u> invertible.