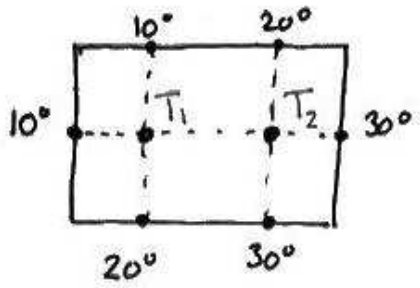
1.6 Applications

Steady-State Temperature of a plate(For Section 1.1):

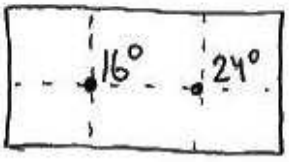


$$\begin{array}{rcl} T_1 = \frac{10+10+T_2+20}{4} & \Rightarrow & 4T_1 - T_2 = 40 \\ T_2 = \frac{T_1+20+30+30}{4} & \Rightarrow & -T_1 + 4T_2 = 80 \end{array}$$

$$\begin{array}{rcl} 4T_1 - T_2 &=& 40 \\ -T_1 + 4T_2 &=& 80 \end{array}$$

$$\left[\begin{array}{cc|c} 4 & -1 & 40 \\ -1 & 4 & 80 \end{array}\right] \backsim \begin{array}{c} rref \\ calc \end{array} \backsim \left[\begin{array}{ccc} 1 & 0 & 16 \\ 0 & 1 & 24 \end{array}\right]$$

$T_1 = 16 \quad T_2 = 24$



Balancing Chemical Equations

$$\underset{(Propane)}{C_3H_8} + \underset{(Oxygen)}{O_2} \rightarrow \underset{(CarbonDioxide)}{CO_2} + \underset{(Water)}{H_2O}$$

How many molecules of C_3H_8 and O_2 are needed to combine, and how many molecules of CO_2 and H_2O results?

$$(X_1)C_3H_8 + (X_2)O_2 \rightarrow (X_3)CO_2 + (X_4)H_2O$$

Carbon(C), Hydrogen(H), Oxygen(O)

Vector representation of a molecule

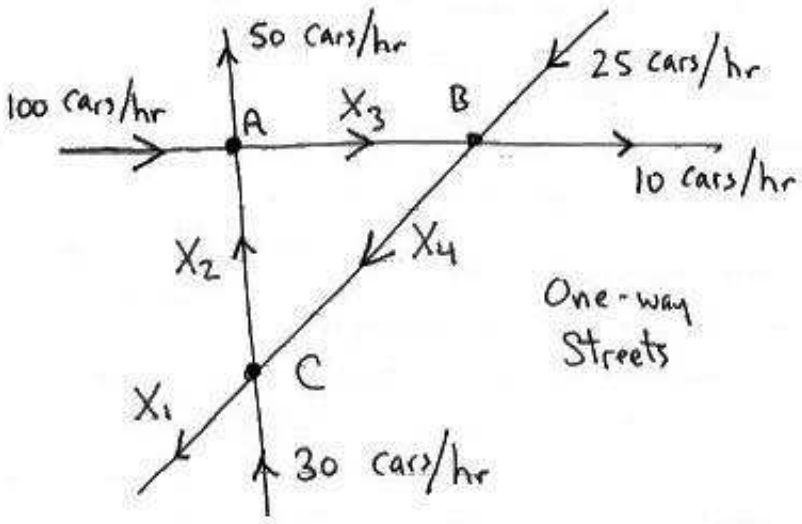
$$\begin{bmatrix} O \\ H \\ C \end{bmatrix} \; atoms$$

$$X_1 \begin{bmatrix} 0 \\ 8 \\ 3 \end{bmatrix} + X_2 \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = X_3 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + X_4 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$X_1 \begin{bmatrix} 0 \\ 8 \\ 3 \end{bmatrix} + X_2 \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} - X_3 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} - X_4 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

[Solve to find X_1, X_2, X_3, X_4,... See book]

Network Flow



Goal: find traffic flow rates along all streets.

$$\begin{array}{lllll} & & In & & out \\ Intersection & A & : & 100 + X_2 & = & 50 + X_3 \\ Intersection & C & : & 30 + X_4 & = & X1 + X_2 \\ Intersection & B & : & 25 + X_3 & = & 10 + X_4 \end{array}$$

$$\begin{array}{rcrcrcr} & X_2 & - & X_3 & & = & -50 \\ -X_1 & - & X_2 & & + & X_4 & = & -30 \\ & & & X_3 & - & X_4 & = & -15 \end{array}$$

$$\left[\begin{array}{ccccc|c} 0 & 1 & -1 & 0 & | & -50 \\ -1 & -1 & 0 & 1 & | & -30 \\ 0 & 0 & 1 & -1 & | & -15 \end{array}\right] \backsim \begin{array}{c} Calcu \\ Rref \end{array} \left[\begin{array}{ccccc} (1) & 0 & 0 & 0 & 95 \\ 0 & (1) & 0 & -1 & -65 \\ 0 & 0 & (1) & -1 & -15 \end{array}\right]$$

$$\begin{array}{rrrrrrr} X_1 & & & & = & 95 \\ & X_2 & & - & X_4 & = & -65 \\ & & X_3 & - & X_4 & = & -15 \\ & & & & X_4 & & free \end{array} \Rightarrow \begin{array}{rrrrr} X_1 & = & 95 & & \\ X_2 & = & -65 & + & X_4 \\ X_3 & = & -15 & + & X_4 \\ X_4 & & free & & \end{array}$$

$$\begin{array}{lllll} Need & X_4 \geqslant 0, & X_3 \geqslant 0, & X_2 \geqslant 0, \\ & & (X_4 \geqslant 15) & (X_4 \geqslant 65) \end{array}$$

So,

$$\begin{array}{rcrl} X_1 & = & 95 & \\ X_2 & = & -65 & + X_4 \\ X_3 & = & -15 & + X_4 \\ X_4 & \geqslant & 65 & \end{array}$$

describes all possible traffic flow patters .