1.5 Vector Form of Solution Sets

Ex: (a)
$$\begin{array}{rcrcl} X_1 & + & 2X_2 & = & 0 \\ 2X_1 & + & 4X_2 & = & 0 \end{array}$$

- **Right sides are all 0; homogeneous system**
- **Consistent ;** $(X_1, X_2) = (0, 0)$ **is a solution.**
  Are there nontrivial solutions?

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \end{bmatrix} \backsim \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad -2R_1 + R_2$$

$$\begin{array}{rcrcl} X_1 & + & 2X_2 & = & 0 \\ & & (0X_2 & = & 0) \quad X_2 \;\; free \end{array}$$

The solutions are
$X_1 = -2X_2,$
X_2 **free**

In vector form (parametric vector form)
$$\vec{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -2X_2 \\ X_2 \end{bmatrix} = X_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

(Note: Solution set=Span$\left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$**).**

(b) $$\begin{array}{rcrcl} X_1 & + & 2X_2 & = & 3 \\ 2X_1 & + & 4X_2 & = & 6 \end{array}$$

- **Right sides are all 0; nonhomogeneous system**
- **Consistent is harder to determine by inspection.**

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix} \backsim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \quad -2R_1 + R_2$$

The solutions are:

$$\begin{array}{rcrcc} X_1 & + & 2X_2 & = & 3 \\ & & X_2 & & free \end{array} \Rightarrow \begin{array}{l} X_1 = 3 - 2X_2 \\ X_2 \quad free \end{array}$$

In Parametric vector form.

$$\vec{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 3 - 2X_2 \\ X_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} + \begin{bmatrix} -2X_2 \\ X_2 \end{bmatrix}$$

$$\vec{X} = \underset{(1)}{\begin{bmatrix} 3 \\ 0 \end{bmatrix}} + X_2 \underset{(2)}{\begin{bmatrix} -2 \\ 1 \end{bmatrix}}$$

(1): A particular solution of the nonhom system

(2): The solution of the associated homogeneous system