Compressive Sensing for Noisy Video Reconstruction

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Abstract: In this paper, a compressing and reconstruction method for a noise video based on Compressed Sensing (CS) theory is proposed. At first, the CS theory is presented. Then the noise video is estimated from noisy measurement by solving the convex minimization problem. The video recovery algorithms based on gradient-based method is used to compressing and reconstructing the noise signal. And a compressive sensing algorithm with gradient-based method is proposed. At last, the performance of the proposed approach is shown and compared with some conventional algorithms. Our method can obtain best results in terms of peak signal noise ratio (PSNR) than those achieved by common methods with only a little runtime.

Index Terms—*Noisy Video, Compressed Sensing, signal recovery algorithms, Restricted Isometry Property, gradient-based*

1. INTRODUCTION

The well-known Nyquist/Shannon sampling theorem that the sampling rate must be at least twice the maximum frequency of the signal is a golden rule used in visual and audio electronics, medical imaging devices and so on. Compressed Sensing (CS) is a sampling paradigm that provides the signal compression at a rate significantly below the Nyquist/Shannon rate. Based on the CS theory, a sparse or compressible signal can be represented by the fewer number of bases than the one required by Nyquist/Shannon theorem, when it is mapped to the space with bases incoherent to the sparse data space [1, 2]. The major algorithmic challenge in compressive sampling is to approximate a signal given a vector of samples. The literature describes a huge number of approaches to solving this problem. Such as Orthogonal Matching Pursuit (OMP) [3], Greedy Basis Pursuit (GBP)[4], Iteratively Reweighted Least Square (IRLS)[5], CoSaMP[6] , Suspace Pursuit(SP)[7], and so on.

The contents of most references are about imagery and raw data compressing and reconstruction based on CS theory. CS has been successfully applied to MRI [8], with consistent benefits in a clinical setting [9]. In [10], an iterative image reconstruction method in X-ray CT is proposed based on compressive sensing (CS). Reference [11] proposes a new method of fast encoding for Synthetic Aperture Radar (SAR) raw data by using the CS theory to complete SAR raw data compressing and reconstruction. But, there are so many problems in compressive sampling to approximate a noise signal [12, 13]. And noise video compressive sampling is also a challenge

Compressive Sensing IV, edited by Fauzia Ahmad, Proc. of SPIE Vol. 9484
94840C · © 2015 SPIE · CCC code: 0277-786X/15/$18
doi: 10.1117/12.2180358

research problem. In this paper, a compressive sensing method for noisy video reconstruction based on gradient-based are studied.

This paper is organized as follows. In Section 2, the CS theory is presented. The processing of noisy video compressive sensing is introduced in Section 3. In Section 4, Noisy video reconstruction base on compressive sensing with gradient-based method is presented, and fast gradient-based compressed sensing reconstruction algorithm (FGB-CS)recovery algorithm are presented in detail. Experimental
results are obtained with the different methods in Section 5. Finally, the conclusions are summarized at the end of this paper.

2. Compressed Sensing theory

CS is based on the assumption of the sparse property of signal and incoherency between the bases of sparse domain and the bases of measurement vectors. CS has three major steps: the construction of k-sparse representation, the compression, and the reconstruction. The first step is the construction of k-sparse representation, where k is the number of the non-zero entries of sparse signal. Most natural signal can be made sparse by applying orthogonal transforms such as Wavelet Transform, Fast Fourier Transform, and Discrete Cosine Transform. This step is represented as [2].

$$s = \Psi^T x \tag{1}$$

where x is an N-dimensional non-sparse signal; s is a weighted N-dimensional vector (sparse signal with *k* nonzero elements), and Ψ is an *N* × *N* orthogonal basis matrix. The second step is compression. In this step, the random measurement matrix is applied to the sparse signal according to the following equation

$$y = \Phi s = \Phi \Psi^T x \tag{2}$$

where Φ is an *M* × *N* random measurement matrix (*M* < *N*).

Let *M* be the number of measurements (the row dimension of **y**) sufficient for high probability of successful reconstruction, and *M* is determined by

$$M \geq C\mu^2(\Phi, \Psi) k \log N \, . \tag{3}$$

For some positive constant *C*, $\mu(\Phi, \Psi)$ is the coherence between Φ and Ψ, and defined by

$$\mu(\Phi, \Psi) = \sqrt{N} \max_{i,j} |< \phi_i, \psi_j >| \cdot \tag{4}$$

If the elements in ϕ and ψ are correlated, the coherence is large. Otherwise, it is small. From linear algebra, it is known that $\mu(\Phi, \Psi) \in [1, \sqrt{N}]$. In the measurement process, the noise may occur. The noise is added into the compressed measurement vector as follows

$$y = \Phi s + noise \tag{5}$$

where *noise* is an *M*-dimensional noise vector.

As expected, signal ***x*** in (5) may be estimated from noisy measurement ***y*** by solving the convex minimization problem, called second-order cone program (SOCP), as follows.

$$\begin{aligned}&minimize \ \|x\|_1\\&subject\ to: \|\Phi\Psi^T x - y\|_2 \le \varepsilon\end{aligned} \tag{6}$$

where ε is a bound of the amount of noise in the data.

3 The processing of noisy video compressive sensing

The framework of our approach is shown in Figure 1. For a video, it mainly contains the following steps:

(i) Read each frame, make it sparse by orthogonal transformation, consider its compression and recovery as a convex optimization problem, and then apply for the gradient-based method. Here, we assume the problem to be convex with the Lipschitz gradient.

(ii) Aiming at improving the efficiency, we replace an iteration parameter by the Lipschitz constant and a fast compressive sensing algorithm with gradient-based method is proposed.

(iii) Reconstruct each frame based on compressive compressing.

The processing of noisy video compressive sensing approach is shown in Figure 1.

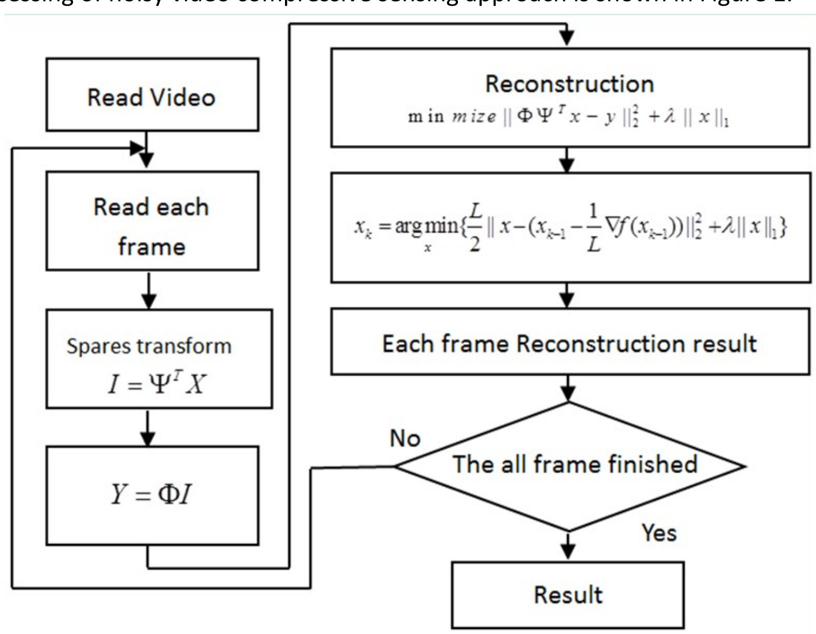


Fig 1 The processing of noisy video compressive sensing

4 Noisy video reconstruction compressive sensing

4.1 Convex optimization problems based on Gradient-based methods

At first, we consider the unconstrained minimization problem of a convex function $g(x)$

$$\min\{g(x): x \in R^n\}. \tag{7}$$

One of the simplest methods for solving eq.(8) is the gradient algorithm which generates a sequence x_k via

$$x_0 \in R^n, x_k = x_{k-1} - t_k \nabla g(x_{k-1}), \quad (8)$$

where $t_k > 0$ is a suitable step size. It is very well known that the gradient iteration Eq.8 can be viewed as a proximal regularization[13] of the linearized function g at x_{k-1}, and written equivalently as

$$x_k = \arg\min_x \{g(x_{k-1}) + < (x - x_{k-1}), \nabla g(x_{k-1}) > + \frac{1}{2t_k} \| x - x_{k-1} \|_2^2\} \cdot \quad (9)$$

Adopting this same basic gradient idea to the nonsmooth l_1 regularized problem

$$\min\{g(x) + \lambda \| x \|_1 : x \in R^n\} \quad (10)$$

leads to the iterative scheme [14]

$$x_k = \arg\min_x \{g(x_{k-1}) + < x - x_{k-1}, \nabla g(x_{k-1}) > + \frac{1}{2t_k} \| x - x_{k-1} \|_2^2 + \lambda \| x \|_1\}. \quad (11)$$

After ignoring constant terms, this can be rewritten as

$$x_k = \arg\min_x \{\frac{1}{2t_k} \| x - (x_{k-1} - t_k \nabla g(x_{k-1})) \|_2^2 + \lambda \| x \|_1\}. \quad (12)$$

4.2 Noise signal optimization with Lipschitz gradient

The noise is added into the compressed measurement vector as follows

$$y = \Phi s + noise \quad (13)$$

where $noise$ is an *M*-dimensional noise vector. We think about an objective function $F(x) = g(x) + n(x)$, which is a composite type convex function, the Lipschitz gradient is named[15]

$$\| \nabla g(x) - \nabla g(y) \|_2 \leq L(g) \| x - y \|_2 \ \textit{for every } x, y, \quad (14)$$

where $L(g)$ > 0 is a (Lipschitz) constant. where $\|.\|$ denotes the standard Euclidean norm and L(g) > 0 is the Lipschitz constant of ∇g. And we approximate the function $F(x)$ at point x_{k-1} by quadratic function

$$Q_L(x, x_{k-1}) = g(\mathrm{x}) + < x - x_{k-1}, \nabla g(x_{k-1}) > + \frac{L}{2} \| x - x_{k-1} \|_2^2 + n(x) \quad (15)$$

which admits a unique minimizer

$$PL(x_{k-1}) = \arg\min_{x} \{Q_L(x, x_{k-1}), x \in R^n\} \cdot \tag{16}$$

Simple algebra shows that (ignoring constant terms in x_{k-1})

$$PL(x_{k-1}) = \arg\min_{x} \{\frac{L}{2} \| x - (x_{k-1} - \frac{1}{L} \nabla g(x_{k-1})) \|_2^2 + n(x)\} \cdot \tag{17}$$

Clearly, the basic step in Eq.9 is replaced by

$$x_k = PL(x_{k-1}) \tag{18}$$

with *L* set to $1/t_k$. Note that as long as the constant *L* in Eq.14 is taken to be no less than Lipschitz Constant $L(g)$, it follows that [16]

$$g(x) + n(x) \le g(x_{k-1}) + < \nabla g(x_{k-1}), x - x_{k-1} > + \frac{L}{2} \| x - x_{k-1} \|_2^2 + n(x) . \tag{19}$$

During our work, we replaced $1/t_k$ by a constant L which will be related to the Lipschitz constant $L(g)$. We can find that the right-hand side of Eq.19 is precisely equal to $Q_L(x, y)$ in Eq.15. In other words, $Q_L(x, y)$ is an easier-to-deal-with convex upper bound of the objective function *F*(***x***) and by minimizing the upper bound, $Q_L(x, y)$ with x_k given by Eq.18 offers a tight upper bound of *F*(***x***),provided that *L* ≥ *L*(*f*) , and

$$F(x_k) - F(x) \le \frac{2L(f) \|x_2 - x\|_2^2}{(k+1)^2} . \tag{20}$$

So the convergence is $O(1/k^2)$.

4.3 Compressed sensing for noisy video reconstruction

The major algorithmic challenge in compressive sampling is to approximate a noise signal given a vector of samples. The literature describes few numbers of approaches to solving this problem. In our method, Eq.(6) is often found more natural to study the closely related problem

$$\min mize \| \Phi \Psi^T x - y \|_2^2 + \lambda \| x \|_1 . \tag{21}$$

We begin with considering the problem of Eq.21 assumed to be smooth and convex with smooth Lipschitz gradient. For any L > 0, compressed sensing of noisy signals formulated by Eq.21 becomes,

$$x_k = \arg\min_x \{\frac{L}{2} \| x - x_{k-1} \|^2 + \lambda \| x \|_1\} \quad (22)$$

where $x_k = PL(x_{k-1})$, so

$$x_k = \arg\min_x \{\frac{L}{2} \| x - (x_{k-1} - \frac{1}{L} \nabla f(x_{k-1})) \|_2^2 + \lambda \| x \|_1\} \quad (23)$$

or equivalently

$$x_k = \arg\min_x \{\frac{L}{2} \| x - d_k \|_2^2 + \lambda \| x \|_1\} \quad (24)$$

where $d_k = x_{k-1} - \frac{1}{L} \nabla f(x_{k-1})$. According to Eq.22, this d_k can be rewritten as

$$d_k = x_{k-1} - \frac{1}{L} (\Phi \Psi^T)^T (\Phi \Psi^T x_{k-1} - y). \quad (25)$$

Because both the 1-norm and square of the 2-norm are *separable*, i.e. each of them is mere the sum of *n nonnegative* terms and each of these terms involves only a *single* (scalar) variable, the iterate x_k in Eq.24 can be computed exactly by a straightforward shrinkage step (assuming d_k in Eq.24 has been calculated) as

$$x_k = \Gamma_{\lambda L}(d_k) \quad (26)$$

where Γ_α is an shrinkage operator which maps R^n to R^n with the i-th entry of the output vector given by

$$\Gamma_\alpha(d)|_i = (| d_i | - \alpha)_+ \operatorname{sgn}(d_i) \quad (27)$$

where $(u)_+ = \max(u, 0)$.

4.3 FGBCS algorithm

In our study, a gradient-based compressive sensing algorithm (FGBCS) for noisy video reconstruction method is proposed, in which FGBCS algorithm is used to noise signal recovery, the detail algorithm is shown as follows.

FGBCS(Φ, Ψ, s, λ, K**)**

Input:

(i) $L = L(g) - a\ Lipschitz\ cons\tan t\ of\ \nabla g(x)$ in Eq.14.;

(ii)a video and its length is P. Each frame is s. $\Psi \in R^{N \times N}$ is a signal sparse transform matrix,

(iii)a measurement matrix $\Phi \in R^{M \times N}$, $x = \Phi s_p$; the iteration counter K and noise parameter λ.

Procedure.

For frame=1 to P

Initialize, $y_1 = x_0 \in R^n, t_1 = 1 . s_p = \Psi s$;

If ($1 \le k \le K$) Compute

(i) $x_k = PL(y_k)$ by solving the problem in Eq.18.

(ii) $t_{k+1} = \dfrac{1+\sqrt{1+4t_k^2}}{2}$ $\quad y_{k+1} = x_k + (\dfrac{t_k - 1}{t_{k+1}})(x_k - x_{k-1})$

End

Output:

A sparse approximation x_k of the each frame s then reconstruction result signal $s^{'} = \Psi^T x_k$

End

5 Experiment Result

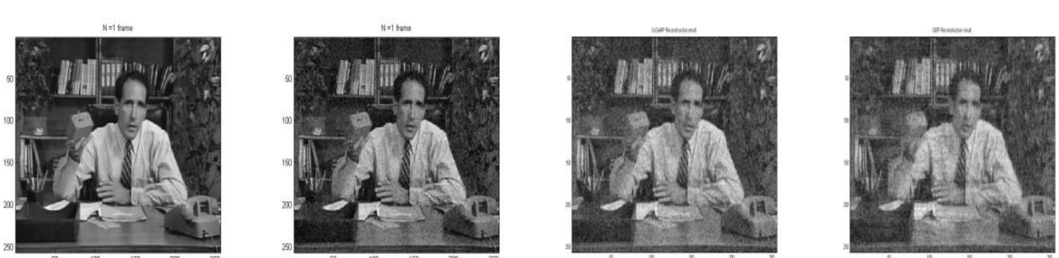
In order to evaluate the quality of the reconstructed results, the mean square error (MSE) and peak signal noise ratio (PSNR) can be utilized. They are defined as [17]

$$MSE = \frac{1}{M \times N} \sum_{i=1}^{M} \sum_{j=1}^{N} (\hat{f}(i,j) - f(i,j))^2 \qquad (28)$$

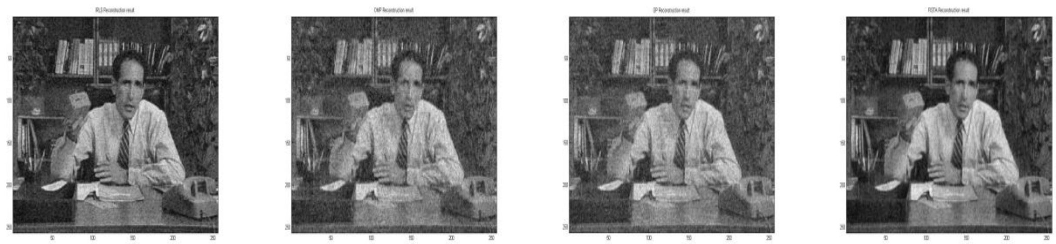
$$\mathrm{PSNR} = 10 \lg \left(\frac{255^2}{MSE} \right) \qquad (29)$$

Where *M* and *N* are the image dimensions, $\hat{f}$ is the denoised image, and f is the original noiseless image. During formula (28) ,255 means the pixel values is 0 to 255 in a optical gray image. Many researchers used PSNR to estimate the result in image processing, In our study, the PSNR is used to compare the experiment result.

The video gsalesmang15.avi(total 48 frames)[18] was used as a test data in our experiment. CoSaMP, GBP, IRLS and OMP are chosen as the comparison methods. The video was degraded by rand noise (σ=15). The 1th frame and its noisy image are shown as in figure 2(a) and figure 2(b). The reconstruction results based on different methods with the matrix's rows M=230 can be shown as figure 2(c-g). During FGbCS ,we set λ=20 and K=40, and the result can be shown as figure 2(h).



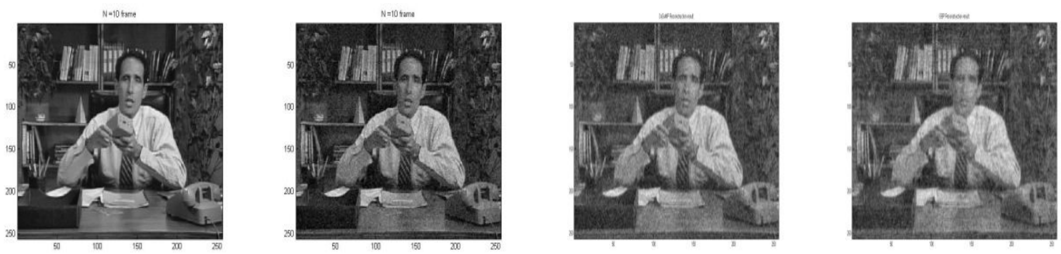
(a)frame=1 (b) Noise (c)CoSaMP(25.97) (d)GBP(25.34)



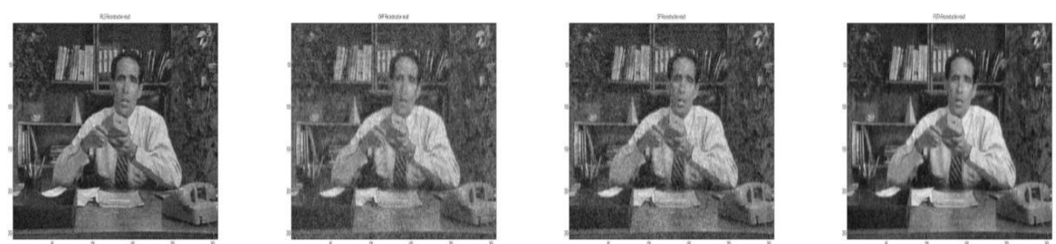
(e)IRLS(27.94) (f)OMP(24.83) (g)SP(25.72) (h) FGbCS(28.95)

Fig.2 Noisy video reconstructed results in terms of PSNR in frame 1

The 10th frame and its noisy image are shown as in figure 3(a) and figure 3(b). The reconstruction results based on different methods with the matrix's rows M=230 can be shown as figure 3(c-g). During FGbCS ,we set λ=20 and K=40, and the result can be shown as figure 3(h).



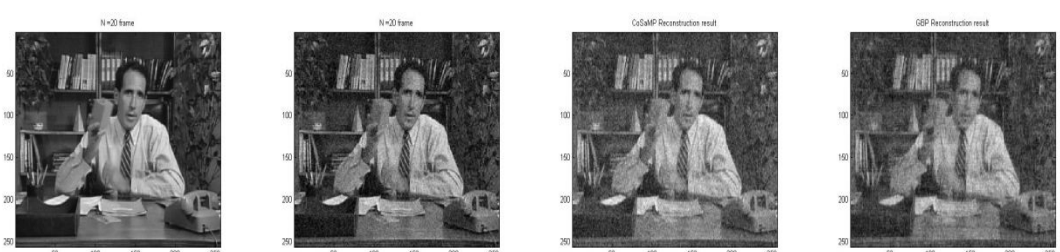
(a)frame =10 (b)Noise (c)CoSaMP(25.97) (d)GBP(25.28)



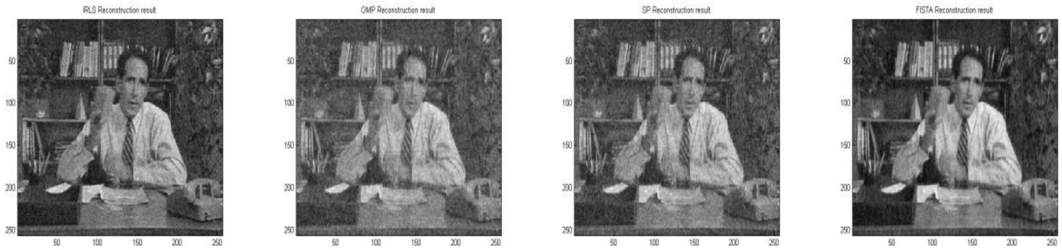
(e)IRLS(27.70) (f)OMP(24.70) (g)SP(25.64) (h)FGbCS(28.89)

Fig.3 Noisy video reconstructed results in terms of PSNR in frame 10

The 20th frame and its noisy image are shown as in figure 3(a) and figure 3(b). The reconstruction results based on different methods with the matrix's rows M=230 can be shown as figure 3(c-g). During FGbCS , we set λ=20 and K=40, and the result can be shown as figure 3(h).



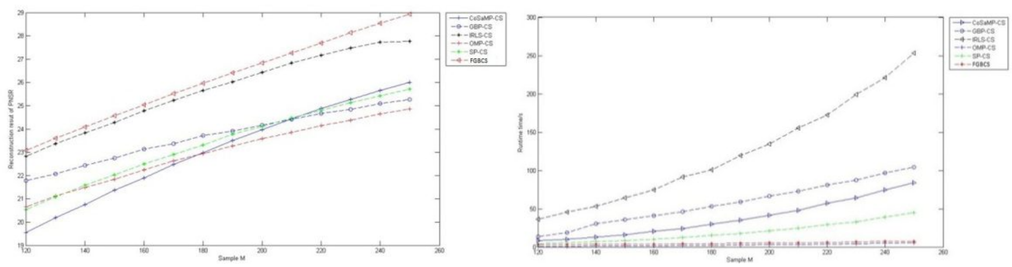
(a)frame =20 (b)Noise (c)CoSaMP(26.07) (d)GBP(25.28)



(e)IRLS(27.76) (f)OMP(24.99) (g)SP(25.71) (h)FGbCS(28.94)

Fig.4 Noisy video reconstructed results in terms of PSNR in frame 20

In order to compare their reconstruction performance in detail, the more experiments were also done. The reconstruction average time and PSNR with different rows of measurement matrix can be shown as figure 5(a,b).



(a) Average PSNR comparisons based on different measurement matrix rows

(b) Average runtime comparisons based on different measurement matrix rows

Fig.5 Quantization comparisons in noisy video reconstruction

We can see from figure 5 that,

(i) The noisy video reconstruction accuracy decrease with the increase of measurement matrix rows. And among of those methods, FGB algorithm can obtain best result than those other methods, and the second is IRLS algorithm.

(ii) The noisy video reconstruction accuracy decrease with the increase of measurement matrix rows. The method based on OMP and FGB algorithm can run the fastest than those other methods in noise signal reconstruction, and SP, CoSaMP, GBP and IRLS is in the second, third and fourth. Among of them, the runtimes decrease with the increase of measurement matrix rows. IRLS method can get good reconstruction result, but it spends the most time.

We can see from figure 5 that, the proposed method can obtain best reconstruction result comparing with OMP ,SP, CoSaMP, GBP and IRLS.

6 Conclusion

There are many ices in compressive sampling to approximate a noise signal given a vector of samples. And there many algorithms, such as OMP, SP, CoSaMP, GBP and IRLS are proposed to solve the problem. But those algorithms are not good for noisy video reconstruction. Among of them, IRLS achieves the good performance on low error, but it spends the most time. This paper presented here has focused on the formulation of the compressed sensing for noise signal reconstruction algorithm. The noisy and video is estimated as a convex minimization problem. And the gradient-based methods is used to solve the problem. In order to improve the run speed, the step size in gradient iteration is replaced by a constant 1/L which is related to the Lipschitz constant. We proposed a new compressive sensing for noisy video reconstruction methods base on gradient-based. The experiments have been shown that the proposed method can obtain best reconstruction result comparing with OMP ,SP, CoSaMP, GBP and IRLS.

References

[1] Donoho, D. L., "Compressed Sensing, " *IEEE Transactions on Information Theory*, Vol.52,no.4, pp.1289-1306, 2006.

[2] David L. Donoho,Michael Elad,Vladimir Temlyakov.," Stable recovery of sparse overcomplete representations in the presence of noise," *IEEE Trans. Inf. Theory.*, vol. 52, pp. 6-18, 2006.

[3]T. Tony Cai and Lie Wang,"Orthogonal Matching Pursuit for Sparse Signal Recovery With Noise," *IEEE TRANSACTIONS ON INFORMATION THEORY*, vol.57, no.7,pp.4680-4688, 2011.

[4]Huggins, Patrick S.; Zucker, Steven W."Greedy Basis Pursuit." *IEEE Transactions on Signal Processing,* vol. 55, issue 7, pp. 3760-3772.2007.

[5]Ingrid Daubechies,, Ronald DeVore, Massimo Fornasier,and C. Sinan G¨unt¨urk, "Iteratively Reweighted Least Squares Minimization for Sparse Recovery ".*Communications on Pure and Applied Mathematics,*Volume 63, Issue 1, pp.1-38, January 2010

[6]D. Needell J.A. Tropp,"CoSaMP: iterative signal recovery from incomplete and inaccurate samples," *Communications of the ACM*,Vol.53,no.12, pp.93-100,2010.

[7]Wei Dai and Olgica Milenkovic."Subspace Pursuit for Compressive Sensing Signal Reconstruction."*IEEE Transactions on Information Theory* , Volume:55 , Issue.5,2230-2249 ,2009.

[8]Lustig M, Donoho D, Pauly JM. "Sparse MRI: The application of compressed sensing for rapid MR imaging,"*Magn Reson Med*,vol.58,no.6,pp.1182-1195, 2007.

[9] Vasanawala SS, Alley MT, Hargreaves BA, Barth RA, Pauly JM, Lustig M. "Improved pediatric MR imaging with compressed sensing," *Radiology*,vol.256,no.2,pp.607-616, 2010.

[10] Jakob H. Jørgensen, Emil Y. Sidky, and Xiaochuan Pan,"Quantifying admissible undersampling for sparsity-exploiting iterative image reconstruction in X-ray CT, " *IEEE Transactions on Medical Imaging,* Vol.32,no.2 ,pp. 460-473,2012.

[11] Bhattacharya, S., T. Blumensath, B. Mulgrew, et al., "Fast encoding of synthetic aperture radar raw data using Compressed Sensing," *IEEE/SP 14th Workshop on Statistical Signal Processing*, Madison, WI, USA,26-29 Aug. 2007,pp.448-452.

[12]Jeevan K. Pant, Wu-Sheng Lu, and Andreas Antoniou,"A New Algorithm for Compressive Sensing Based on Total-Variation Norm." *2013 IEEE International Symposium on Circuits and Systems*, pp.1352-1355, 19-23 May,2013.

[13] D. L. Donoho, Tsaig, Y. , Drori, I. and Starck, J.-L. . "Sparse Solution of Underdetermined Systems of Linear Equations by Stagewise Orthogonal Matching Pursuit," *IEEE Transactions on Information Theory*, vol.58,no.2, pp.1094-1121, 2012.

[14] I. Daubechies, M. Defrise, and C. D. Mol, "An iterative thresholding algorithm for linear inverse problems with a sparsity constraint," Comm. Pure Appl. Math., vol. 57, pp. 1413-1457, 2004.

[15]Nesterov, Y. (1983). A method of solving a convex programming problem with convergence rate O (1/k2). In Soviet Mathematics Doklady (Vol. 27, No. 2, pp. 372-376).

[16] A. Beck and M. Teboulle, "A fast iterative shrinkage-thresholding algorithm for linear inverse problems," SIAM J. Imaging Sciences, vol. 2, no. 1, pp. 183-202, 2009.

[17] Huynh-Thu, Q.,Ghanbari, M.. "Scope of validity of PSNR in image/video quality assessment". Electronics Letters, vol 44 ,no.13.pp.800-801. 2008.

[18]http://157.193.140.25/~vzlokoli/PHD/Grey_scale/progressive/noisy/