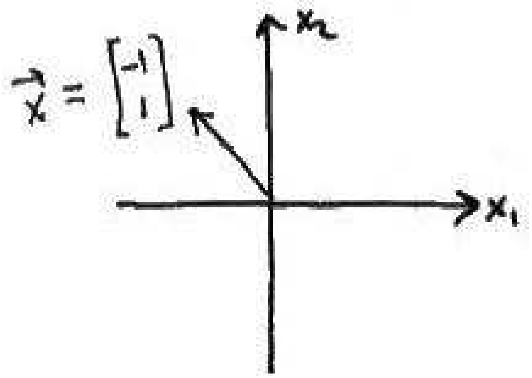
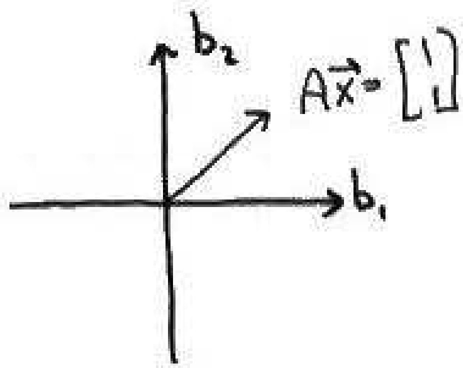
1.8 The Transformation Interpretation of $A\vec{X}$

<u>EX</u> $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\quad A \quad \vec{X} \quad = \quad \vec{b}$

The matrix A has "transformed" $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ *into* $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$





Terminology

1. **A transformation T from $\Re^n$ to $\Re^m$ is a rule that converts each $\vec{X}$ in $\Re^n$ into another vector in $\Re^m$** $\quad T : \Re^n \to \Re^m$

2. **$\Re^n$ is called the domain.**
   $\Re^m$ is called the codomain.

3. **$T(\vec{X})$ is called the image of $\vec{X}$.**
   If we find all images, the resolutions set in $\Re^m$ is called the range.

4. **A matrix transformation is**
   $T(\vec{X}) = A\vec{X}$
   for some matrix A.

EX

$$\begin{array}{ccc} T:\Re^2 & \rightarrow & \Re^2 \\ \uparrow & & \uparrow \\ Domain & & codomain \end{array} \quad by\ T(\vec{X}) = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \vec{X}$$

(a) Image of $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$$T\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 14 \end{bmatrix}$$

(b) Find all $\vec{X}$ in the domain whose image is $\vec{0}$.

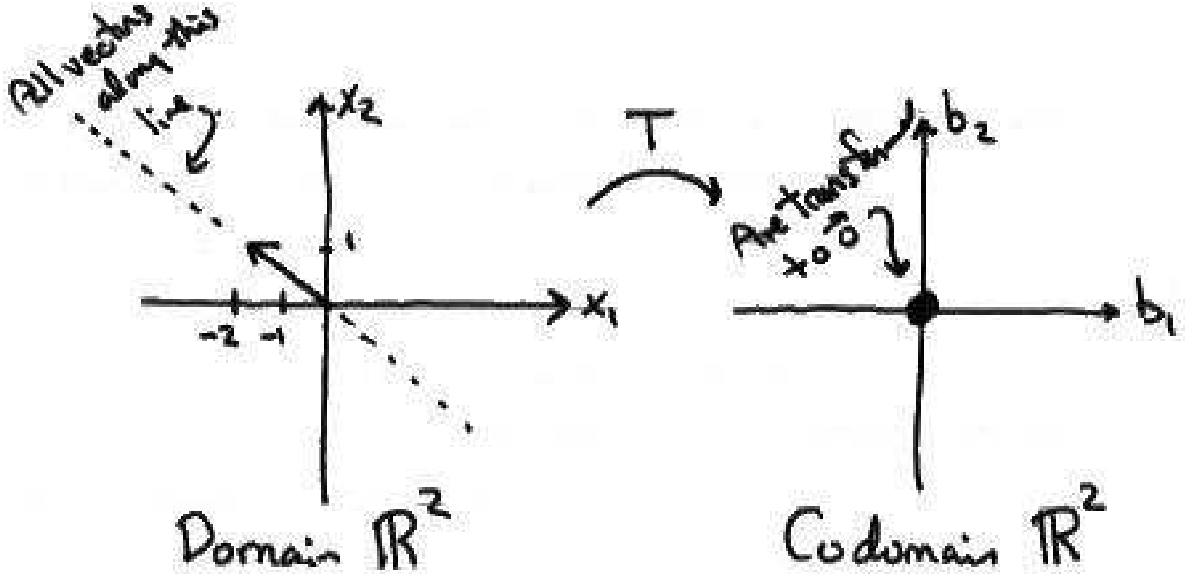
$$T(\vec{X}) = \vec{0}$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \end{bmatrix} \backsim \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \; -2R_1 + R_2$$

$$\begin{array}{cccc} X_1 & + & 2X_2 & = & 0 \\ & & X_2 & & free \end{array} \Rightarrow \begin{array}{ccc} X_1 & = & -2X_2 \\ X_2 & = & X_2 \end{array}$$

$$\vec{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -2X_2 \\ X_2 \end{bmatrix} = X_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$


All vectors along this line
x2
1
-2
-1
x1
Domain R^2
T
Are transformed to 0
b2
b1
Codomain R^2